# Program Analysis and Verification

0368-4479

http://www.cs.tau.ac.il/~maon/teaching/2013-2014/paav/paav1314b.html

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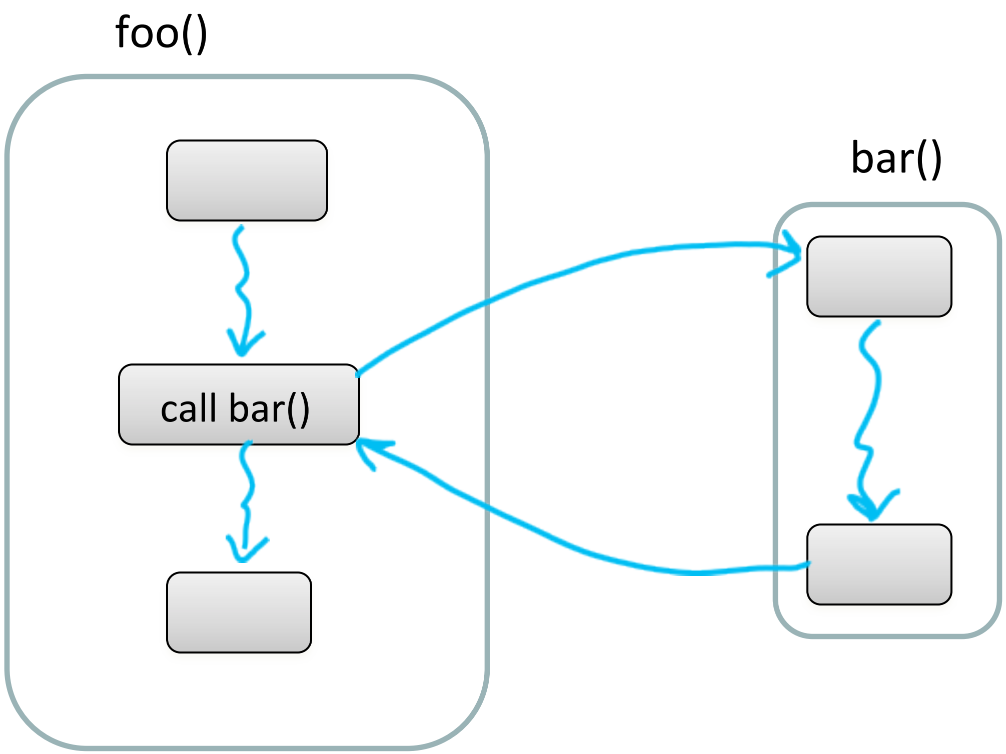
## Lecture 13: Interprocedural Shape Analysis

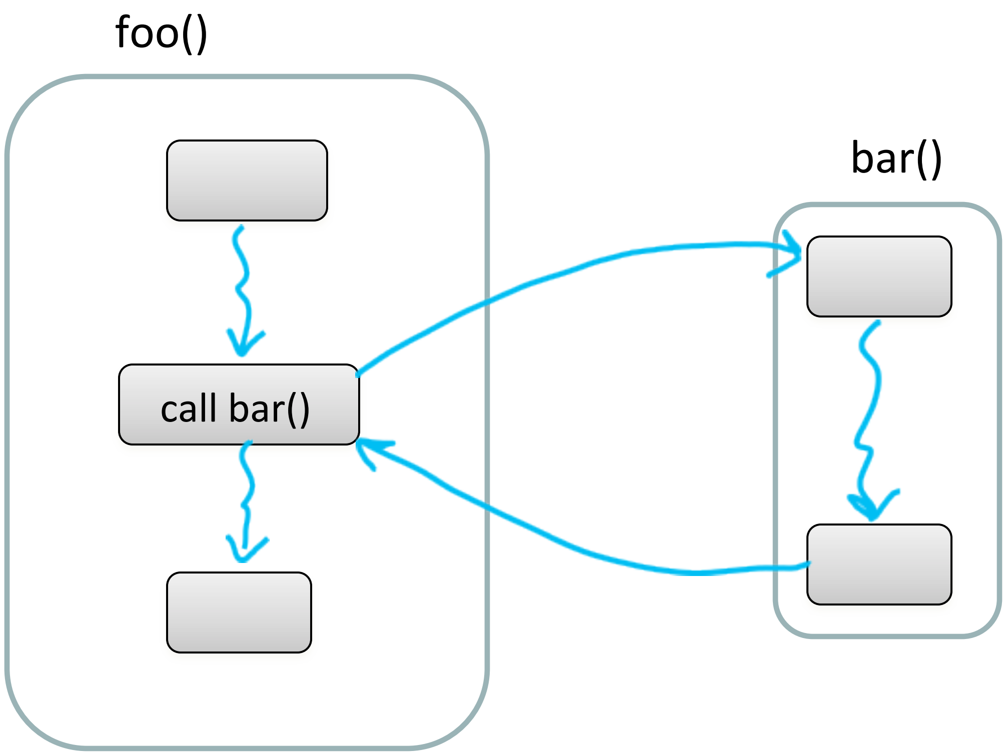
Slides credit: Roman Manevich, Mooly Sagiv, Eran Yahav

# Effect of procedures




The effect of calling a procedure is the effect of executing its body

# Interprocedural Analysis




goal: compute the abstract effect of calling a procedure

# A trivial treatment of procedure

- Analyze a single procedure
- After every call continue with conservative information
  - Global variables and local variables which “may be modified by the call” have unknown values
- Can be easily implemented
- Procedures can be written in different languages
- Procedure inline can help

