Compilation
0368-3133 (Semester A, 2013/14)

Lecture 5: Syntax Analysis

(Bottom-Up Parsing)

Modern Compiler Design: Chapter 2.2

Slides credit: Roman Manevich, Mooly Sagiv, Jeff Ullman, Eran Yahav

Admin

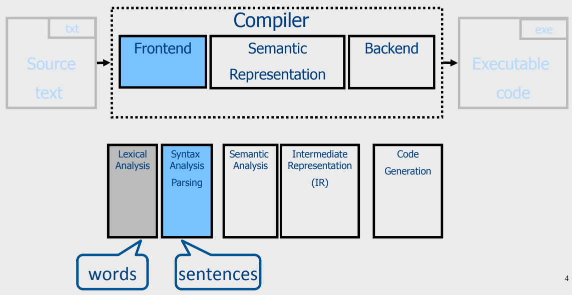
- Next weeks: Trubowicz 101 (Law school)
- Mobiles ...

What is a Compiler?

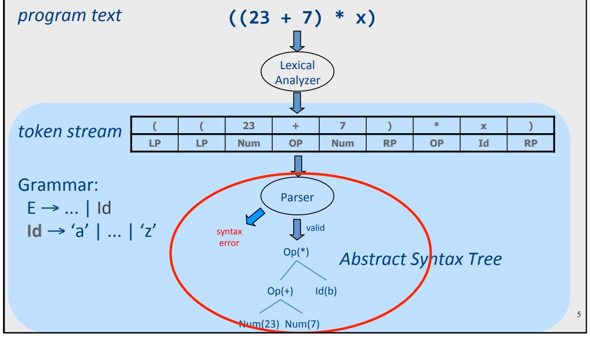
- “A compiler is a computer program that transforms source code written in a programming language (source language) into another language (target language).
- The most common reason for wanting to transform source code is to create an executable program.”

--Wikipedia

Conceptual Structure of a Compiler



From scanning to parsing



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Context Free Grammars

G = (V,T,P,S)

- V – non terminals (syntactic variables)
- T – terminals (tokens)
- P – derivation rules
  - Each rule of the form V → (T ∪ V)*
- S – start symbol

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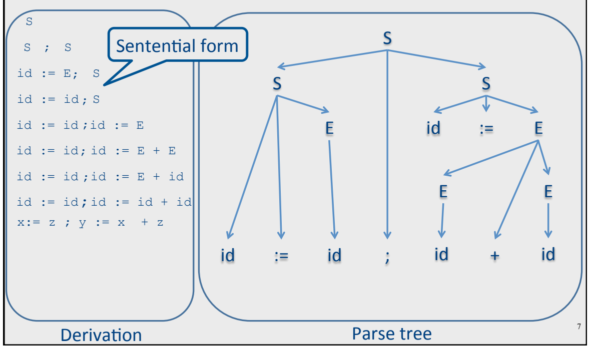
Derivations & Parse Trees



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Leftmost/rightmost Derivation

- Leftmost derivation
  - always expand leftmost non-terminal
- Rightmost derivation
  - Always expand rightmost non-terminal

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Broad kinds of parsers

- Parsers for arbitrary grammars
  - Earley's method, CYK method
  - Usually, not used in practice (though might change)
- Top-Down parsers
  - Construct parse tree in a top-down matter
  - Find the leftmost derivation
- Bottom-Up parsers
  - Construct parse tree in a bottom-up manner
  - Find the rightmost derivation in a reverse order

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Intuition: Top-Down Parsing

- Begin with start symbol
- "Guess" the productions
- Check if parse tree yields user's program

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Recursive Descent

- Blind exhaustive search
  - Goes over all possible production rules
  - Read & parse prefixes of input
  - Backtracks if guesses wrong
- Implementation
  - A (recursive) function for every production rule
  - Backtracks

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Predictive parsing

- Predicts which rule to use based on
  - Non terminal
  - Next k input symbols (look ahead)
    - Restricted grammars (LL(k))
- Implementation:
  - Predication stack: Expected future derivation
  - Transition table: Non terminal x terminal → Rule
    - FIRST(α) = The terminals that can appear first in some derivation for α
    - FOLLOW(X) = The tokens that can immediately follow X in some sentential form

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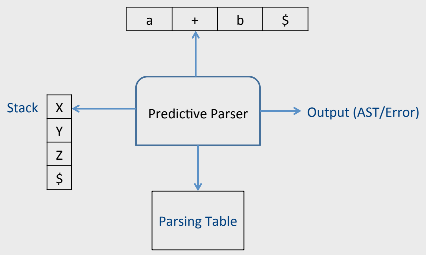
Stack-based Predictive Parser



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A reminder: Predictive parsing

- Predication stack: Expected future derivation
- Transition table: Non terminal x terminal → Rule
- Moves:
  - Predict
  - match

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Running parser example

aacbb$

A ⟶ aAb | c

Input suffix	Stack content	Move
aacbb$	A$	predict(A,a) = A ⟶ aAb
aacbb$	aAb$	match(a,a)
acbb$	Ab$	predict(A,a) = A ⟶ aAb
acbb$	aAbb$	[illegible]
cbb$	Abb$	predict(A,c) = A ⟶ c
cbb$	cbb$	[illegible]
bb$	bb$	match(b,b)
b$	b$	match(b,b)
$	$	match($,$) – success

	a	b	c
A	A ⟶ aAb		A ⟶ c

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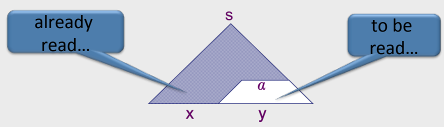
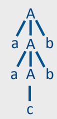
Top-Down Predictive Parsing

