Compilation

0368-3133 (Semester A, 2013/14)

Lecture 4: Syntax Analysis

(Top-Down Parsing)

Modern Compiler Design: Chapter 2.2

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Admin

- Next week: Trubowicz 101 (Law school)
- Mobiles ...

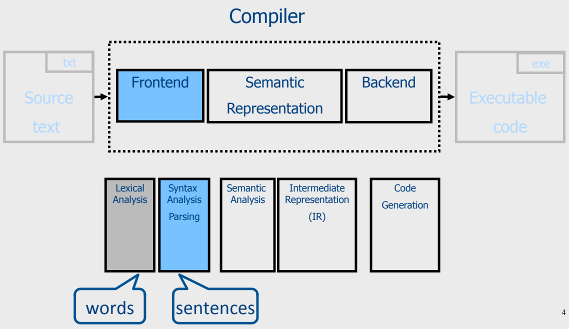
What is a Compiler?

"A compiler is a computer program that transforms source code written in a programming language (source language) into another language (target language).

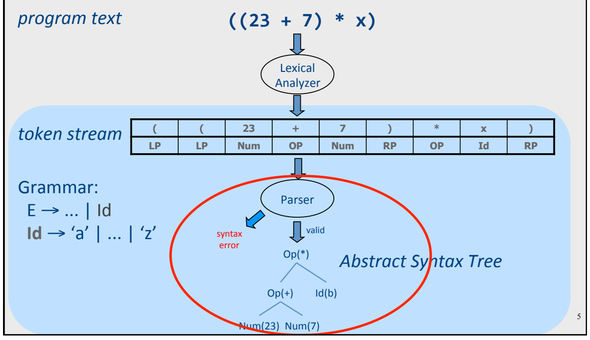
The most common reason for wanting to transform source code is to create an executable program."

--Wikipedia

Conceptual Structure of a Compiler



From scanning to parsing



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Context Free Grammars

G = (V,T,P,S)

- V – non terminals (syntactic variables)
- T – terminals (tokens)
- P – derivation rules
  - Each rule of the form V ⟶ (T ∪ V)*
- S – start symbol

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CFG terminology

S → S ; S
S → id := E
E → id
E → num
E → E + E

Symbols:
Terminals (tokens): **; := ( ) id num print**
Non-terminals: S E L

Start non-terminal: S
Convention: the non-terminal appearing in the first derivation rule

Grammar productions (rules)
N → μ

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CFG terminology

- Derivation - a sequence of replacements of non-terminals using the derivation rules
- Language - the set of strings of terminals derivable from the start symbol
- Sentential form - the result of a partial derivation in which there may be non-terminals

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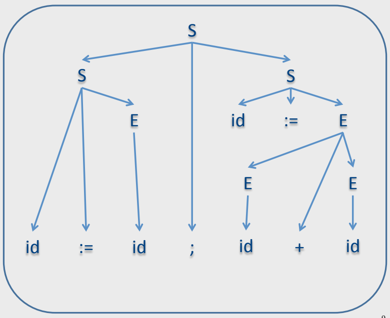
Parse Tree

```
S
S ; S
id := E; S
id := id; S
id := id;id := E
id := id; id := E + E
id := id;id := E + id
id := id;id := id + id
x:= z ; y := x  + z
```



Leftmost/rightmost Derivation

- Leftmost derivation
  - always expand leftmost non-terminal
- Rightmost derivation
  - Always expand rightmost non-terminal
- Allows us to describe derivation by listing the sequence of rules
  - always know what a rule is applied to

Leftmost Derivation

```
x := z;
y := x + z
```

```
S → S;S
S → id := E
E → id | E + E | E * E | ( E )
```

```
            S                    S → S;S
          S ; S                  S → id := E
  id := E ; S                    E → id
  id := id; S                    S → id := E
  id := id; id := E              E → E + E
  id := id; id := E + E          E → id
  id := id; id := id + E         E → id
  id := id; id := id + id
   x := z  ;  y  := x + z
```


