Termination

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Partial vs Total Correctness

{P} S {Q}

{input provided} PROGRAM {output correct} → partial correctness

Total correctness: {P} S {⇓ Q}

Given precondition P, statement S terminates *and* satisfies the post-condition.

Safety vs Liveness

Safety property "this bad event cannot happen"

Liveness property "this good event eventually happens"

image: wikipedia

News

Zune chokes on leap-year bug

The bug disabled the players on Dec. 31, the last day of a leap year

By Robert McMillan

December 31, 2008 12:00 PM ET Add a comment

IDG News Service - Microsoft Corp.'s Zune 30GB music player just wasn't ready for a leap year.

That's what owners of the devices discovered Wednesday morning when they awoke to find their players frozen and unworkable.

The problem turned out to be "a bug in the internal clock driver related to the way the device handles a leap year," Microsoft Zune spokesman Matt Akers said in a posting to Zune forums Wednesday. The issue does not affect all Zune players, but all models of the Zune 30GB are potentially affected, he said.

Zune is Microsoft's alternative to Apple's popular iPod devices.

The bug disabled the players on Dec. 31, the last day of a leap year. Microsoft expects that the bug will resolve itself by Jan. 1, when the device's internal clock will reset itself.

"By [Thursday] you should allow the battery to fully run out of power before the unit can restart successfully, then simply ensure that your device is recharged, then turn it back on," Akers said. "If you're a Zune Pass subscriber, you may need to sync your device with your PC to refresh the rights to the subscription content you have downloaded to your device."

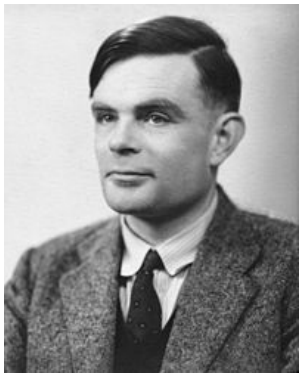
source: IDG

What happened?

```
year = ORIGINYEAR; /* = 1980 */

while (days > 365)
{
   if (IsLeapYear(year))
   {
      if (days > 366)
      {
         days -= 366;
         year += 1;
      }
   }
   else
   {
      days -= 365;
      year += 1;
   }
}
```

Proving termination



Turing Told us that it is Undecidable

But Turing also introduced the practical way of proving termination using a **ranking function** aka "progress measure"

Finally the checker has to verify that the process comes to an end. Here again he should be assisted by the programmer giving a further definite assertion to be verified. This may take the form of a quantity which is asserted to decrease continually and vanish when the machine stops. To the [illegible] it is natural to give an ordinal number. In this problem

Proving termination for a WHILE loop

```
while ( x ≤ y) {
   x := x+1
}
```

$\rho(x,y) = y - x$

A ranking function ρ is:

- A function of the state variables $\rho(\sigma)$
- Maps a state to a value in a set *W* (example: $\mathbb{N}$)
- Every iteration decreases the value $\rho(\sigma) > \rho(\sigma')$

  - Termination proof: every execution generates a decreasing sequence

Generalizations:

- Other sets *W* can be used as long as there are *no infinite decreasing chains* - *well-founded set*
- Other types of loops, *recursion*

Termination proofs in Dafny

```
method Mul(x: int, y: int) returns (r: int)
  requires 0 <= x && 0 <= y
  decreases x
{
  if x == 0 {
    r := 0;
  } else {
    if y > 0 {
      r := 0;
    //  r := Mul(x, y-1);
    }
    var m := Mul(x-1, y);
    r := m + x;
  }
}
```

Ranking functions in Dafny

```
function gcd(m:int, n: int) : int
// gcd of positive numbers
  requires n >= 1 && m >= 1
{
  if n == 1 || m == 1 then 1
 else if n == m then m
  else if m > n then
    gcd(m-n, n)
  else
    gcd(n-m, m)
}
```