A→aAb | c

aacbb$





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Earley Parsing

- Parse arbitrary grammars in O(|input|3)
- Dynamic programming implementation of a recursive descent parser
  - S[N+1] Sequence of sets of "Earley states"
    - N = |INPUT|
    - Earley states is a sentential form + aux info
  - S[i] All parse tree that can be produced (by an RDP) after reading the first i tokens
    - S[i+1] built using S[0] … S[i]

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Earley States

- s = < constituent, back >
  - constituent (dotted rule) for A→αβ
    - A→•αβ predicated constituents
    - A→α•β in-progress constituents
    - A→αβ• completed constituents
  - back previous Early state in derivation

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Earley Parser

```
Input = x[1…N]
S[0] = <E'→ •E, 0>; S[1] = … S[N] = {}
for  i = 0 … N do
   until S[i] does not change do
      foreach s ∈ S[i]
         if  s = <A→…•a…, b>  and  a=x[i+1] then                    // scan
            S[i+1] =  S[i+1]  U  {<A→…a•…, b>  }
         if  s = <A→ … •X …, b> and X→α then                          // predict
            S[i] = S[i]  U  {<X→•α, i >  }
         if  s = < A→ … • , b>  and  <X→…•A…,  k>  ∈ S[b] then     // complete
            S[i] = S[i]  U {<X→…A•…,  k>  }
```

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Bottom-Up Parsing

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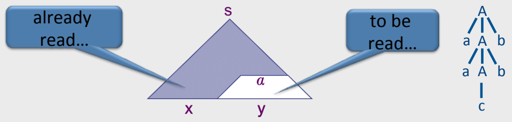
Top-Down vs Bottom-Up

A→aAb | c

aacbb$

- Top-down (predict match/scan-complete)



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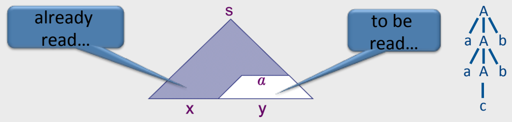
Top-Down vs Bottom-Up

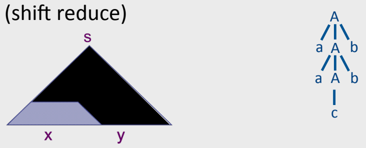
A→aAb | c

aacbb$

- Top-down (predict match/scan-complete)



- Bottom-up (shift reduce)



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Bottom-Up parsing: LR(k) Grammars

- A grammar is in the class LR(K) when it can be derived via:
  - Bottom-up derivation
  - Scanning the input from left to right (L)
  - Producing the rightmost derivation (R)
  - With lookahead of k tokens (k)
- A language is said to be LR(k) if it has an LR(k) grammar
- The simplest case is LR(0), which we will discuss

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Terminology: Reductions & Handles

- The opposite of derivation is called *reduction*
  - Let A→α be a production rule
  - Derivation: βAμ ➔ βαμ
  - Reduction: βαμ ➔ βAμ
- A *handle* is the reduced substring
  - α is the handles for βαμ

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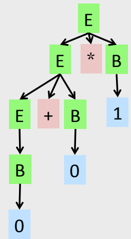
Goal: Reduce the Input to the Start Symbol

E → E * B | E + B | B
B → 0 | 1

Example:

0 + 0 * 1
B + 0 * 1
E + 0 * 1
E + B * 1
E * 1
E * B
E



Go over the input so far, and upon seeing a right-hand side of a rule, "invoke" the rule and replace the right-hand side with the left-hand side (reduce)

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Use Shift & Reduce

In each stage, we
shift a symbol from the input to the stack, or
reduce according to one of the rules.

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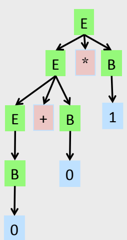
Use Shift & Reduce

In each stage, we
shift a symbol from the input to the stack, or
reduce according to one of the rules.

Example: "0+0*1"

E → E * B | E + B | B
B → 0 | 1



Stack	Input	action
	0+0*1$	shift
0	+0*1$	reduce
B	+0*1$	reduce
E	+0*1$	shift
E+	0*1$	shift
E+0	*1$	reduce
E+B	*1$	reduce
E	*1$	shift
E*	1$	shift
E*1	$	reduce
E*B	$	reduce
E	$	accept

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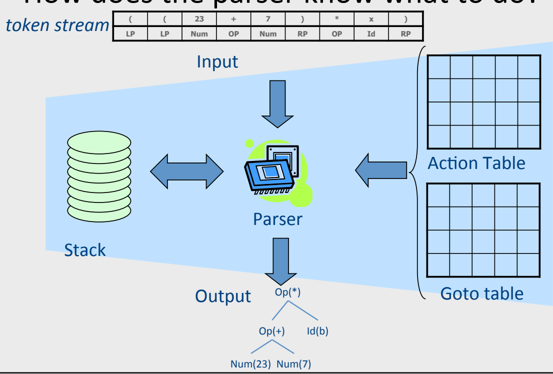
How does the parser know what to do?



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How does the parser know what to do?

