Compilation

0368-3133 (Semester A, 2013/14)

Lecture 6a: Syntax

(Bottom–up parsing)

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Slides credit: Roman Manevich, Mooly Sagiv, Eran Yahav

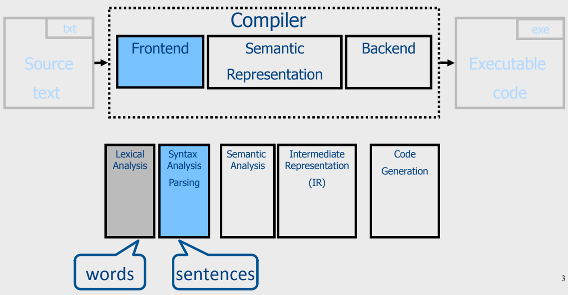
What is a Compiler?

"A compiler is a computer program that transforms source code written in a programming language (source language) into another language (target language).

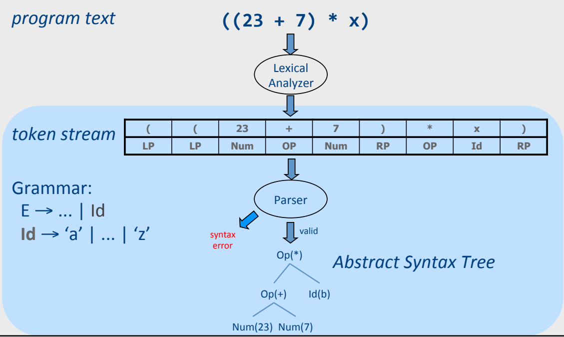
The most common reason for wanting to transform source code is to create an executable program."

--Wikipedia

Conceptual Structure of a Compiler



From scanning to parsing

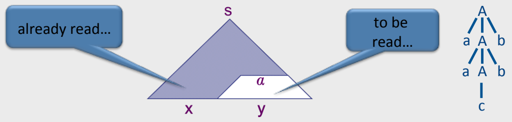


Top-Down vs Bottom-Up

A→aAb | c

aacbb$

- Top-down (predict match/scan-complete)



Top-Down vs Bottom-Up

A→aAb | c

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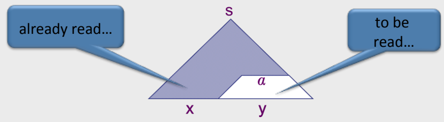
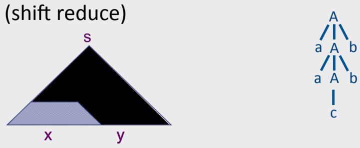
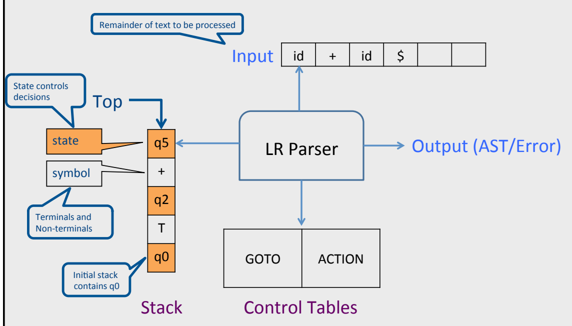
- Top-down (predict match/scan-complete)



- Bottom-up (shift reduce)



Model of an LR parser



LR(0) parser tables

Empty cell = error move

GOTO Table

ACTION Table

State	i	+	(	)	$	E	T	action
q0	q5		q7			q1	q6	shift
q1		q3			q2			shift
q2								reduce: Z→E$
q3	q5		q7				q4	shift
q4								reduce: E→E+T
q5								reduce: T→i
q6								reduce: E→T
q7	q5		q7			q8	q6	shift
q8		q3		q9				shift
q9								reduce: T→E

LR(0) parser tables

- Shift action row
  - Tells which state to GOTO for current token
  - Blank entry indicates an error
- Reduce action row
  - Tells which rule to reduce with
    - Independent of current token
  - GOTO entries are blank