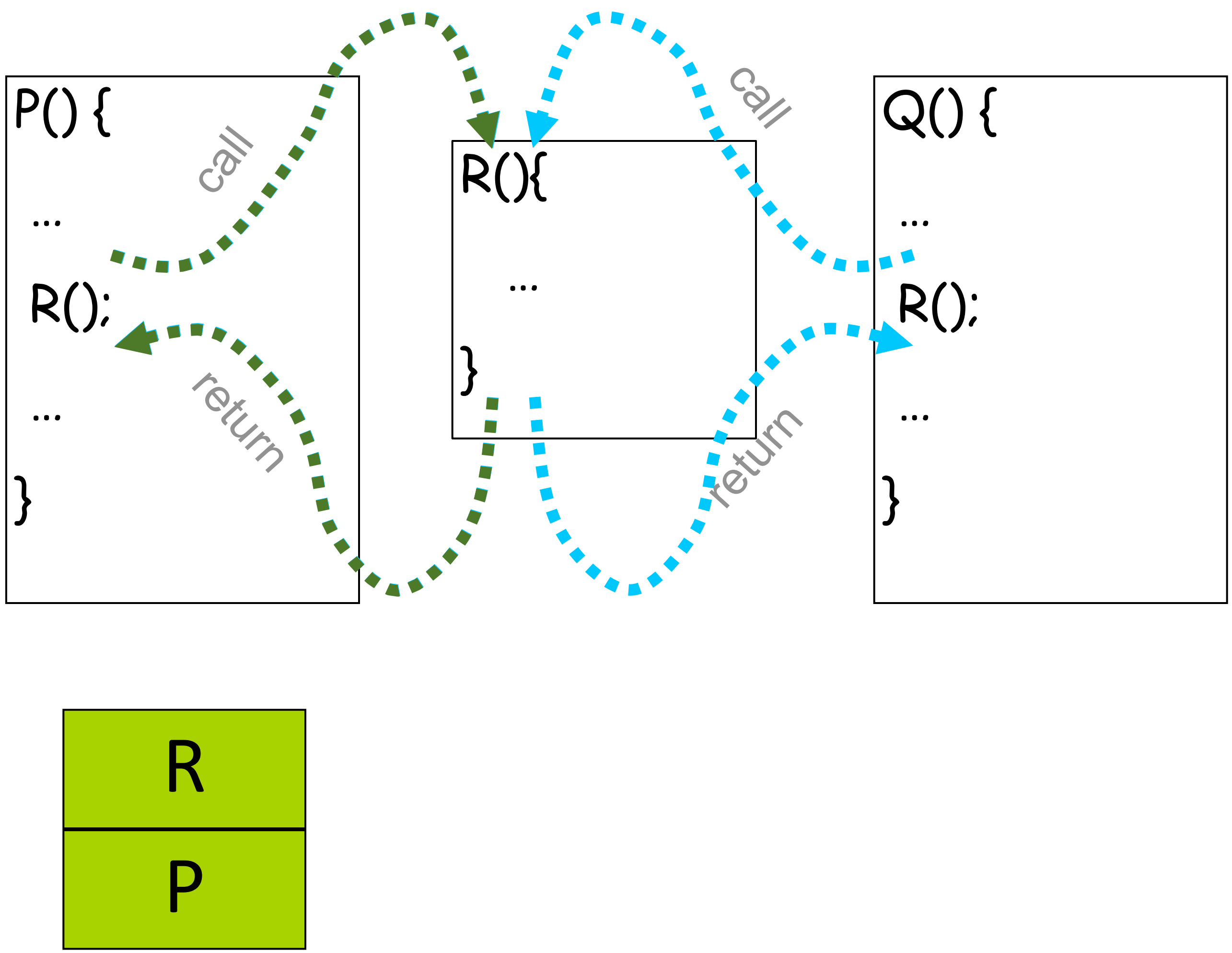
# Reduction to intraprocedural analysis

- Procedure inlining
- Naive solution: call-as-goto

# Interprocedural analysis: Guiding light

- Exploit stack regime
  - ➔Precision
  - ➔Efficiency

# Stack regime



# Simplifying Assumptions

- Parameter passed by value
- No procedure nesting
- No concurrency

✓ Recursion is supported

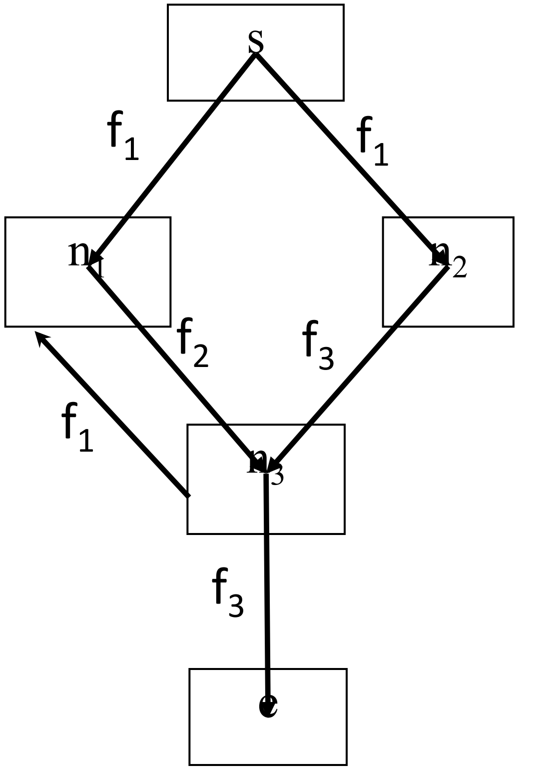
# The collecting lattice

- Lattice for a given control-flow node *v*:
  $L_v=(2^{\mathbf{State}}, \subseteq, \cup, \cap, \emptyset, \mathbf{State})$
- Lattice for entire control-flow graph with nodes *V*:

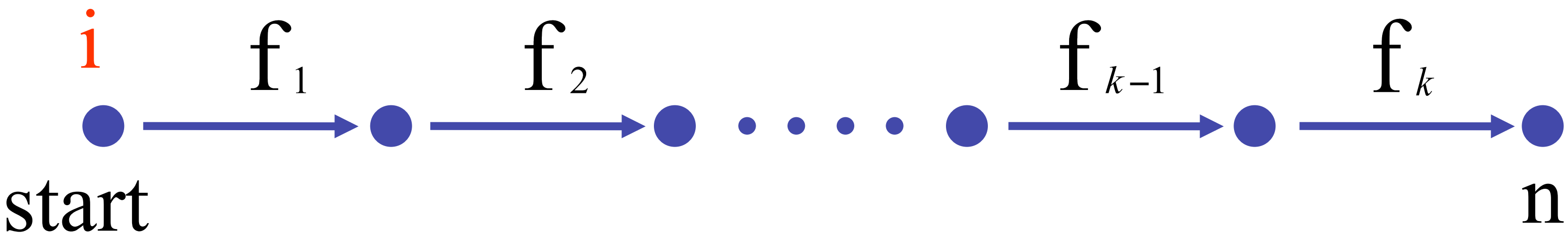
  $$L_{CFG} = Map(V, L_v)$$
- We will use this lattice as a baseline for static analysis and define abstractions of its elements

# Paths

- paths(n) the set of paths from s to n
  - ( $(s,n_1)$, $(n_1,n_3)$, $(n_3,n_1)$ )



# Join Over All Paths (JOP)



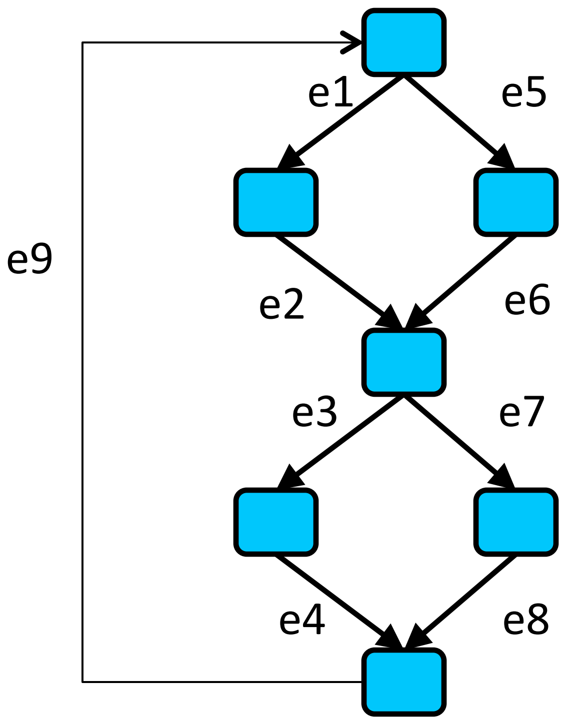

$$[\![ f_k \circ \ldots \circ f_1 ]\!] \in L \rightarrow L$$

- $JOP[v] = \sqcup\{[\![e_1, e_2, \ldots, e_n]\!](\iota) \mid (e_1, \ldots, e_n) \in paths(v)\}$
- $JOP \sqsubseteq LFP$
  - Sometimes JOP = LFP
    - precise up to "symbolic execution"
    - Distributive problem

# Join-Over-All-Paths (JOP)

- Let paths(v) denote the potentially infinite set paths from start to v (written as sequences of edges)
- For a sequence of edges $[e_1, e_2, \ldots, e_n]$ define $f\,[e_1, e_2, \ldots, e_n]: L \rightarrow L$ by composing the effects of basic blocks
  $f\,[e_1, e_2, \ldots, e_n](l) = f(e_n)\,(\ldots\,(f(e_2)\,(f(e_1)\,(l))\,\ldots)$
- $JOP[v] = \sqcup\{f\,[e_1, e_2, \ldots, e_n](\iota) \mid [e_1, e_2, \ldots, e_n] \in paths(v)\}$

# Join-Over-All-Paths (JOP)




Paths transformers:
f[e1,e2,e3,e4]
f[e1,e2,e7,e8]
f[e5,e6,e7,e8]
f[e5,e6,e3,e4]
f[e1,e2,e3,e4,e9, e1,e2,e3,e4]
f[e1,e2,e7,e8,e9, e1,e2,e3,e4,e9,…]
…

JOP:
f[e1,e2,e3,e4](initial) $\sqcup$
f[e1,e2,e7,e8](initial) $\sqcup$
f[e5,e6,e7,e8](initial) $\sqcup$
f[e5,e6,e3,e4](initial) $\sqcup$ …