Ranking functions in Dafny

```
method gcd(x: int, y: int) returns (g: int)
  requires 0 < x && 0 < y
{
  var n: int := x;
  var m: int := y;
  m := y;
  while m != n && m > 1 && n > 1
  {
    if m < n {
      n := n - m;
    } else {
      m := m - n;
    }
  }
  if m == 1 {
    g := m;
  } else {
    g := n;
  }
}
```

Termination proofs in Dafny

```
function Ackermann(m: int, n: int): int
{
  if m <= 0 then
    n + 1
  else if n <= 0 then
    Ackermann(m - 1, 1)
  else
    Ackermann(m - 1, Ackermann(m, n - 1))
}
```

Kinds of ranking function

Numeric: $\rho(\sigma)$ is a **number** and decreases at least by 1

Lexicographic: $\rho(\sigma)$ is a **tuple** and decreases lexicographically

Theorem: if W is a well-founded set than W^n (under lexicographic order) is well-founded

Automatic termination provers

- Well-studied subject
- Different methods, different types of programs, many tools

(at least two called *Terminator*)

Logic programs: since the 1980's. **Termilog** - Sagiv & Lindenstrauss, ~1990

Functional programs: the 1990's, in particular in connection with program transformation tools

Imperative programs: the 2000's

Java: **Julia**, COSTA

C : many, including Microsoft tools Terminator and T2 - used to verify device drivers

- Another class of tools - non-termination finders

Finding ranking functions automatically

Given: a loop *transition relation* (in a particular form)

Find: a ranking function (of a specific kind)

DECIDABLE cases:

- transition relation expressed by *linear constraints*
- variables *integer*, *rational* or *real*
- ranking functions are *linear* or *lexicographic-linear*

$$\rho(x_1,\ldots,x_4) = \langle x_1+x_2,\ x_3-2x_4\rangle$$

```
while (1 <= z) {
  x := x + 1;
  y := y - x;
  z := z - 1;
}
```

transition relation

$$z \geq 1 \wedge$$
$$x' = x+1 \wedge$$
$$y' = y - x - 1 \wedge$$
$$z' = z - 1$$

The template method

Given: the transition relation τ as a formula on $(\mathbf{x}, \mathbf{x}')$ $\mathbf{x} = \langle x_1, \ldots, x_n \rangle$

A template for a desired ranking function E.g.: $\rho(\mathbf{x}) = \mathbf{a} \cdot \mathbf{x} + a_0$