State	id	+	(	)	$	E	T	action
q0	q5		q7			q1	q6	shift
q5								reduce: T→i

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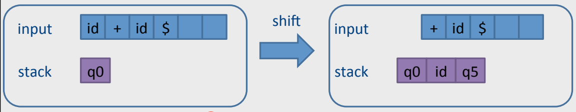
Shift Move

- Remove first token from input
- Push it on the stack
- Compute next state based on GOTO table
- Push new state on the stack
- If new state is error – report error



State	id	+	(	)	$	E	T	action
q0	q5		q7			q1	q6	shift
q5								reduce: T→i

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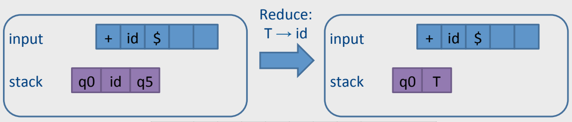
Reduce Move (using N→α)

- Symbols in α and their following states are removed from stack
- New state computed based on GOTO table (using top of stack, before pushing N)
- N is pushed on the stack
- New state pushed on top of N



State	id	+	(	)	$	E	T	action
q0	q5		q7			q1	q6	shift
q5								reduce: T→i

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Reduce Move (using N→α)

- Symbols in α and their following states are removed from stack
- New state computed based on GOTO table (using top of stack, before pushing N)
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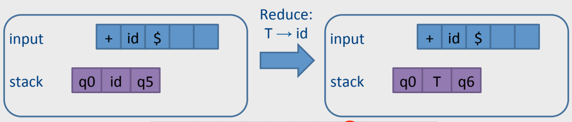

State	id	+	(	)	$	E	T	action
q0	q5		q7			q1	q6	shift
q5								reduce: T→i

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GOTO/ACTION table

State	i	+	(	)	$	E	T
q0	s5		s7			s1	s6
q1		s3			s2		
q2	r1	r1	r1	r1	r1	r1	r1
q3	s5		s7				s4
q4	r3	r3	r3	r3	r3	r3	r3
q5	r4	r4	r4	r4	r4	r4	r4
q6	r2	r2	r2	r2	r2	r2	r2
q7	s5		s7			s8	s6
q8		s3		s9			
q9	r5	r5	r5	r5	r5	r5	r5

(1) Z → E $
(2) E → T
(3) E → E + T
(4) T → i
(5) T → (E)

Warning: numbers mean different things!
rn = reduce using rule number n
sm = shift to state m

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Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5

Initialize with state 0

S	action					goto	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

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Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5

Initialize with state 0

S	action					goto	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

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Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4

S	action					goto	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

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Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4

pop id 5

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 ~~id 5~~	+ id $	r4

push T 6

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2
0 E 1	+ id $	s3

S	*action*					*goto*	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2
0 E 1	+ id $	s3
0 E 1 + 3	id $	s5

S	action					goto	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

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Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2
0 E 1	+ id $	s3
0 E 1 + 3	id $	s5
0 E 1 + 3 id 5	$	r4

S	action					goto	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

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Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2
0 E 1	+ id $	s3
0 E 1 + 3	id $	s5
0 E 1 + 3 id 5	$	r4
0 E 1 + 3 T 4	$	r3

S	action					goto	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

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Parsing id+id$

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

Stack	Input	Action
0	id + id $	s5
0 id 5	+ id $	r4
0 T 6	+ id $	r2
0 E 1	+ id $	s3
0 E 1 + 3	id $	s5
0 E 1 + 3 id 5	$	r4
0 E 1 + 3 T 4	$	r3
0 E 1	$	s2

S	action					goto	
	id	+	(	)	$	E	T
0	s5		s7			g1	g6
1		s3			acc		
2							
3	s5		s7				g4
4	r3	r3	r3	r3	r3		
5	r4	r4	r4	r4	r4		
6	r2	r2	r2	r2	r2		
7	s5		s7			g8	g6
8		s3		s9			
9	r5	r5	r5	r5	r5		

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Constructing an LR parsing table