Number of program paths is unbounded due to loops

# The lfp computation approximates JOP

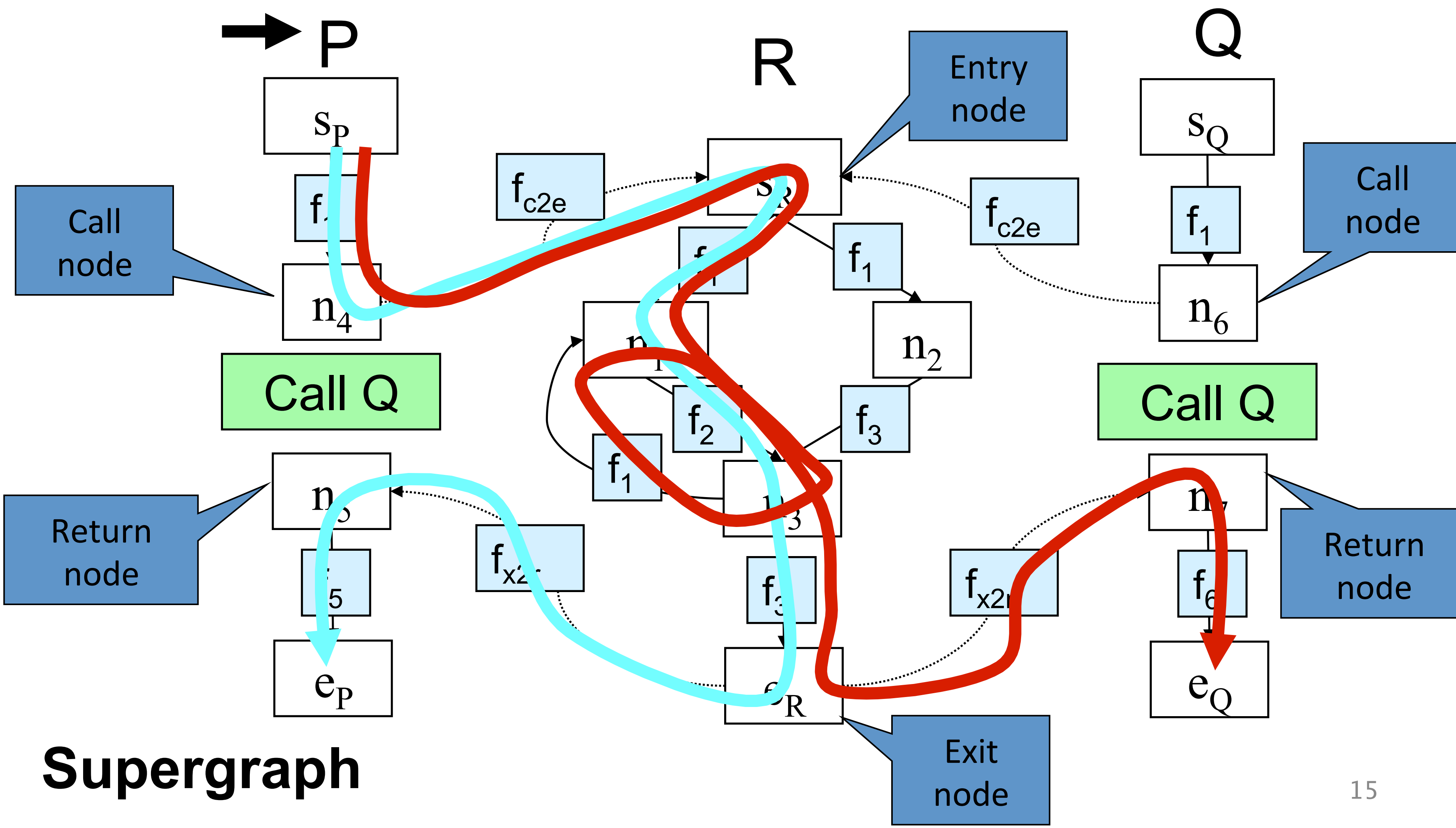
- JOP[v] $= \sqcup\{f\,[e_1, e_2, \ldots, e_n](\iota) \mid [e_1, e_2, \ldots, e_n] \in \text{paths}(v)\}$
- LFP[v] $= \sqcup\{f\,[e](\text{LFP}[v']) \mid e = (v', v)\}$

  LFP[$v_0$] $= \iota$

- JOP $\sqsubseteq$ LFP - for a monotone function
  - $f(x \sqcup y) \sqsupseteq f(x) \sqcup f(y)$
- JOP = LFP - for a distributive function
  - $f(x \sqcup y) = f(x) \sqcup f(y)$

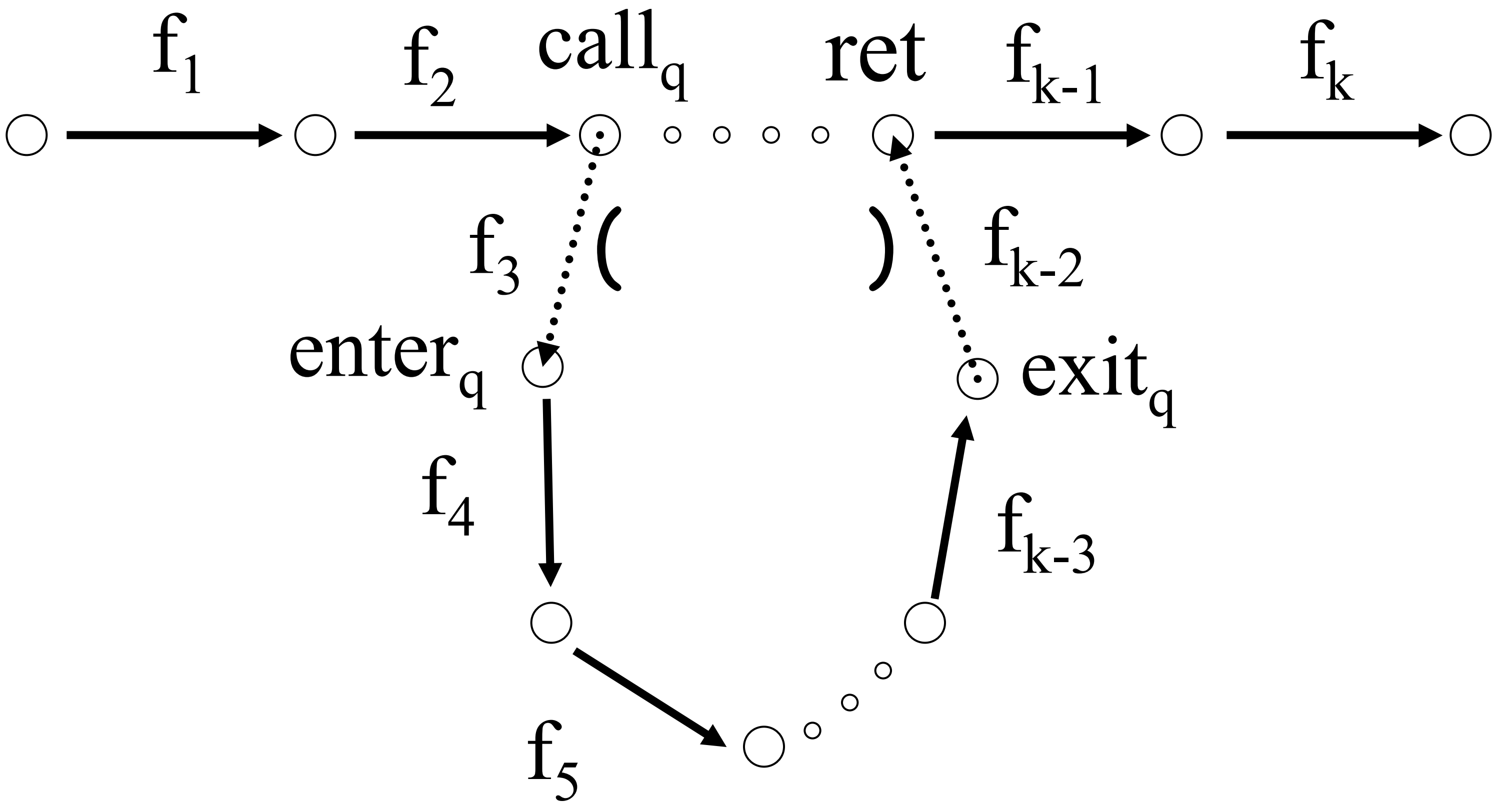
JOP may not be precise enough for interprocedural analysis!