- A state will keep the info gathered on handle(s)
  - A state in the "control" of the PDA
  - Also (part of) the stack alpha beit — Set of LR(0) items
- A table will tell it "what to do" based on current state and next token
  - The transition function of the PDA
- A stack will records the "nesting level"
  - Prefixes of handles

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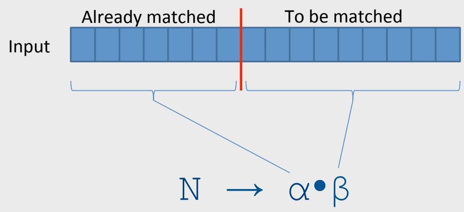
LR item



Hypothesis about αβ being a possible handle, so far we've matched α, expecting to see β

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Example: LR(0) Items

- All items can be obtained by placing a dot at every position for every production:

Grammar

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

LR(0) items

1: S → •E$
2: S → E • $
3: S → E $ •
4: E → • T
5: E → T •
6: E → • E + T
7: E → E • + T
8: E → E + • T
9: E → E + T •
10: T → • i
11: T → i •
12: T → • (E)
13: T → (• E)
14: T → (E •)
15: T → (E) •

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LR(0) items

N → α•β Shift Item

N → αβ• Reduce Item

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States and LR(0) Items

E → E * B | E + B | B
B → 0 | 1

- The state will “remember” the potential derivation rules given the part that was already identified
- For example, if we have already identified E then the state will remember the two alternatives:

(1) E → E * B, (2) E → E + B

- Actually, we will also remember where we are in each of them:

(1) E → E ● * B, (2) E → E ● + B

- A derivation rule with a location marker is called LR(0) item.
- The state is actually a set of LR(0) items. E.g., q_{13} = { E → E ● * B , E → E ● + B}

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Intuition

- Gather input token by token until we find a right-hand side of a rule and then replace it with the non-terminal on the left hand side
  - Going over a token and remembering it in the stack is a shift
    - Each shift moves to a state that remembers what we’ve seen so far
  - A reduce replaces a string in the stack with the non-terminal that derives it

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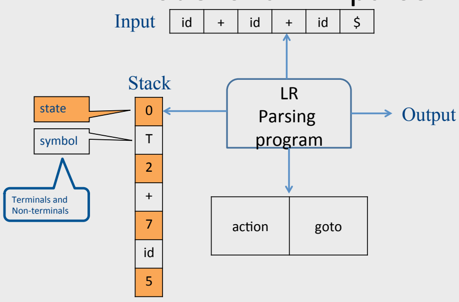
Model of an LR parser



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LR parser stack

- Sequence made of state, symbol pairs
- For instance a possible stack for the grammar

S → E $
E → T
E → E + T
T → id
T → (E)
could be: 0 T 2 + 7 id 5

Stack grows this way

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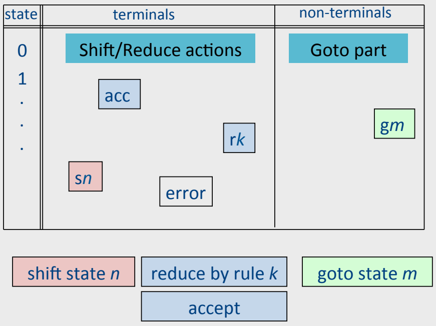
Form of LR parsing table



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LR parser table example

STATE	action					goto	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

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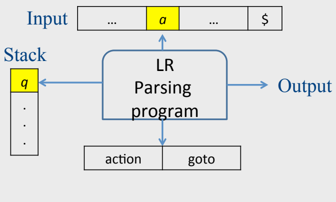
Shift move



- If action[*q*, *a*] = *sn*

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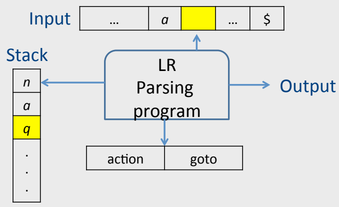
Result of shift



- If action[*q*, *a*] = *sn*

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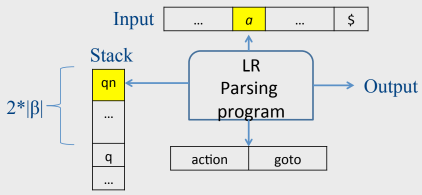
Reduce move



- If action[qn, a] = rk
- Production: (k) A →β
- If β= σ1… σn
  Top of stack looks like q1 σ1… qn σn
- goto[q, A] = qm

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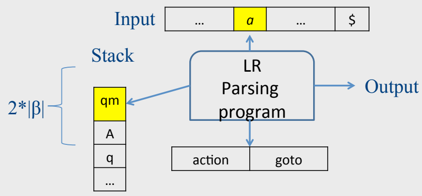
Result of reduce move



- If action[qn, a] = rk
- Production: (k) A →β
- If β= σ1… σn
  Top of stack looks like q1 σ1… qn σn
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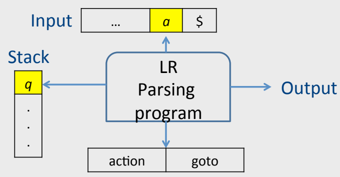
Accept move



If action[*q*, *a*] = accept

parsing completed

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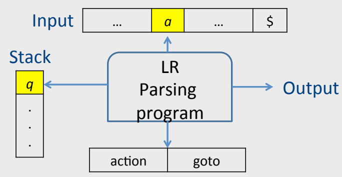
Error move



If action[*q*, *a*] = error (usually empty)

parsing discovered a syntactic error

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Example

Z → E **$**

E → T | E **+** T

T → **i** | **(** E **)**

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Example: parsing with LR items

Z → E **$**
E → T | E **+** T
T → **i** | **(** E **)**

	i		+		i		$

Z → •E **$**

E → •T

E → •E **+** T

T → •**i**

T → •**(** E **)**

Why do we need these additional LR items?
Where do they come from?
What do they mean?