- Construct a transition diagram (deterministic FSM)
  - States = sets of LR(0) items
  - Transitions = one-step derivation
- If there are conflicts – stop
- Fill table entries from diagram

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Terminology: Reductions & Handles

- The opposite of derivation is called *reduction*
  - Let A→α be a production rule
  - Derivation: βAµ ➔ βαµ
  - Reduction: βαµ ➔ βAµ
- A *handle* is the reduced substring
  - α is the handles for βαµ

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LR(0) Items

- The items of a grammar are obtained by placing a dot at every position in every production

Grammar

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

LR(0) items

1: S → •E$
2: S → E • $
3: S → E $ •
4: E → • T
5: E → T •
6: E → • E + T
7: E → E • + T
8: E → E + • T
9: E → E + T •
10: T → • id
11: T → id •
12: T → • (E)
13: T → (• E)
14: T → (E •)
15: T → (E) •

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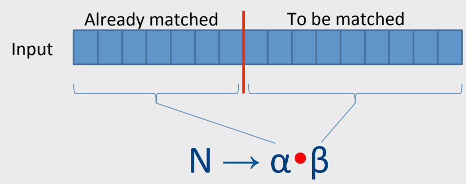
LR(0) Item - Intuition



Hypothesis about αβ is the rule being reduced, and so far we've matched α and we expect to see β

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Types of LR(0) items

$N \longrightarrow \alpha \bullet \beta$ Shift Item

$N \longrightarrow \alpha\beta \bullet$ Reduce Item

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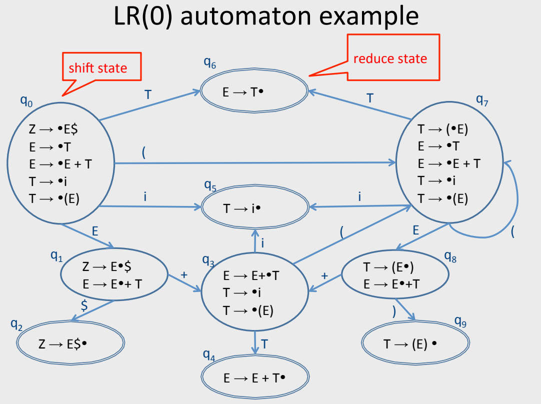
LR(0) automaton example



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Computing item sets

- Initial set
  - Z is in the start symbol
  - ε-closure({ Z⟶•α | Z⟶α is in the grammar })
- Next set from a set S and the next symbol X
  - step(S,X) = { N→αX•β | N→α•Xβ in the item set S}
  - nextSet(S,X) = ε-closure(step(S,X))

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Operations for transition diagram construction

- Initial = {S'⟶•S$}
- For an item set I
  Closure(I) = Closure(I) ∪
  {X⟶•μ is in grammar| N⟶α•Xβ in I}
- Goto(I, X) = { N⟶αX•β | N⟶α•Xβ in I}

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Initial example

Grammar

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

- Initial = {S → •E $}

Closure example

Grammar

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

- Initial = {S → •E $}
- Closure({S → •E $}) = {
  S → •E $
  E → •T
  E → •E + T
  T → •id
  T → •(E) }

Goto example

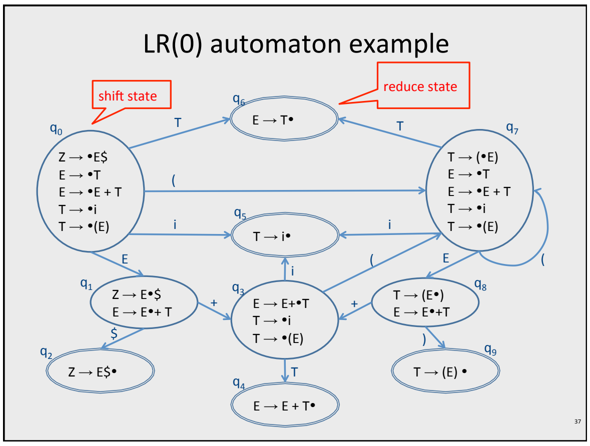
Grammar

(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)