# Interprocedural analysis




**Supergraph**

# Interprocedural Valid Paths




- IVP: all paths with matching calls and returns
  – And prefixes

# Interprocedural Valid Paths

- IVP set of paths
  - Start at program entry
- Only considers matching calls and returns
  - aka, valid
- Can be defined via context free grammar
  - matched ::= matched $(_i$ matched $)_i$ | ε
  - valid ::= valid $(_i$ matched | matched
    - paths can be defined by a regular expression

# The Join-Over-Valid-Paths (JVP)

- vpaths(n) all valid paths from program start to n
- JVP[n] $= \sqcup\{[[e_1, e_2, \ldots, e]](\iota)$
  $(e_1, e_2, \ldots, e) \in$ vpaths(n)$\}$
- JVP $\sqsubseteq$ JOP
  - In some cases the JVP can be computed
  - (Distributive problem)

# The Call String Approach

- The data flow value is associated with sequences of calls (call string)
- Use Chaotic iterations over the supergraph

# Summary Call String

- Easy to implement
- Efficient for very small call strings
- Limited precision
  - Often loses precision for recursive programs
  - For finite domains can be precise even with recursion (with a bounded callstring)

- Order of calls can be abstracted
- Related method: procedure cloning

# The Functional Approach

- The meaning of a procedure is mapping from states into states
- The abstract meaning of a procedure is function from an abstract state to abstract states
- Relation between input and output
- In certain cases can compute JVP

# The Functional Approach

- Two phase algorithm
  - Compute the dataflow solution at the exit of a procedure as a function of the initial values at the procedure entry (functional values)
  - Compute the dataflow values at every point using the functional values

# Phase 1

```
void main() {

  p(7);

}
```

p($a_0$,$x_0$) = $[a \mapsto a_0, x \mapsto -2a_0 + 5]$

```
int p(int a) {
```

$[a \mapsto a_0, x \mapsto x_0]$

```
 if (…) {
```

$[a \mapsto a_0, x \mapsto x_0]$

```
   a = a -1 ;
```

$[a \mapsto a_0\text{-}1, x \mapsto x_0]$

```
   p (a);
```

$[a \mapsto a_0\text{-}1, x \mapsto \text{-}2a_0+7]$

```
   a = a + 1;
```

$[a \mapsto a_0, x \mapsto \text{-}2a_0+7]$

```
 }
```

$[a \mapsto a_0, x \mapsto x_0]$ $[a \mapsto a_0, x \mapsto \top]$

```
 x = -2*a + 5;
```

$[a \mapsto a_0, x \mapsto \text{-}2*a_0+5]$

```
}
```

# Phase 2

```
void main() {
  p(7);
  [x ↦ -9]
}
```

$p(a_0,x_0) = [a \mapsto a_0,\ x \mapsto -2a_0 + 5]$

```
int p(int a) {
  [a ↦7, x ↦0]          [a ↦⊤, x ↦0]
  if (…) {
    [a ↦7, x ↦0]        [a ↦⊤, x ↦0]
    a = a -1 ;
    [a ↦6, x ↦0]        [a ↦⊤, x ↦0]
    p (a);
    [a ↦6, x ↦-7]       [a ↦⊤, x ↦⊤]
    a = a + 1;
    [a ↦7, x ↦-7]       [a ↦⊤, x ↦⊤]
  }
  [a ↦7, x ↦0]          [a ↦⊤, x ↦⊤]
  x = -2*a + 5;
  [a ↦7, x ↦-9]         [a ↦⊤, x ↦⊤]
}
```

# Summary Functional approach

- Computes procedure abstraction
- Sharing between different contexts
- Rather precise
- Recursive procedures may be more precise/efficient than loops
- But requires more from the implementation
  - Representing (input/output) relations
  - Composing relations

# Issues in Functional Approach

- How to guarantee that finite height for functional lattice?
  - It may happen that L has finite height and yet the lattice of monotonic function from L to L do not
- Efficiently represent functions
  - Functional join
  - Functional composition
  - Testing equality

# Tabulation

- Special case: L is finite
- Data facts: $d \in L \times L$
- Initialization:
  - $f_{start,start} = (\top,\top)$ ; otherwise $(\bot,\bot)$
  - $S[start, \top] = \top$

- Propagation of (x,y) over edge e = (n,n')
  - Maintain summary: $S[n',x] = S[n',x] \sqcup [\![n]\!](y))$
  - n intra-node: ➔ n' : $(x, [\![n]\!](y))$
  - n call-node:
    - ➔ n' : (y,y) if $S[n',y] = \bot$ and n' = entry node
    - ➔ n' : (x,z) if S[exit(call(n),y] = z and n' = ret-site-of n
- n return-node: ➔ n' : (u,y) ; $n_c$ = call-site-of n', $S[n_c,u]=x$

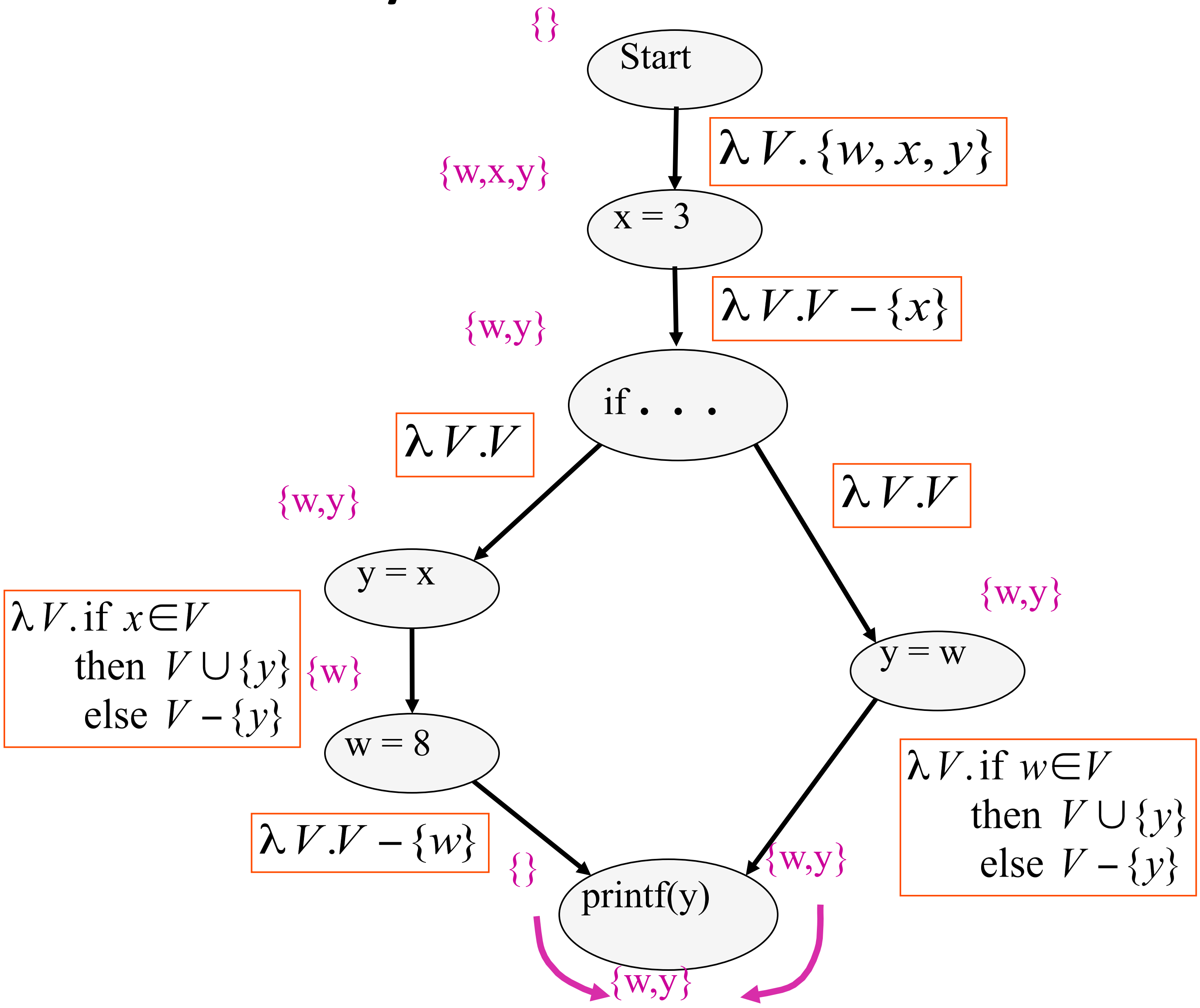
# CFL-Graph reachability

- Special cases of functional analysis
- Finite distributive lattices
- Provides more efficient analysis algorithms
- Reduce the interprocedural analysis problem to finding context free reachability