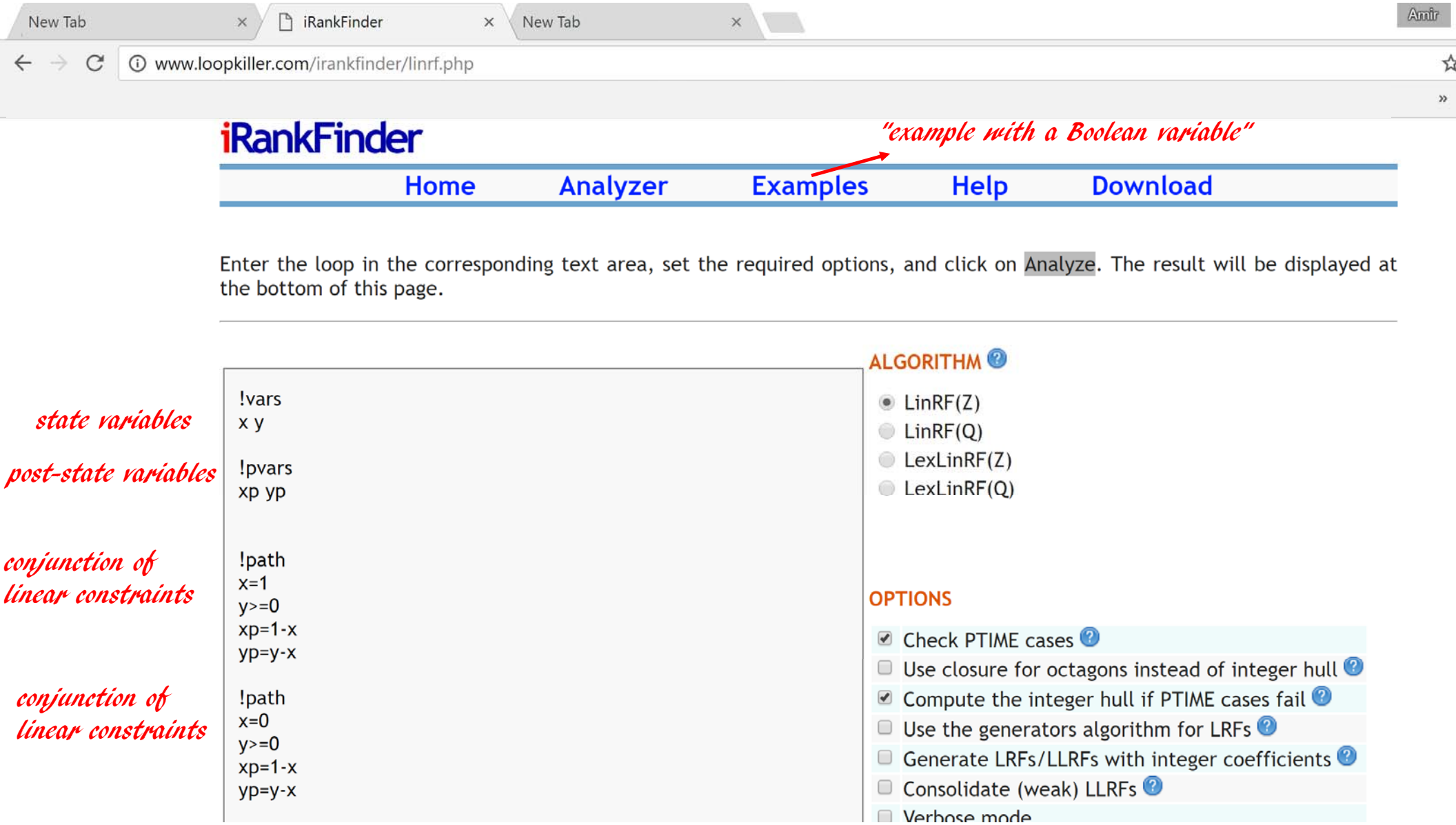
- the coefficients $\mathbf{a}$ are symbolic variables

Solve: $\forall(\mathbf{x}, \mathbf{x}') \quad \tau(\mathbf{x}, \mathbf{x}') \rightarrow \rho(\mathbf{x}) \geq 0$

$\wedge\ \tau(\mathbf{x}, \mathbf{x}') \rightarrow \rho(\mathbf{x}') \leq \rho(\mathbf{x}) - 1$

Algorithms & complexity (for linear constraints)