Broad kinds of parsers

- Parsers for arbitrary grammars
  - Earley's method, CYK method
  - Usually, not used in practice (though might change)
- Top-Down parsers
  - Construct parse tree in a top-down matter
  - Find the leftmost derivation
- Linear Bottom-Up parsers
  - Construct parse tree in a bottom-up manner
  - Find the rightmost derivation in a reverse order

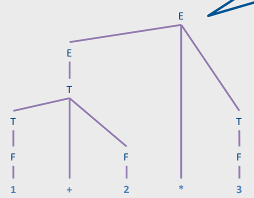
Intuition: Top-Down Parsing

- Begin with start symbol
- “Guess” the productions
- Check if parse tree yields user's program

Intuition: Top-Down parsing

Unambiguous grammar
E → E * T
E → T
T → T + F
T → F
F → id
F → num
F → (E)

Recall: Non standard precedence …



Intuition: Top-Down parsing

Unambiguous grammar
E → E * T
E → T
T → T + F
T → F
F → id
F → num
F → (E)

We need this rule to get the *

Intuition: Top-Down parsing

Unambiguous grammar
E → E * T
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F → id
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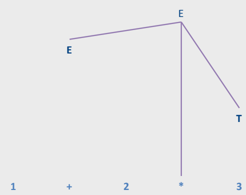

Intuition: Top-Down parsing

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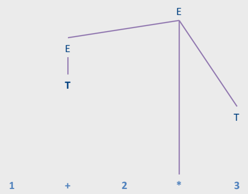

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Intuition: Top-Down parsing

Unambiguous grammar

E → E * T
E → T
T → T + F
T → F
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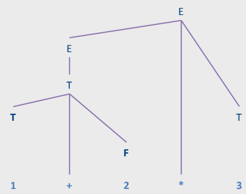

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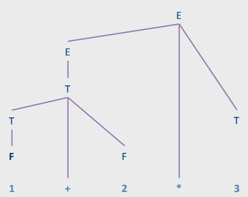
Intuition: Top-Down parsing

Unambiguous grammar

E → E * T
E → T
T → T + F
T → F
F → id
F → num
F → (E)



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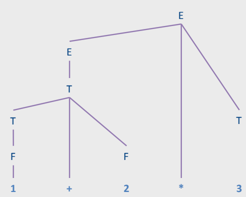
Intuition: Top-Down parsing

Unambiguous grammar

E → E * T
E → T
T → T + F
T → F
F → id
F → num
F → (E)



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Intuition: Top-Down parsing

Unambiguous grammar
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E → T
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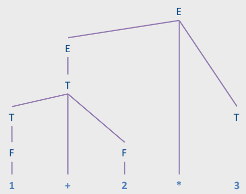

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Intuition: Top-Down parsing

Unambiguous grammar
E → E * T
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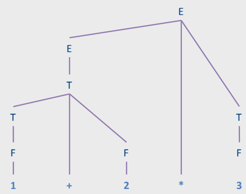
T → F
F → id
F → num
F → (E)



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Intuition: Top-Down parsing

Unambiguous grammar
E → E * T
E → T
T → T + F
T → F
F → id
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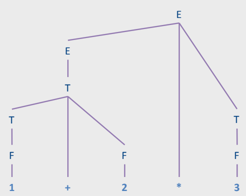

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Challenges in top-down parsing

- Top-down parsing begins with virtually no information
  - Begins with just the start symbol, which matches every program
- How can we know which productions to apply?

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Recursive Descent

- Blind exhaustive search
  - Goes over all possible production rules
  - Read & parse prefixes of input
  - Backtracks if guesses wrong
- Implementation
  - Uses (possibly recursive) functions for every production rule
  - Backtracks → "rewind" input

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Recursive descent

```
bool A(){ // A → A1 | … | An
  pos = recordCurrentPosition();

  for (i = 1; i ≤ n; i++){
    if (Ai())
      return true;
    rewindCurrent(pos);
  }
  return false;
}

bool Ai(){  // Ai = X1X2…Xk
  for (j=1; j ≤ k; j++)
    if (Xj is a terminal)
      if (Xj == current) match(current);
      else return false;
    else if (! Xj()) return false;
  return true;
}
```

current ↓

token stream

(	(	23	+	7	)	*	x	)
LP	LP	Num	OP	Num	RP	OP	Id	RP

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Example

- Grammar
  - E → T | T + E
  - T → int | int * T | (E)
- Input: (5)
- Token stream: LP int RP