# IDFS / IDE

- IDFS Interprocedural Distributive Finite Subset Precise interprocedural dataflow analysis via graph reachability. Reps, Horowitz, and Sagiv, POPL'95
- IDE Interprocedural Distributive Environment Precise interprocedural dataflow analysis with applications to constant propagation. Reps, Horowitz, and Sagiv, FASE'95, TCS'96
  - More general solutions exist

# Possibly Uninitialized Variables



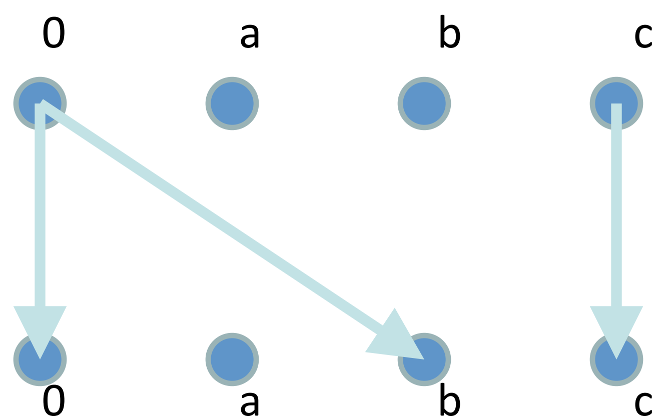
# IFDS Problems

- Finite subset distributive
  - Lattice L = $\wp$(D)
  - $\sqsubseteq$ is $\subseteq$
  - $\sqcup$ is $\cup$
  - Transfer functions are distributive

- Efficient solution through formulation as CFL reachability

# Encoding Transfer Functions

- Enumerate all input space and output space
- Represent functions as graphs with 2(D+1) nodes
- Special symbol "0" denotes empty sets (sometimes denoted $\Lambda$)
- Example: D = { a, b, c }
  f(S) = (S – {a}) U {b}

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# Efficiently Representing Functions

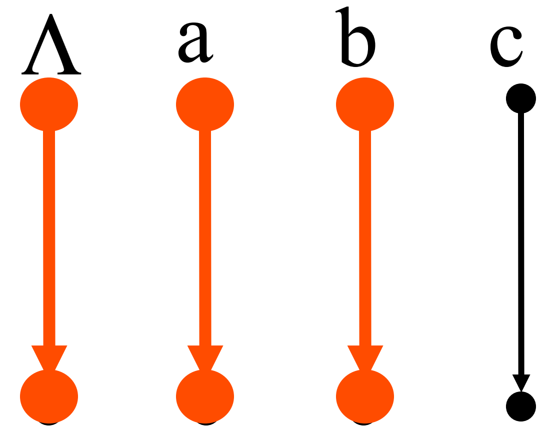
- Let $f: 2^D \rightarrow 2^D$ be a distributive function
- Then:
  - $f(X) = f(\emptyset) \cup (\cup \{ f(\{z\}) \mid z \in X \})$
  - $f(X) = f(\emptyset) \cup (\cup \{ f(\{z\}) \setminus f(\emptyset) \mid z \in X \})$