Function kind \ Domain	$\mathbb{Q}$ / $\mathbb{R}$	$\mathbb{Z}$
linear	Polynomial time Colón and Sipma 2001; Feautrier 1992; Mesnardand Serebrenik 2008; Podelski & Rybalchenko 2004; Sohn and Van Gelder 1991	coNP-complete Ben-Amram and Genaim 2013
lexicographic - linear	Polynomial time Alias et al., 2010; Ben-Amram and Genaim 2014	coNP-complete Ben-Amram and Genaim 2014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Enter the loop in the corresponding text area, set the required options, and click on Analyze. The result will be displayed at the bottom of this page.
state variables
post-state variables
conjunction of linear constraints
conjunction of linear constraints
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x y
!pvars
xp yp
!path
x=1
y>=0
xp=1-x
yp=y-x
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LinRF(Z)
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OPTIONS
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Use closure for octagons instead of integer hull
Compute the integer hull if PTIME cases fail
Use the generators algorithm for LRFs
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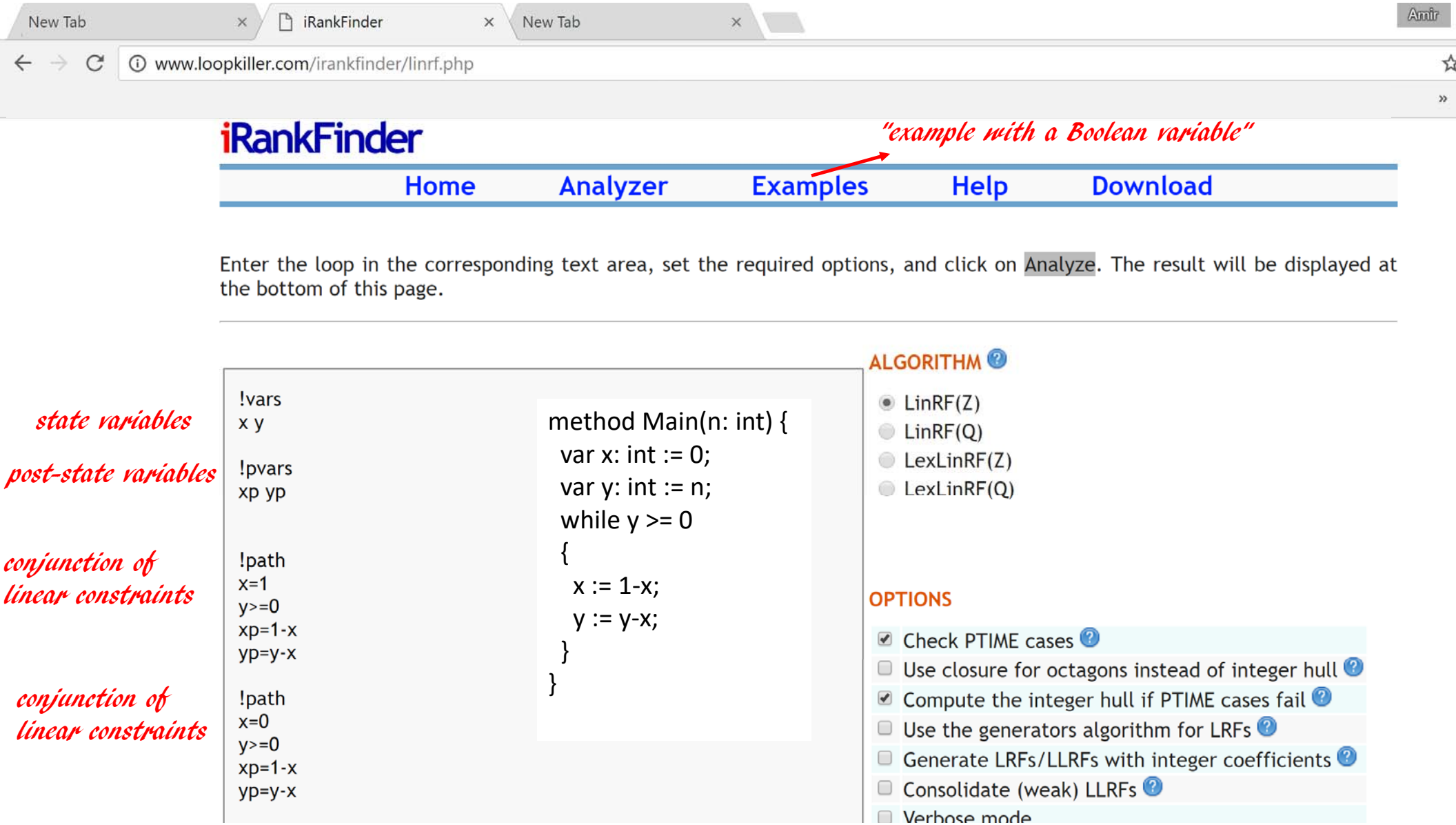
Verbose mode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y>=0
xp=1-x
yp=y-x
method Main(n: int) {
var x: int := 0;
var y: int := n;
while y >= 0
{
x := 1-x;
y := y-x;
}
}
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$\mathbb{Z}$ vs $\mathbb{Q}$

Treating integer variables as if they are rational (or real) is safe - but may be too safe

$$x_2 - x_1 \leq 0, \;\; x_1 + x_2 \geq 1$$

$$x_2' = x_2 - 2x_1 + 1, \quad x_1' = x_1$$

Non-terminating over $\mathbb{Q}$: (½, ½)