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Problem: Left Recursion

E → E - term | term

```
int E() {
  return E() && match(token('-')) && term();
}
```

- What happens with this procedure?
- Recursive descent parsers cannot handle left-recursive grammars

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Left recursion removal

p. 130

N → Nα | β ⇒ N → βN′, N′ → αN′ | ε

G_1 G_2

Can be done algorithmically.
Problem: grammar becomes mangled beyond recognition

- $L(G_1)$ = β, βα, βαα, βααα, …
- $L(G_2)$ = same

- For our 3rd example:

E → E - term | term ⇒ E → term TE | term, TE → - term TE | ε

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Challenges in top-down parsing

- Top-down parsing begins with virtually no information
  - Begins with just the start symbol, which matches every program
- How can we know which productions to apply?
- Wanted: Top-Down parsing without backtracking

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Predictive parsing

- Given a grammar G and a word w derive w using G
  - Apply production to leftmost nonterminal
  - Pick production rule based on next input token
- General grammar
  - More than one option for choosing the next production based on a token
- Restricted grammars (LL)
  - Know exactly which single rule to apply based on
    - Non terminal
    - Next (k) tokens (lookahead)

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Boolean expressions example

E → LIT | (E OP E) | not E
LIT → true | false
OP → and | or | xor

not (not true or false)

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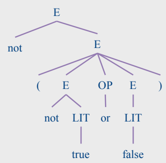
Boolean expressions example

E → LIT | (E OP E) | **not** E
LIT → **true** | **false**
OP → **and** | **or** | **xor**

not (not true or false)

production to apply known from next token

E =>
not E =>
not (E OP E) =>
not (not E OP E) =>
not (not LIT OP E) =>
not (not true OP E) =>
not (not true or E) =>
not (not true or LIT) =>
not (not true or false)



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Problem: productions with common prefix

term → ID | ID [expr]

- Cannot tell which rule to use based on lookahead (ID)

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Solution: left factoring

- Rewrite the grammar to be in LL(1)

term → ID | ID [expr]

⬇

term → ID after_ID
After_ID → [expr] | ε

Intuition: just like factoring x*y + x*z into x*(y+z)

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Left factoring – another example

S → if E then S else S
  | if E then S
  | T

⬇

S → if E then S S'
  | T
S' → else S | ε

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LL(k) Parsers

- Predictive parser
  - Can be generated automatically
  - Does not use recursion
  - Efficient
- In contrast, recursive descent
  - Manual construction
  - Recursive
  - Expensive

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LL(k) parsing via pushdown automata and prediction table

- Pushdown automaton uses
  - Prediction stack
  - Input token stream
  - Transition table
    - nonterminals x tokens -> production alternative
    - Entry indexed by nonterminal N and token t contains the alternative of N that must be predicated when current input starts with t

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Example transition table

(1) E → LIT
(2) E → (E OP E)
(3) E → not E
(4) LIT → true
(5) LIT → false
(6) OP → and
(7) OP → or
(8) OP → xor

Which rule should be used

Input tokens

Nonterminals

	(	)	not	true	false	and	or	xor	$
E	2		3	1	1				
LIT				4	5				
OP						6	7	8	

Table

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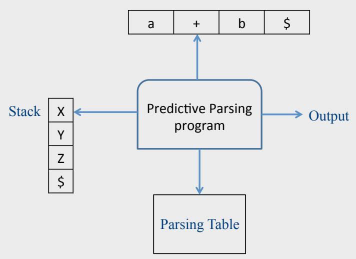
Non-Recursive Predictive Parser



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LL(k) parsing via pushdown automata and prediction table