# Representing Dataflow Functions

Identity Function

$f = \lambda V.V$

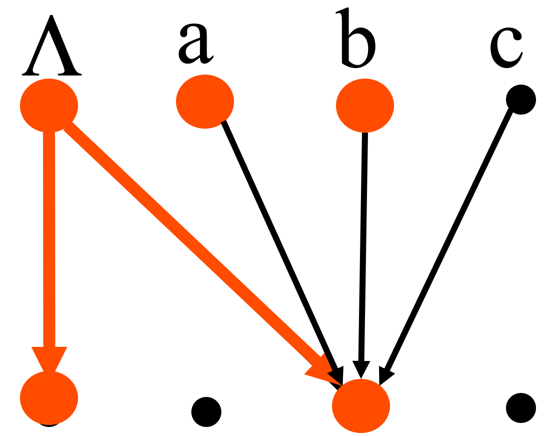
$f(\{a,b\}) = \{a,b\}$




Constant Function

$f = \lambda V.\{b\}$

$f(\{a,b\}) = \{b\}$

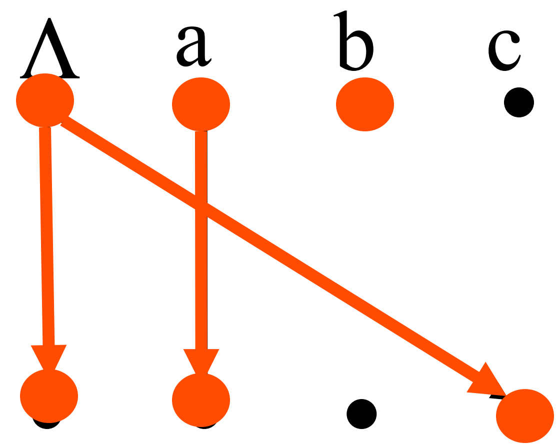


# Representing Dataflow Functions

"Gen/Kill" Function

$$f = \lambda V.(V - \{b\}) \cup \{c\}$$

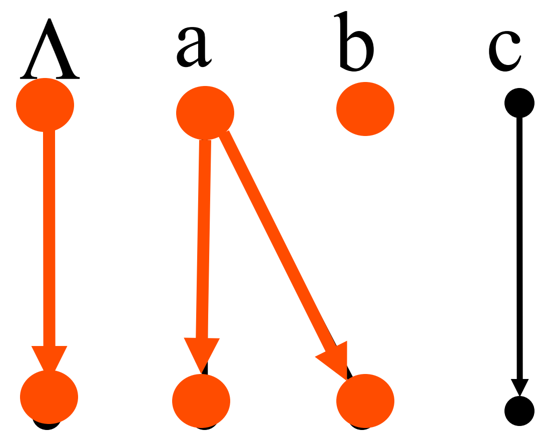
$$f(\{a,b\}) = \{a,c\}$$

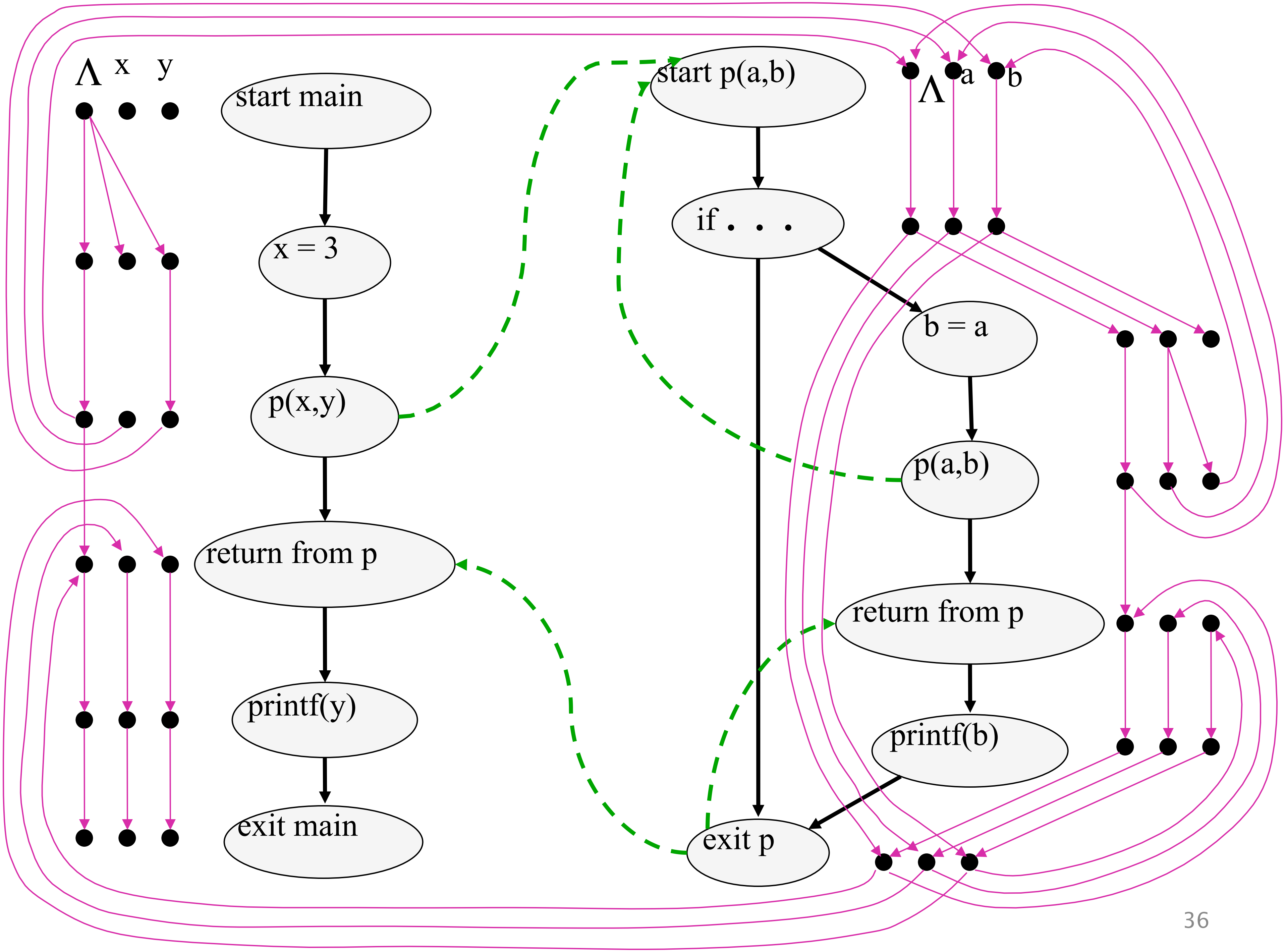



Non-"Gen/Kill" Function

$$f = \lambda V.\text{if } a \in V \text{ then } V \cup \{b\} \text{ else } V - \{b\}$$

$$f(\{a,b\}) = \{a,b\}$$

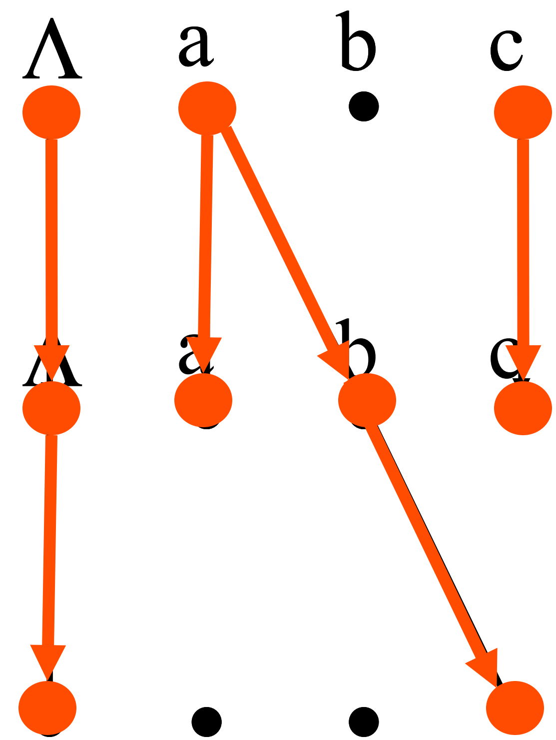



Λ
x
y
start main
x = 3
p(x,y)
return from p
printf(y)
exit main
start p(a,b)
if . . .
b = a
p(a,b)
return from p
printf(b)
exit p
Λ
a
b

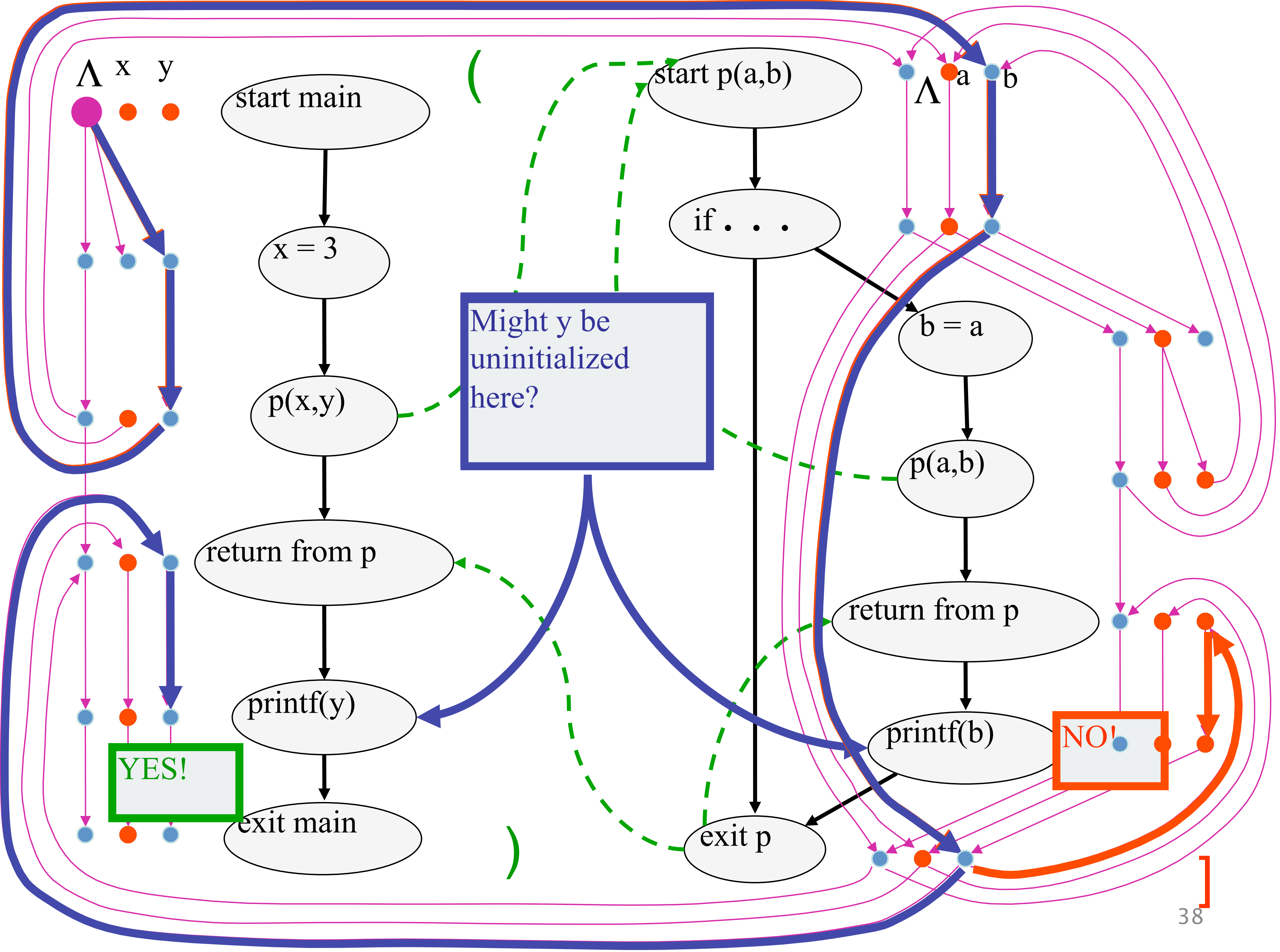
# Composing Dataflow Functions

$$f_1 = \lambda V.\text{if } a \in V \text{ then } V \cup \{b\} \text{ else } V - \{b\}$$

$$f_2 = \lambda V.\text{if } b \in V \text{ then } \{c\} \text{ else } \phi$$




$$f_2 \circ f_1(\{a, c\}) = \boxed{\{c\}}$$



Λ x y
start main
x = 3
p(x,y)
return from p
printf(y)
exit main
YES!
Might y be uninitialized here?
start p(a,b)
Λ a b
if . . .
b = a
p(a,b)
return from p
printf(b)
exit p
NO!