- Initial = {S → •E $}
- Closure({S → •E $}) = {
  S → •E $
  E → •T
  E → •E + T
  T → •id
  T → •(E) }
- Goto({S → •E $, E → •E + T, T → •id}, E) =
  {S → E• $, E → E• + T}

Constructing the transition diagram

- Start with state 0 containing item Closure({S → •E $})
- Repeat until no new states are discovered
  - For every state p containing item set Ip, and symbol N, compute state q containing item set Iq = Closure(goto(Ip, N))


LR(0) automaton example
shift state
reduce state
q0
Z → •E$
E → •T
E → •E + T
T → •i
T → •(E)
q6
E → T•
q7
T → (•E)
E → •T
E → •E + T
T → •i
T → •(E)
q5
T → i•
q1
Z → E•$
E → E•+ T
q3
E → E+•T
T → •i
T → •(E)
q8
T → (E•)
E → E•+T
q2
Z → E$•
q4
E → E + T•
q9
T → (E) •
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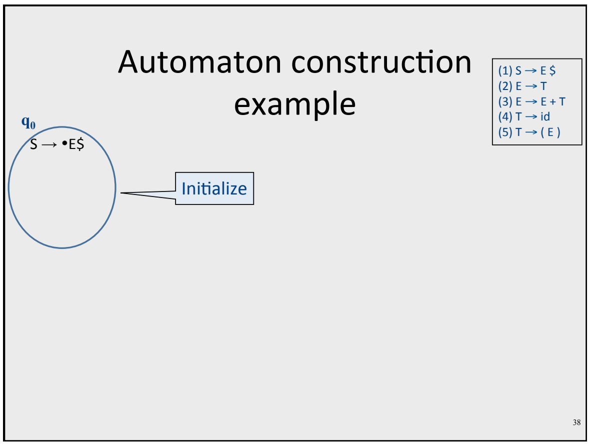
Automaton construction example
(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)
q0
S → •E$
Initialize
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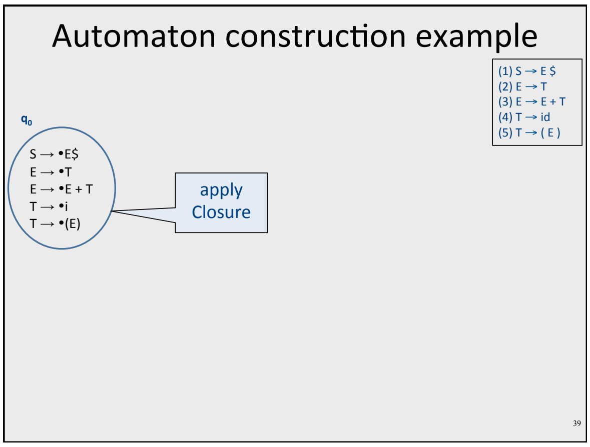
Automaton construction example
(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)
q0
S → •E$
E → •T
E → •E + T
T → •i
T → •(E)
apply Closure
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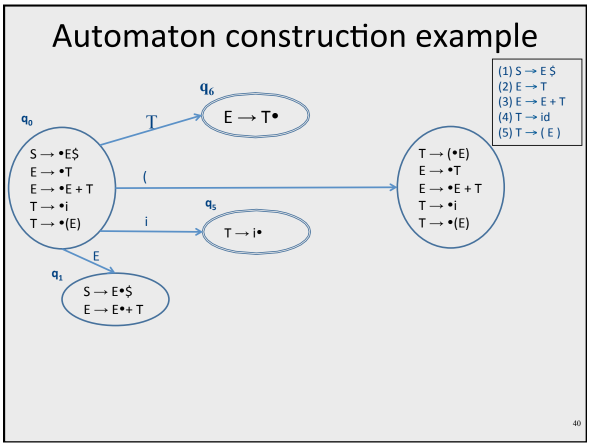
Automaton construction example
(1) S → E $
(2) E → T
(3) E → E + T
(4) T → id
(5) T → (E)
q0
S → •E$
E → •T
E → •E + T
T → •i
T → •(E)
q6
E → T•
T → (•E)
E → •T
E → •E + T
T → •i
T → •(E)
q5
T → i•
q1
S → E•$
E → E•+ T
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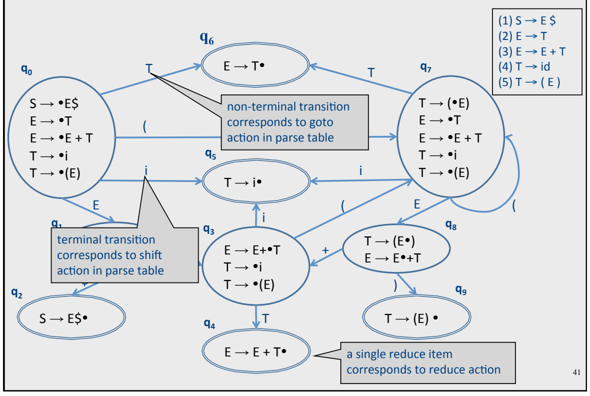
Automaton construction example