Terminating over $\mathbb{Z}$

$\mathbb{Z}$ vs $\mathbb{Q}$

$$x_2 - x_1 \leq 0, \;\; x_1 + x_2 \geq 1$$

$$x_2' = x_2 - 2x_1 + 1, \quad x_1' = x_1$$

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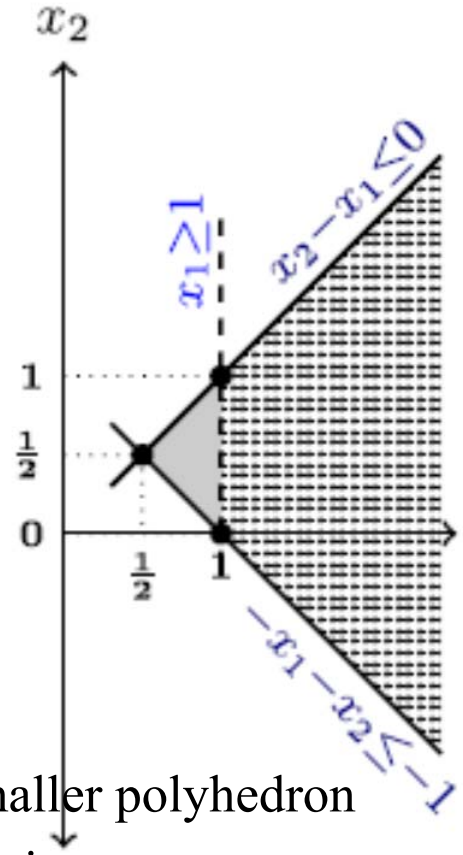

Geometrically: the set of states is a *polyhedron*

To avoid false alarms for integer variables, we have to compute a smaller polyhedron

The Integer Hull: smallest polyhedron that includes all the integer points

Termination & ranking functions

- Turing proposed ranking functions as a way to prove termination
- For certain kinds of loops (linear constraints) and certain kind of ranking functions (linear, lexicographic-linear) we have *complete solution*

- End-to-end tools that prove termination:

T2	https://www.microsoft.com/en-us/research/publication/t2-temporal-property-verification/
Julia	http://www.juliasoft.com/
AProVE	http://aprove.informatik.rwth-aachen.de/
COSTA	http://costa.ls.fi.upm.es/~costa/costa/costa.php

Obtaining the linear-constraint loop

- Symbolic evaluation / SSA form
- Non-numeric variables:
  - Ignore them if they do not affect the loop conditions
  - Abstract them if they are important

```
while (1 <= z) {
  x := x + 1;
  y := y – x;
  z := z – 1;
}
```

$z \geq 1 \wedge$
$x' = x+1 \wedge$
$y' = y - x - 1 \wedge$
$z' = z - 1$

```
function RecursiveSearch(a: array<int>, value: int) =

    if (a[1] == value) then 1
    else if (a.len < 2) then None
    else 1+RecursiveSearch( a[2:], value )
```

$a\_len \geq 1 \ \wedge \ a\_len' = a\_len - 1$

Obtaining the linear-constraint loop

- Symbolic evaluation / SSA form
- Non-numeric variables:
  - Ignore them if they do not affect the loop conditions
  - Abstract them if they are important
- Non-linear operations:
  - Use constraints (inequalities) to represent them

```
while (1 <= z) {
  x := x + 1;
  y := y – x;
  z := z – 1;
}
```

$$z \geq 1 \wedge$$
$$x' = x+1 \wedge$$
$$y' = y - x - 1 \wedge$$
$$z' = z - 1$$

```
function BinarySearch(low: int, hi: int) =
    mid := (low + hi) / 2
    …  BinarySearch( mid, hi )
    …
```

$$2*mid' \leq low+hi$$
$$\wedge \quad 2*mid' \geq low+hi-1$$

Project suggestions

Write a termination prover for a small language, using Z3

Extend a symbolic-checking tool (CBMC?) to check "decreases" assertions

Think about the converse problem: how to find a non-termination bug using tools like BMC and symbolic execution. Implement something.