# The Tabulation Algorithm

- Worklist algorithm, start from entry of "main"
- Keep track of
  - Path edges: matched paren paths from procedure entry
  - Summary edges: matched paren call-return paths
- At each instruction
  - Propagate facts using transfer functions; extend path edges
- At each call
  - Propagate to procedure entry, start with an empty path
  - If a summary for that entry exits, use it
- At each exit
  - Store paths from corresponding call points as summary paths
  - When a new summary is added, propagate to the return node

# Interprocedural Dataflow Analysis via CFL-Reachability

- Graph: Exploded control-flow graph
- L: L(unbalLeft)
  - unbalLeft = valid
- Fact d holds at n iff there is an L(unbalLeft)-path from $\langle start_{main}, \Lambda \rangle$ to $\langle n, d \rangle$

# Asymptotic Running Time

- CFL-reachability
  - Exploded control-flow graph: ND nodes
  - Running time: $O(N^3D^3)$
- Exploded control-flow graph → Special structure

Running time: $O(ED^3)$

Typically: $E \approx N$, hence $O(ED^3) \approx O(ND^3)$

"Gen/kill" problems: $O(ED)$

# IDE

- Goes beyond IFDS problems
  – Can handle unbounded domains
- Requires special form of the domain
- Can be much more efficient than IFDS

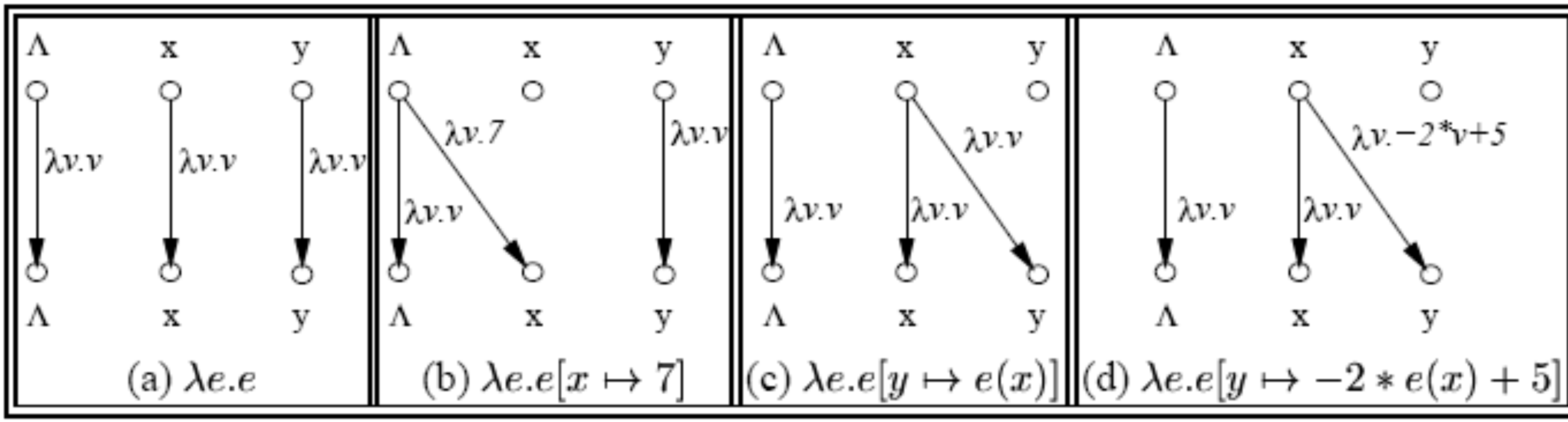
# Example Linear Constant Propagation

- Consider the constant propagation lattice
- The value of every variable y at the program exit can be represented by:

  $y = \sqcup \{(a_x x + b_x) \mid x \in Var_* \} \sqcup c$

  $a_x, c \in Z \cup \{\bot, \top\} \quad b_x \in Z$
- Supports efficient composition and "functional" join
  - [z := a * y + b]
  - What about [z:=x+y]?

