Techniques for Improving Software Productivity
16/17 Fall Semester

Homework Assignment 4: Dafny
The code skeleton for the exercise can be found in: https://bitbucket.org/tausigplan/soft-prod16 under exercises/ex4/, and the code from the demos can be found there under demos/.

1. Consider the Dafny specification and code below, which is supposed to calculate the product of two numbers. Annotate the code so that it checks.

```dafny
method Product (m: nat, n: nat) returns (res:nat)
    ensures res == m * n;
{
    var m1: nat := m; res := 0;
    while (m1 != 0)
    {
        var n1: nat := n;
        while (n1 != 0)
        {
            res := res + 1;
            n1 := n1 - 1;
        }
        m1 := m1 - 1;
    }
}
```

2. Prove, using Dafny, that the following algorithm satisfies its postconditions.

```dafny
method Divide(x : nat, y : nat) returns (q : nat, r : nat)
    requires y > 0;
    ensures q * y + r == x && r >= 0 && r < y;
{
    q := 0;
    r := x;
    while (r >= y)
    {
        r := r - y;
        q := q + 1;
    }
}
3. Prove the correctness of the following program, which performs bitwise addition of two numbers \( a_0 \) and \( b_0 \). You have to supply the appropriate invariant and ranking function.

```plaintext
method Bitwise_add(a0 : int, b0 : int) returns (c : int)
  requires a0 >= 0 && b0 >= 0;
  ensures c == a0+b0;
{
  c := 0;
  var a, b, g, m := a0, b0, 1, 0;
  while (a > 0 || b > 0)
  {
    m := m + a % 2 + b % 2;
    a := a / 2;
    b := b / 2;
    c := c + g * (m % 2);
    g := 2 * g;
    m := m / 2;
  }
  c := c + g * m;
}
```

4. Prove the correctness of the following implementation of Linear Search.

```plaintext
method find(a : array<int>, key : int) returns (index : int)
  requires a != null;
  ensures 0 <= index <= a.Length;
  ensures index < a.Length ==> a[index] == key;
{
  index := 0;
  while (index < a.Length && a[index] != key)
  {
    index := index + 1;
  }
}
```
5. Below, we show the functional definition of the factorial function, and a loopy program that supposedly computes the factorial of a number as well. Write suitable annotations to show that the latter in fact computes the factorial function.

```plaintext
function Factorial(n: nat): nat
{
    if n == 0 then 1 else n * Factorial(n-1)
}

method AdditiveFactorial(n: nat) returns (u: nat)
ensures u == Factorial(n);
{
    u := 1;
    var r := 0;
    while (r < n)
    {
        var v := u;
        var s := 1;
        while (s <= r)
        {
            u := u + v;
            s := s + 1;
        }
        r := r + 1;
    }
}
```

6. Consider the following implementation of Binary Search, annotated with pre and post-conditions. As you will find by pasting this code into Dafny, there is something wrong with either the annotations or the code. Explain what the problem is.

```plaintext
method BinarySearch(a: array<int>, value: int) returns (index: int)
requires a != null && 0 <= a.Length;
requires forall j, k :: 0 <= j < k < a.Length ==> a[j] <= a[k];
ensures 0 <= index ==> index < a.Length && a[index] == value;
ensures index < 0 ==> forall k :: 0 <= k < a.Length ==> a[k] != value;
{
    var low, high := 0, a.Length;
    while (low < high)
    {Invariant 0 <= low <= high <= a.Length;
        invariant forall i :: 0 <= i < a.Length && !(low <= i < high) ==> a[i] != value;
        var mid := (low + high) / 2;
        if (a[mid] < value)
        {
            low := mid;
        }
        else if (value < a[mid])
        {
            high := mid - 1;
        }
        else
        {
            return mid;
        }
    }
    return -1;
}
```