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ε-closure

- Given a set S of LR(0) items
- If P → α•Nβ is in S
- then for each rule N → γ in the grammar S must also contain N → •γ

ε-closure ({Z → •E $}) = { Z → •E $,
E → •T,
E → •E + T,
T → •i ,
T → •(E) }

Z → E **$**
E → T | E **+** T
T → **i** | **(** E **)**

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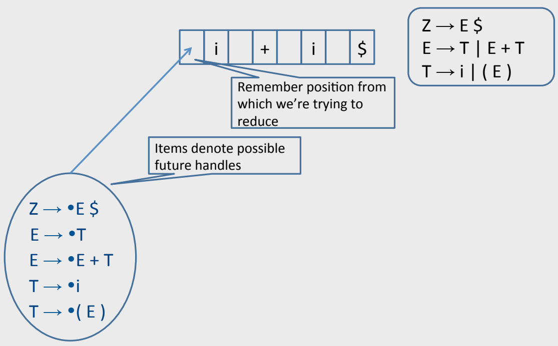
Example: parsing with LR items



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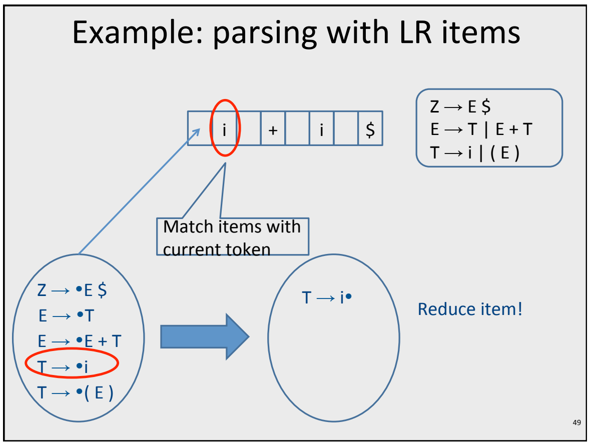
Example: parsing with LR items
i
+
i
$
Z → E $
E → T | E + T
T → i | (E)
Match items with current token
Z → •E $
E → •T
E → •E + T
T → •i
T → •(E)
T → i•
Reduce item!
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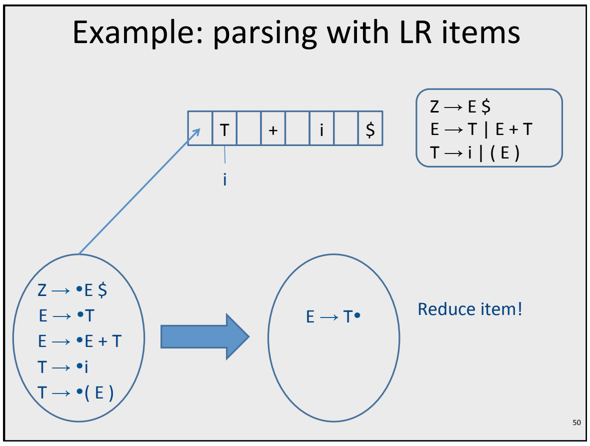
Example: parsing with LR items
T
+
i
$
i
Z → E $
E → T | E + T
T → i | (E)
Z → •E $
E → •T
E → •E + T
T → •i
T → •(E)
E → T•
Reduce item!
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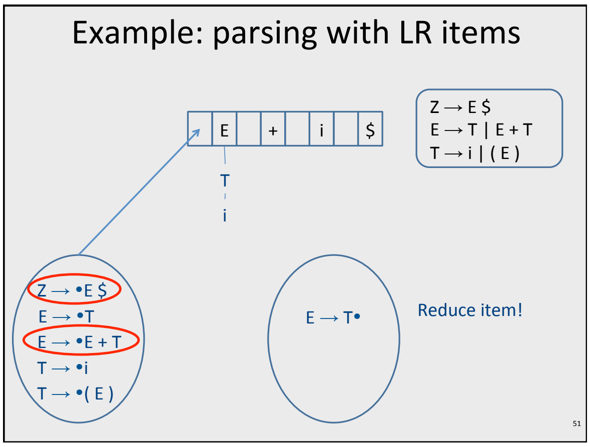
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E
+
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i
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T → •(E)
E → T•
Reduce item!
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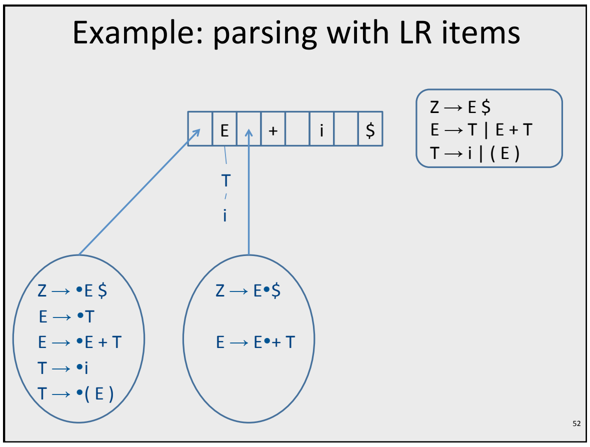
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E
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i
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T → i | (E)
Z → •E $
E → •T
E → •E + T
T → •i
T → •(E)
Z → E•$
E → E•+ T
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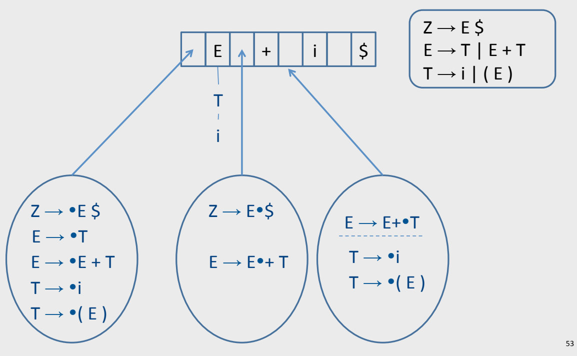
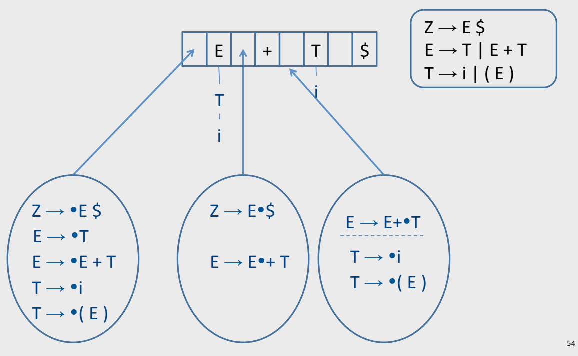
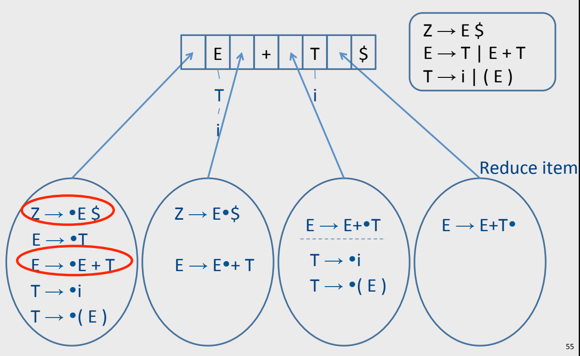
Example: parsing with LR items