- Two possible moves
  - Prediction
    - When top of stack is nonterminal N, pop N, lookup table[N,t]. If table[N,t] is not empty, push table[N,t] on prediction stack, otherwise – syntax error
  - Match
    - When top of prediction stack is a terminal T, must be equal to next input token t. If (t == T), pop T and consume t. If (t ≠ T) syntax error
- Parsing terminates when prediction stack is empty
  - If input is empty at that point, success. Otherwise, syntax error

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Running parser example

aacbb$

A → aAb | c

Input suffix	Stack content	Move
aacbb$	A$	predict(A,a) = A → aAb
aacbb$	aAb$	match(a,a)
acbb$	Ab$	predict(A,a) = A → aAb
acbb$	aAbb$	match(a,a)
cbb$	Abb$	predict(A,c) = A → c
cbb$	cbb$	match(c,c)
bb$	bb$	match(b,b)
b$	b$	match(b,b)
$	$	match($,$) – success

	a	b	c
A	A → aAb		A → c

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FIRST sets

- FIRST(α) = { t | α →* t β} ∪ {X →* ℇ }
  - FIRST(α) = all terminals that α can appear as first in some derivation for α
    - + ℇ if can be derived from X
- Example:
  - FIRST(LIT) = { true, false }
  - FIRST((E OP E)) = { ‘(‘ }
  - FIRST(not E) = { not }

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Computing FIRST sets

- FIRST (t) = { t } // “t” non terminal
- ℇ ∈ FIRST(X) if
  - X → ℇ or
  - X → A_1 .. A_k and ℇ ∈ FIRST(A_i) i=1…k
- FIRST(α) ⊆ FIRST(X) if
  - X → A_1 .. A_k α and ℇ ∈ FIRST(A_i) i=1…k

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FIRST sets computation example

STMT → if EXPR then STMT
| while EXPR do STMT
| EXPR ;
EXPR → TERM
| zero? TERM
| not EXPR
| ++ id
| -- id
TERM → id
| constant

STMT	EXPR	TERM

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1. Initialization

STMT → if EXPR then STMT
| while EXPR do STMT
| EXPR ;
EXPR → TERM
| zero? TERM
| not EXPR
| ++ id
| -- id
TERM → id
| constant

STMT	EXPR	TERM
if while	zero? Not ++ --	id constant

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2. Iterate 1

STMT → if EXPR then STMT
| while EXPR do STMT
| EXPR ;
EXPR → TERM
| zero? TERM
| not EXPR
| ++ id
| -- id
TERM → id
| constant

STMT	EXPR	TERM
if while	zero? Not ++ --	id constant
zero? Not ++ --		

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2. Iterate 2

STMT → if EXPR then STMT
| while EXPR do STMT
| EXPR ;
EXPR → TERM -> id
| zero? TERM
| not EXPR
| ++ id
| -- id
TERM → id
| constant

STMT	EXPR	TERM
if while	zero? Not ++ --	id constant
zero? Not ++ --	id constant	

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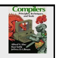
2. Iterate 3 – fixed-point

STMT → if EXPR then STMT
| while EXPR do STMT
| EXPR ;
EXPR → TERM
| zero? TERM
| not EXPR
| ++ id
| -- id
TERM → id
| constant

STMT	EXPR	TERM
if while	zero? Not ++ --	id constant
zero? Not ++ --	id constant	
id constant		



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FOLLOW sets

- FOLLOW(X) = { t | S→* α X t β}
  - FOLLOW(X) = set of tokens that can immediately follow X in some sentential form
  - NOT related to what can be derived from X
- Intuition: X → A B
  - FIRST(B) ⊆ FOLLOW(A)
  - FOLLOW(B) ⊆ FOLLOW(X)
  - If B→*ε then FOLLOW(A) ⊆ FOLLOW(X)

FOLLOW sets: Constraints

- $ ∈ FOLLOW(S)
- FIRST(β) – {ε} ⊆ FOLLOW(X)
  - For each A → α X β
- FOLLOW(A) ⊆ FOLLOW(X)
  - For each A → α X β and ε ∈ FIRST(β)