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Are we done?

- Can make a transition diagram for any grammar
- Can make a GOTO table for every grammar
- Cannot make a deterministic ACTION table for every grammar

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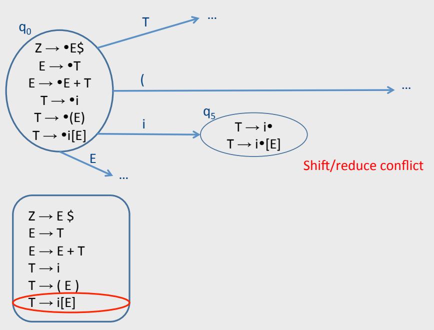
LR(0) conflicts



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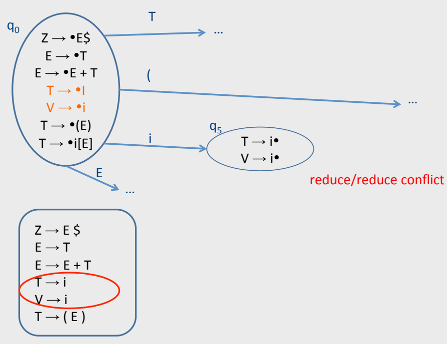
LR(0) conflicts



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LR(0) conflicts

- Any grammar with an ε-rule cannot be LR(0)
- Inherent shift/reduce conflict
  - A→ ε• – reduce item
  - P →α•Aβ – shift item
  - A→ ε• can always be predicted from P →α•Aβ

Conflicts

- Can construct a diagram for every grammar but some may introduce conflicts
- shift-reduce conflict: an item set contains at least one shift item and one reduce item
- reduce-reduce conflict: an item set contains two reduce items

LR variants

- LR(0) – what we've seen so far
- SLR(0)
  - Removes infeasible reduce actions via FOLLOW set reasoning
- LR(1)
  - LR(0) with one lookahead token in items
- LALR(0)
  - LR(1) with merging of states with same LR(0) component

LR (0) GOTO/ACTIONS tables

GOTO table is indexed by state and a grammar symbol from the stack

GOTO Table

ACTION Table

State	i	+	(	)	$	E	T	action
q0	q5		q7			q1	q6	shift
q1		q3			q2			shift
q2								Z→E$
q3	q5		q7				q4	Shift
q4								E→E+T
q5								T→i
q6								E→T
q7	q5		q7			q8	q6	shift
q8		q3		q9				shift
q9								T→E

ACTION table determined only by state, ignores input

SLR parsing

- A handle should not be reduced to a non-terminal N if the lookahead is a token that cannot follow N
- A reduce item N → α• is applicable only when the lookahead is in FOLLOW(N)
  - If b is not in FOLLOW(N) we just proved there is no derivation S ➔* βNb.
  - Thus, it is safe to remove the reduce item from the conflicted state
- Differs from LR(0) only on the ACTION table
  - Now a row in the parsing table may contain both shift actions and reduce actions and we need to consult the current token to decide which one to take