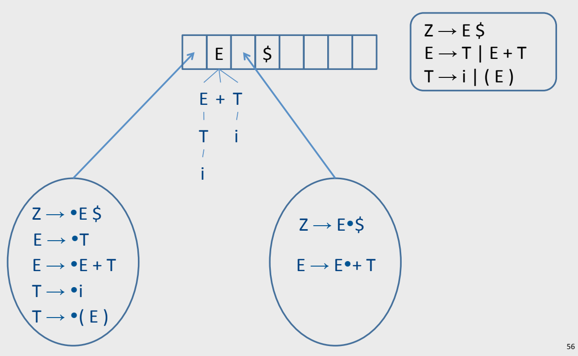
Example: parsing with LR items



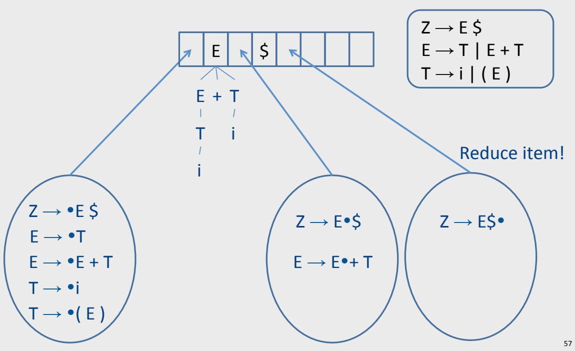
Example: parsing with LR items



Example: parsing with LR items



Example: parsing with LR items



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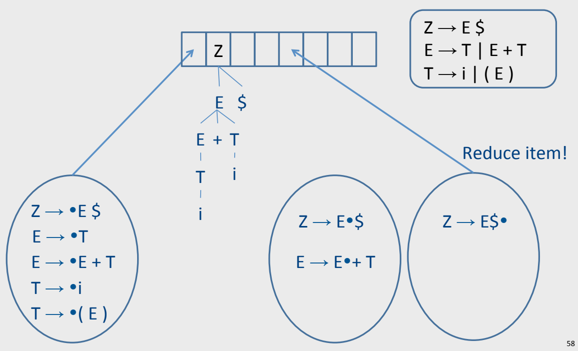
Example: parsing with LR items



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GOTO/ACTION tables

empty – error move

GOTO Table | ACTION Table

State	i	+	(	)	$	E	T	action
q0	q5		q7			q1	q6	shift
q1		q3			q2			shift
q2								Z→E$
q3	q5		q7				q4	Shift
q4								E→E+T
q5								T→i
q6								E→T
q7	q5		q7			q8	q6	shift
q8		q3		q9				shift
q9								T→E

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LR(0) parser tables

- Two types of rows:
  - Shift row – tells which state to GOTO for current token
  - Reduce row – tells which rule to reduce (independent of current token)
    - GOTO entries are blank

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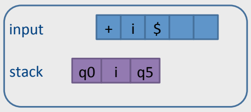
LR parser data structures

- Input – remainder of text to be processed
- Stack – sequence of pairs N, qi
  - N – symbol (terminal or non-terminal)
  - qi – state at which decisions are made



- Initial stack contains q0

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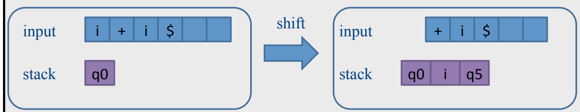
LR(0) pushdown automaton

- Two moves: shift and reduce
- Shift move
  - Remove first token from input
  - Push it on the stack
  - Compute next state based on GOTO table
  - Push new state on the stack
  - If new state is error – report error



State	i	+	(	)	$	E	T	action
q0	q5		q7			q1	q6	shift

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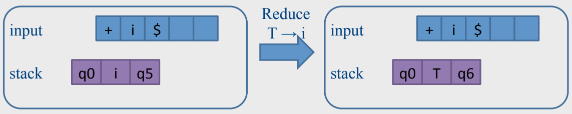
LR(0) pushdown automaton

- Reduce move
  - Using a rule N →α
  - Symbols in α and their following states are removed from stack
  - New state computed based on GOTO table (using top of stack, before pushing N)
  - N is pushed on the stack
  - New state pushed on top of N