Example: FOLLOW sets

- E→ TX X→+ E | ε
- T→ (E) | int Y Y → * T | ε

Terminal	+	(	*	)	int
FOLLOW	int, (	int, (	int, (	_,), $	*,), +, $

Non. Term.	E	T	X	Y
FOLLOW	), $	+,), $	$,)	_,), $

Prediction Table

- A → α
- T[A,t] = α if t ∈ FIRST(α)
- T[A,t] = α if ε ∈ FIRST(α) and t ∈ FOLLOW(A)
  - t can also be $
- T is not well defined → the grammar is not LL(1)

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Problem: Non LL Grammars

```
S → A a b
A → a | ε
```

```
bool S() {
  return A() && match(token('a')) && match(token('b'));
}

bool A() {
  return match(token('a')) || true;
}
```

- What happens for input "ab"?
- What happens if you flip order of alternatives and try "aab"?

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Problem: Non LL Grammars

```
S → A a b
A → a | ε
```

- FIRST(S) = { a } FOLLOW(S) = { $ }
- FIRST(A) = { a ε } FOLLOW(A) = { a }
- FIRST/FOLLOW conflict

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Solution: substitution

```
S → A a b
A → a | ε
```

⇩ Substitute A in S

```
S → a a b | a b
```

⇩ Left factoring

```
S → a after_A
after_A → a b | b
```

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LL(k) grammars

- A grammar is LL(k) if it can be derived via:
  - Top-down derivation
  - Scanning the input from left to right (L)
  - Producing the leftmost derivation (L)
  - With lookahead of k tokens (k)
  - T is well defined
- A language is said to be LL(k) when it has an LL(k) grammar

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LL(k) grammars

- A grammar is not LL(k) if it is
  - Ambiguous
  - Left recursive
  - Not left factored
  - …

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Earley Parsing

- Invented by Earley [PhD. 1968]
- Handles arbitrary CFG
- Can handle ambiguous grammars
- Complexity $O(N^3)$ when N = |input|
- Uses dynamic programming
  - Compactly encodes ambiguity

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Dynamic programming

- Break a problem P into subproblems $P_1 \ldots P_k$
  - Solve P by combining solutions for $P_1 \ldots P_k$
  - Memo-ize (store) solutions to subproblems instead of re-computation
- Bellman-Ford shortest path algorithm
  - Sol(x,y,i) = minimum of
    - Sol(x,y,i-1)
    - Sol(t,y,i-1) + weight(x,t) for edges (x,t)

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Earley Parsing

- Dynamic programming implementation of a recursive descent parser
  - S[N+1] Sequence of sets of “Earley states”
    - N = |INPUT|
    - Earley state (item) s is a sentential form + aux info
  - S[i] All parse tree that can be produced (by a RDP) after reading the first i tokens
    - S[i+1] built using S[0] … S[i]

Earley States

- s = < constituent, back >
  - constituent (dotted rule) for A→αβ
    - A→•αβ predicated constituents
    - A→α•β in-progress constituents
    - A→αβ• completed constituents
  - back previous Early state in derivation

Earley Parser

```
Input = x[1…N]
S[0] = <E'→ •E, 0>; S[1] = … S[N] = {}
for  i = 0 … N do
   until S[i] does not change do
      foreach s ∈ S[i]
         if  s = <A→…•a…, b>  and  a=x[i+1] then
            S[i+1] =  S[i+1] ∪ {<A→…a•…, b> }   // scan
         if  s = <A→ … •X …, b> and X→α then
            S[i] = S[i] ∪ {<X→•α, i > } // predict
         if  s = < A→ … • , b>  and  <X→…•A…,  k>   ∈ S[b] then
            S[i] = S[i] ∪{<X→…A•…,  k> } // complete
```


Example

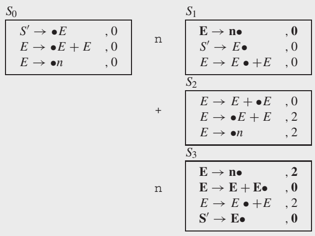

FIGURE 1. Earley sets for the grammar $E \rightarrow E + E \mid n$ and the input n + n. Items in bold are ones which correspond to the input’s derivation.