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SLR action table

Lookahead token from the input

State	i	+	(	)	[	]	$
0	shift		shift				
1		shift					accept
2							
3	shift		shift				
4		E→E+T		E→E+T			E→E+T
5		T→i		T→i	shift		T→i
6		E→T		E→T			E→T
7	shift		shift				
8		shift		shift			
9		T→(E)		T→(E)			T→(E)

SLR – use 1 token look-ahead

vs.

state	action
q0	shift
q1	shift
q2	
q3	shift
q4	E→E+T
q5	T→i
q6	E→T
q7	shift
q8	shift
q9	T→E

LR(0) – no look-ahead

```
… as before…
T → i
T → i[E]
```

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LR(1) grammars

- In SLR: a reduce item N → α• is applicable only when the lookahead is in FOLLOW(N)
- But FOLLOW(N) merges lookahead for all alternatives for N
  - Insensitive to the context of a given production
- LR(1) keeps lookahead with each LR item
- Idea: a more refined notion of follows computed per item

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LR(1) items

- LR(1) item is a pair
  - LR(0) item
  - Lookahead token
- Meaning
  - We matched the part left of the dot, looking to match the part on the right of the dot, followed by the lookahead token
- Example
  - The production L → id yields the following LR(1) items

(0) S' → S
(1) S → L = R
(2) S → R
(3) L → * R
(4) L → id
(5) R → L

LR(0) items

[L → ● id]
[L → id ●]

LR(1) items

[L → ● id, *]
[L → ● id, =]
[L → ● id, id]
[L → ● id, $]
[L → id ●, *]
[L → id ●, =]
[L → id ●, id]
[L → id ●, $]

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LALR(1)

- LR(1) tables have huge number of entries
- Often don't need such refined observation (and cost)
- Idea: find states with the same LR(0) component and merge their lookaheads component as long as there are no conflicts
- LALR(1) not as powerful as LR(1) in theory but works quite well in practice
  - Merging may not introduce new shift-reduce conflicts, only reduce-reduce, which is unlikely in practice

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Summary

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Summary

- Bottom up derivation
- LR(k) can decide on a reduce after seeing the entire right side of the rule plus k look-ahead tokens.
  - We focused on LR(0)
- Using a table and a stack to derive
- LR Items and the automaton
- Creating the table from the automaton
- LR parsing with pushdown automata
- LR(0), SLR, LR(1) – different kinds of LR items, same basic algorithm

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Broad kinds of parsers

- Parsers for arbitrary grammars
  - Earley's method, CYK method
  - Usually, not used in practice (though might change)
- Top-Down parsers
  - Construct parse tree in a top-down matter
  - Find the leftmost derivation
- Bottom-Up parsers
  - Construct parse tree in a bottom-up manner
  - Find the rightmost derivation in a reverse order

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LR is More Powerful than LL

- Any LL(k) language is also in LR(k), i.e., LL(k) ⊂ LR(k).
  - LR is more popular in automatic tools
- But less intuitive
- Also, the lookahead is counted differently in the two cases
  - In an LL(k) derivation the algorithm sees the left-hand side of the rule + k input tokens and then must select the derivation rule
  - In LR(k), the algorithm "sees" all right-hand side of the derivation rule + k input tokens and then reduces
    - LR(0) sees the entire right-side, but no input token

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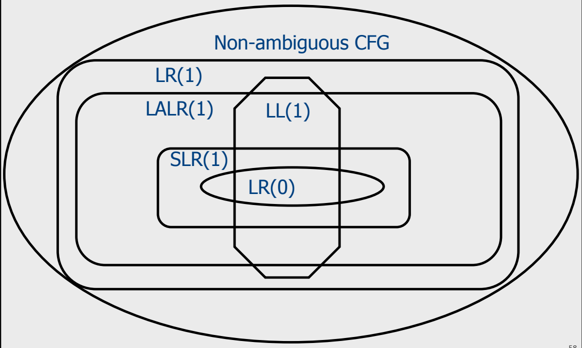
Grammar Hierarchy



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