State	i	+	(	)	$	E	T	action
q0	q5		q7			q1	q6	shift

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GOTO/ACTION table

State	i	+	(	)	$	E	T
q0	s5		s7			s1	s6
q1		s3			s2		
q2	r1	r1	r1	r1	r1	r1	r1
q3	s5		s7				s4
q4	r3	r3	r3	r3	r3	r3	r3
q5	r4	r4	r4	r4	r4	r4	r4
q6	r2	r2	r2	r2	r2	r2	r2
q7	s5		s7			s8	s6
q8		s3		s9			
q9	r5	r5	r5	r5	r5	r5	r5

```
(1) Z → E $
(2) E → T
(3) E → E + T
(4) T → i
(5) T →( E )
```

Warning: numbers mean different things!
rn = reduce using rule number n
sm = shift to state m

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Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5

Initialize with state 0

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

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Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5

Initialize with state 0

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

66

Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

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Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4

pop id 5

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

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Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 ~~id 5~~	+ id $	r4

S	action					goto	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

push T 6

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Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2

S	action					goto	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

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Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2
0 E 1	+ id $	s3

S	action					goto	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

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Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2
0 E 1	+ id $	s3
0 E 1 + 3	id $	s5

S	action					goto	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

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Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2
0 E 1	+ id $	s3
0 E 1 + 3	id $	s5
0 E 1 + 3 id 5	$	r4

S	action					goto	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

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Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2
0 E 1	+ id $	s3
0 E 1 + 3	id $	s5
0 E 1 + 3 id 5	$	r4
0 E 1 + 3 T 4	$	r3

S	action					goto	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

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Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2
0 E 1	+ id $	s3
0 E 1 + 3	id $	s5
0 E 1 + 3 id 5	$	r4
0 E 1 + 3 T 4	$	r3
0 E 1	$	s2

S	action					goto	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

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Constructing an LR parsing table

- Construct a (determinized) transition diagram from LR items
- If there are conflicts – stop
- Fill table entries from diagram

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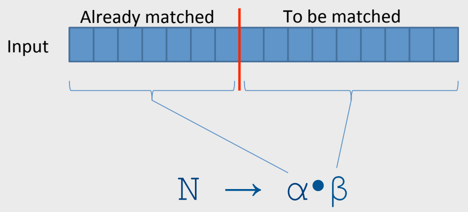
LR item



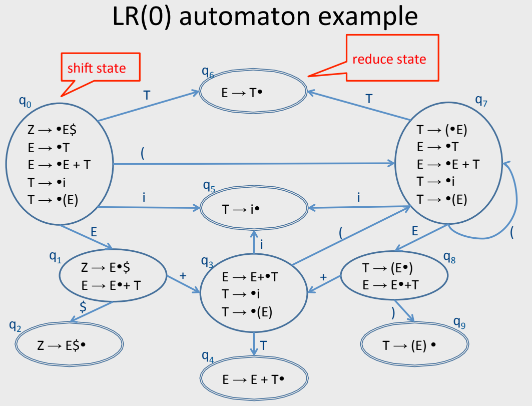
Hypothesis about αβ being a possible handle, so far we've matched α, expecting to see β

Types of LR(0) items

N → α•β Shift Item

N → αβ• Reduce Item

LR(0) automaton example



Computing item sets

- Initial set
  - Z is in the start symbol
  - ε-closure({ Z→•α | Z→α is in the grammar })
- Next set from a set S and the next symbol X
  - step(S,X) = { N→αX•β | N→α•Xβ in the item set S}
  - nextSet(S,X) = ε-closure(step(S,X))

Operations for transition diagram construction

- Initial = {S'→•S$}
- For an item set I
  Closure(I) = Closure(I) ∪
  {X→•μ is in grammar | N→α•Xβ in I}
- Goto(I, X) = { N→αX•β | N→α•Xβ in I}

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Initial example

Grammar
(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

- Initial = {S → •E $}

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Closure example

Grammar
(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

- Initial = {S → •E $}
- Closure({S → •E $}) = {
  S → •E $
  E → •T
  E → •E + T
  T → •id
  T → •(E) }

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Goto example

Grammar
(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

- Initial = {S → •E $}
- Closure({S → •E $}) = {
  S → •E $
  E → •T
  E → •E + T
  T → •id
  T → •(E) }
- Goto({S → •E $, E → •E + T, T → •id}, E) =
  {S → E• $, E → E• + T}

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Constructing the transition diagram

- Start with state 0 containing item Closure({S → •E $})
- Repeat until no new states are discovered
  - For every state p containing item set Ip, and symbol N, compute state q containing item set Iq = Closure(goto(Ip, N))

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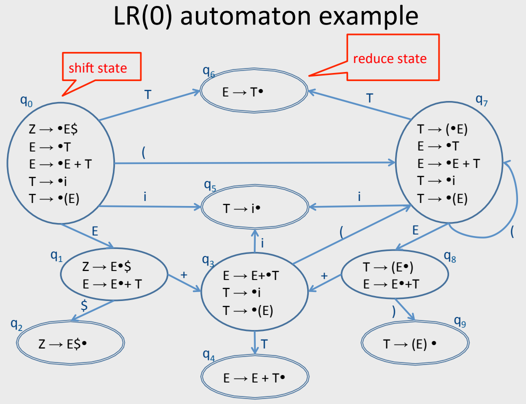
LR(0) automaton example



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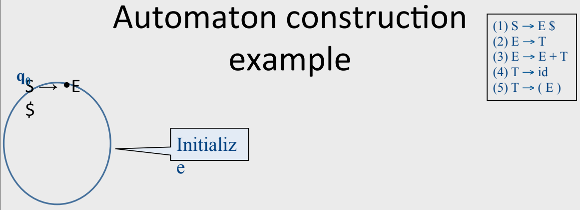
Automaton construction example



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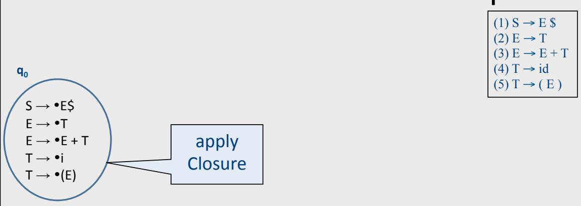
Automaton construction example



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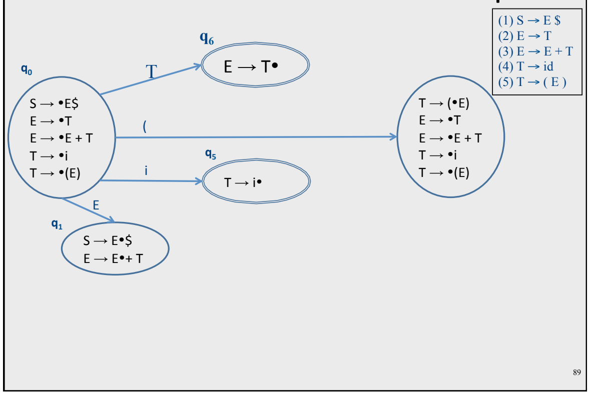
Automaton construction example



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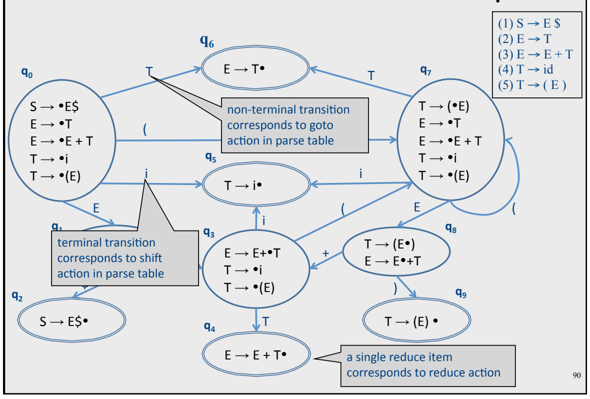
Automaton construction example



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Are we done?

- Can make a transition diagram for any grammar
- Can make a GOTO table for every grammar
- Cannot make a deterministic ACTION table for every grammar

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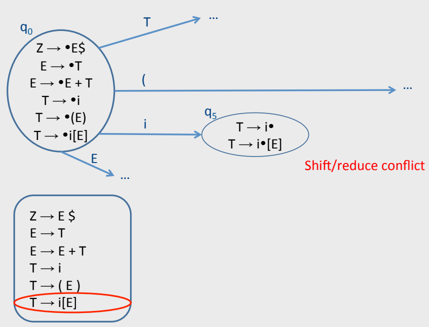
LR(0) conflicts



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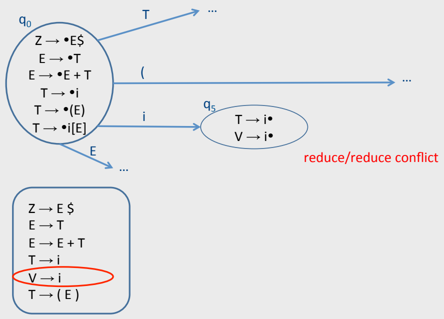
LR(0) conflicts



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LR(0) conflicts

- Any grammar with an ε-rule cannot be LR(0)
- Inherent shift/reduce conflict
  - A→ ε• – reduce item
  - P →α•Aβ – shift item
  - A→ ε• can always be predicted from P →α•Aβ

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Conflicts

- Can construct a diagram for every grammar but some may introduce conflicts
- shift-reduce conflict: an item set contains at least one shift item and one reduce item
- reduce-reduce conflict: an item set contains two reduce items

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LR variants

- LR(0) – what we've seen so far
- SLR(0)
  - Removes infeasible reduce actions via FOLLOW set reasoning
- LR(1)
  - LR(0) with one lookahead token in items
- LALR(0)
  - LR(1) with merging of states with same LR(0) component

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LR (0) GOTO/ACTIONS tables

GOTO table is indexed by state and a grammar symbol from the stack

GOTO Table | ACTION Table

State	i	+	(	)	$	E	T	action
q0	q5		q7			q1	q6	shift
q1		q3			q2			shift
q2								Z⟶E$
q3	q5		q7				q4	Shift
q4								E⟶E+T
q5								T⟶i
q6								E⟶T
q7	q5		q7			q8	q6	shift
q8		q3		q9				shift
q9								T⟶E

ACTION table determined only by state, ignores input

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SLR parsing

- A handle should not be reduced to a non-terminal N if the lookahead is a token that cannot follow N
- A reduce item N ⟶ α• is applicable only when the lookahead is in FOLLOW(N)
  - If b is not in FOLLOW(N) we just proved there is no derivation S ➔* βNb.
  - Thus, it is safe to remove the reduce item from the conflicted state
- Differs from LR(0) only on the ACTION table
  - Now a row in the parsing table may contain both shift actions and reduce actions and we need to consult the current token to decide which one to take

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SLR action table

Lookahead token from the input

State	i	+	(	)	[	]	$
0	shift		shift				
1		shift					accept
2							
3	shift		shift				
4		E⟶E+T		E⟶E+T			E⟶E+T
5		T⟶i		T⟶i	**shift**		T⟶i
6		E⟶T		E⟶T			E⟶T
7	shift		shift				
8		shift		shift			
9		T⟶(E)		T⟶(E)			T⟶(E)

SLR – use 1 token look-ahead

vs.

state	action
q0	shift
q1	shift
q2	
q3	shift
q4	E⟶E+T
q5	T⟶i
q6	E⟶T
q7	shift
q8	shift
q9	T⟶E

LR(0) – no look-ahead

```
… as before…
T ⟶ i
T ⟶ i[E]
```

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LR(1) grammars

- In SLR: a reduce item N ⟶ α• is applicable only when the lookahead is in FOLLOW(N)
- But FOLLOW(N) merges lookahead for all alternatives for N
  - Insensitive to the context of a given production
- LR(1) keeps lookahead with each LR item
- Idea: a more refined notion of follows computed per item

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LR(1) items

- LR(1) item is a pair
  - LR(0) item
  - Lookahead token
- Meaning
  - We matched the part left of the dot, looking to match the part on the right of the dot, followed by the lookahead token
- Example
  - The production L → id yields the following LR(1) items

(0) S' → S
(1) S → L = R
(2) S → R
(3) L → * R
(4) L → id
(5) R → L

LR(0) items
[L → ● id]
[L → id ●]

LR(1) items
[L → ● id, *]
[L → ● id, =]
[L → ● id, id]
[L → ● id, $]
[L → id ●, *]
[L → id ●, =]
[L → id ●, id]
[L → id ●, $]

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LALR(1)

- LR(1) tables have huge number of entries
- Often don't need such refined observation (and cost)
- Idea: find states with the same LR(0) component and merge their lookaheads component as long as there are no conflicts
- LALR(1) not as powerful as LR(1) in theory but works quite well in practice
  - Merging may not introduce new shift-reduce conflicts, only reduce-reduce, which is unlikely in practice

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Summary

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LR is More Powerful than LL

- Any LL(k) language is also in LR(k), i.e., LL(k) ⊂ LR(k).
  - LR is more popular in automatic tools
- But less intuitive
- Also, the lookahead is counted differently in the two cases
  - In an LL(k) derivation the algorithm sees the left-hand side of the rule + k input tokens and then must select the derivation rule
  - In LR(k), the algorithm "sees" all right-hand side of the derivation rule + k input tokens and then reduces
    - LR(0) sees the entire right-side, but no input token

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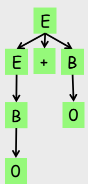
Broad kinds of parsers

- Parsers for arbitrary grammars
  - Earley's method, CYK method
  - Usually, not used in practice (though might change)
- Top-Down parsers
  - Construct parse tree in a top-down matter
  - Find the leftmost derivation
- Bottom-Up parsers
  - Construct parse tree in a bottom-up manner
  - Find the rightmost derivation in a reverse order

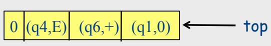
Question

- Why do we need the stack?
- Why can we use FSM to make parsing decisions?

Why do we need a stack?



- Suppose so far we have discovered E → B → 0 and gather information on "E +".
- In the given grammar this can only mean
  E → E + ● B
- Suppose state q_6 represents this possibility.
- Now, the next token is 0, and we need to ignore q_6 for a minute, and work on B → 0 to obtain E+B.
- Therefore, we push q_6 to the stack, and after identifying B, we pop it to continue.



See you next time

- Here!

GOTO/ACTION table

st	i	+	(	)	$	E	T
q0	s5		s7			s1	s6
q1		s3			s2		
q2	r1	r1	r1	r1	r1	r1	r1
q3	s5		s7				s4
q4	r3	r3	r3	r3	r3	r3	r3
q5	r4	r4	r4	r4	r4	r4	r4
q6	r2	r2	r2	r2	r2	r2	r2
q7	s5		s7			s8	s6
q8		s3		s9			
q9	r5	r5	r5	r5	r5	r5	r5

(1) Z → E $
(2) E → T
(3) E → E + T
(4) T → i
(5) T →(E)

top is on the right

Stack	Input	Action
q0	i + i $	s5
q0 i q5	+ i $	r4
q0 T q6	+ i $	r2
q0 E q1	+ i $	s3
q0 E q1 + q3	i $	s5
q0 E q1 + q3 i q5	$	r4
q0 E q1 + q3 T q4	$	r3
q0 E q1	$	s2
q0 E q1 $ q2		r1
q0 Z		

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Example

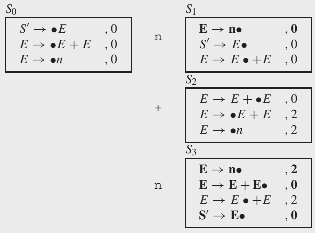

FIGURE 1. Earley sets for the grammar $E \to E + E \mid n$ and the input n + n. Items in bold are ones which correspond to the input's derivation.

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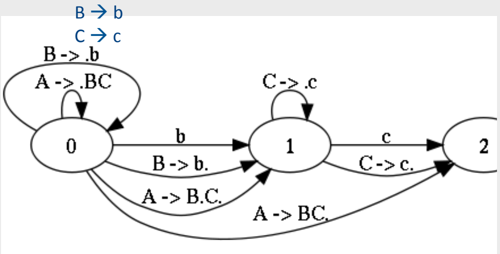
Earley with Pictures

Grammar: A → BC
B → b
C → c

Input: bc



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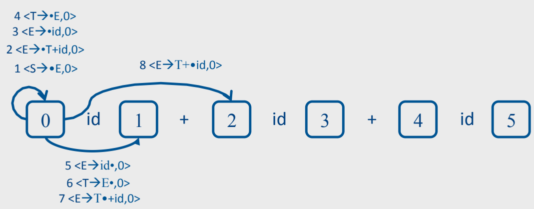
Earley Parsing in Pictures

Grammar: S → E
E → T + id | id
T → E

Input: id + id + id



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