# Linear constant propagation



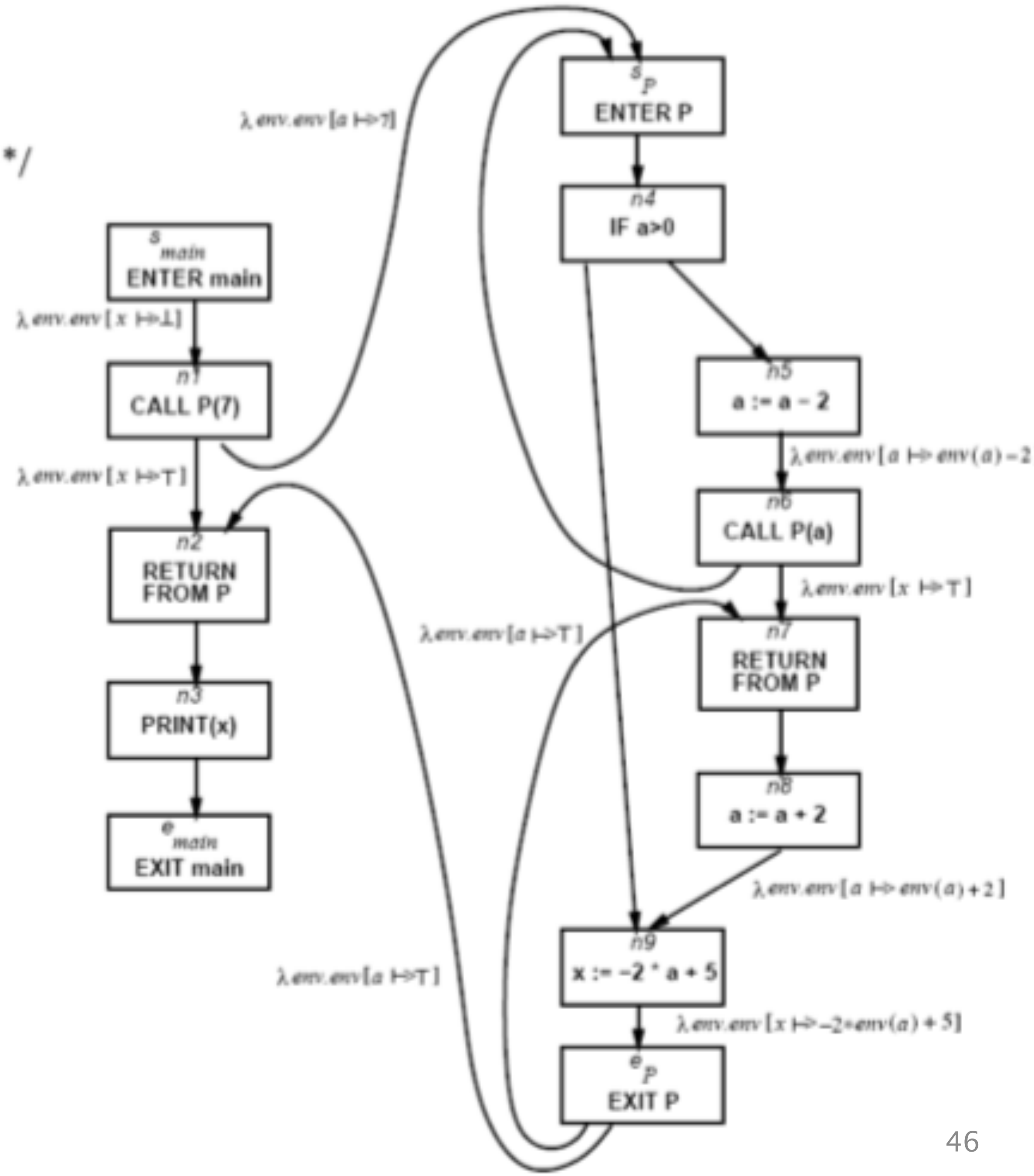

Point-wise representation of environment transformers

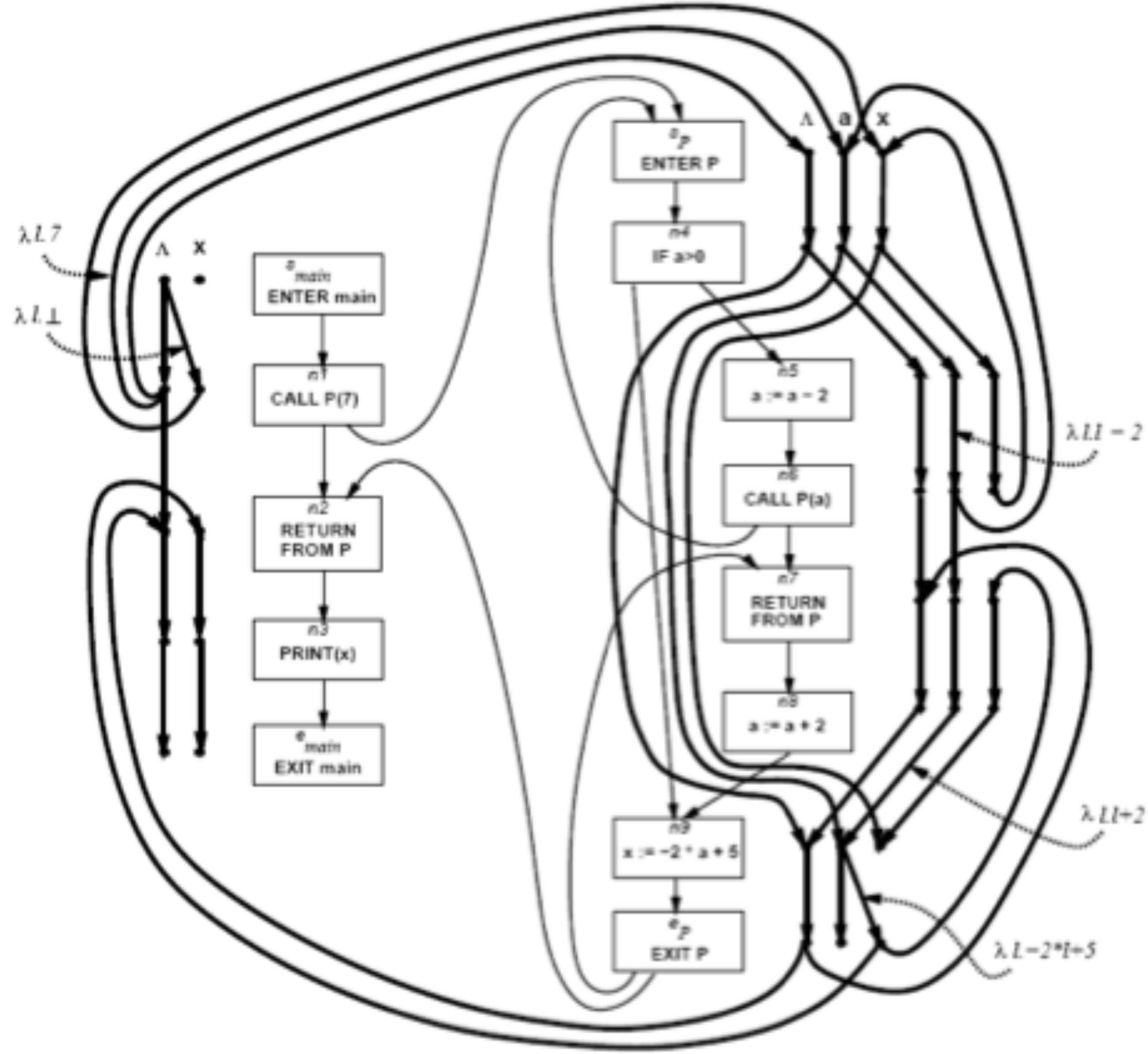
# IDE Analysis

- Point-wise representation closed under composition
- CFL-Reachability on the exploded graph
- Compose functions

```
declare x: integer
program main
begin
        call P(7)
        print (x) /* x is a constant here */
end

procedure P (value a : integer)
begin /* a is not a constant here */
        if a > 0 then
           a := a − 2
           call P (a)
           a := a + 2
        fi
        x := −2 * a + 5
        /* x is not a constant here */
end
```




λl.7
λl.⊥
Λ
x
$e_{main}$
ENTER main
n1
CALL P(7)
n2
RETURN FROM P
n3
PRINT(x)
$e_{main}$
EXIT main
$e_P$
ENTER P
n4
IF a>0
n5
a := a - 2
n6
CALL P(a)
n7
RETURN FROM P
n8
a := a + 2
n9
x := -2 * a + 5
$e_P$
EXIT P
Λ
a
x
λl.l - 2
λl.l+2
λl.-2*l+5

# Costs

- $O(ED^3)$
- Class of value transformers $F \subseteq L \rightarrow L$
  - $id \in F$
  - Finite height
- Representation scheme with (efficient)
    - Application
    - Composition
    - Join
    - Equality
    - Storage

# Conclusion

- Handling functions is crucial for abstract interpretation
- Virtual functions and exceptions complicate things
- But scalability is an issue
  - Small call strings
  - Small functional domains
  - Demand analysis

# Challenges in Interprocedural Analysis

- Respect call-return mechanism
- Handling recursion
- Local variables
- Parameter passing mechanisms
- The called procedure is not always known
- The source code of the called procedure is not always available

# Bibliography


- Textbook 2.5
- Patrick Cousot & Radhia Cousot. Static determination of dynamic properties of recursive procedures In IFIP Conference on Formal Description of Programming Concepts, E.J. Neuhold, (Ed.), pages 237-277, St-Andrews, N.B., Canada, 1977. North-Holland Publishing Company (1978).
- Two Approaches to interprocedural analysis by Micha Sharir and Amir Pnueli
- IDFS Interprocedural Distributive Finite Subset Precise interprocedural dataflow analysis via graph reachability. Reps, Horowitz, and Sagiv, POPL' 95
- IDE Interprocedural Distributive Environment Precise interprocedural dataflow analysis with applications to constant propagation. Sagiv, Reps, Horowitz, and TCS' 96

# Disadvantages of the trivial solution

- Modular (object oriented and functional) programming  encourages small frequently called procedures
- Almost all information is lost

# A Semantics for Procedure Local Heaps and its Abstractions

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Mooly Sagiv Tel Aviv University

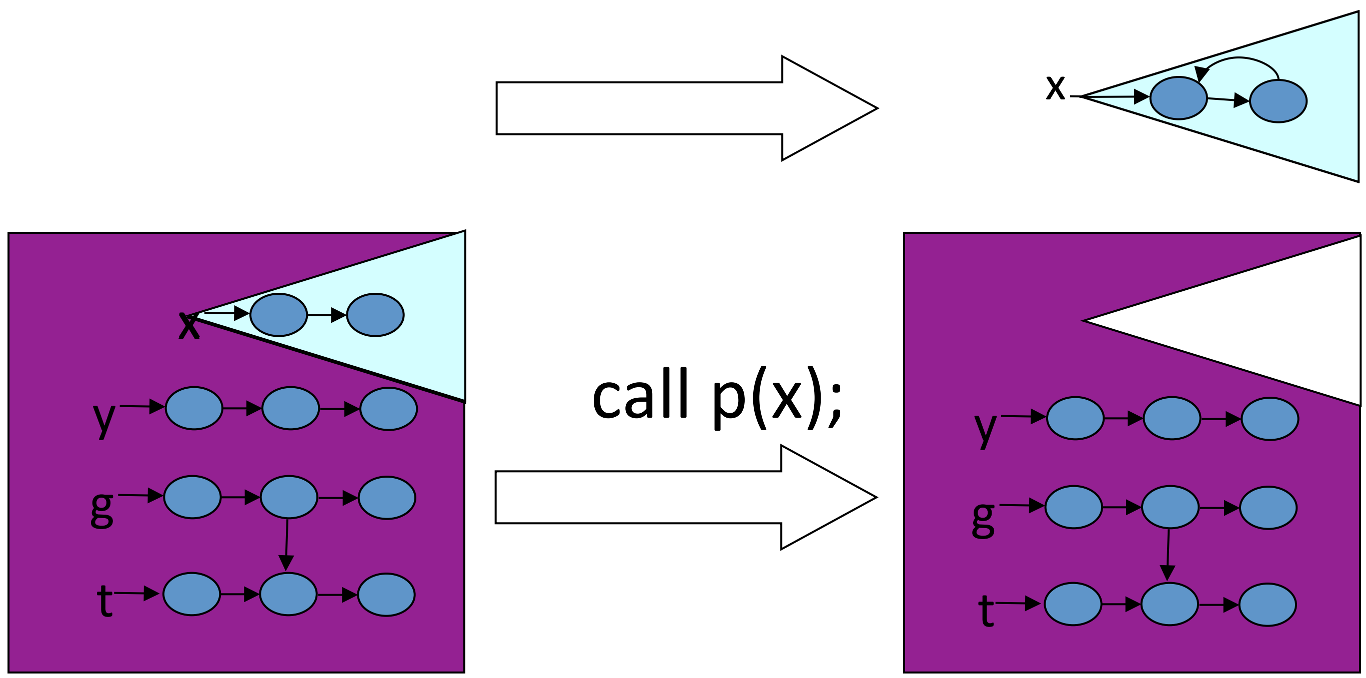
Reinhard Wilhelm Universität des Saarlandes

# Motivation

- Interprocedural shape analysis
  - Conservative static pointer analysis
  - Heap intensive programs
    - Imperative programs with procedures
    - Recursive data structures
- Challenge
  - Destructive update
  - Localized effect of procedures

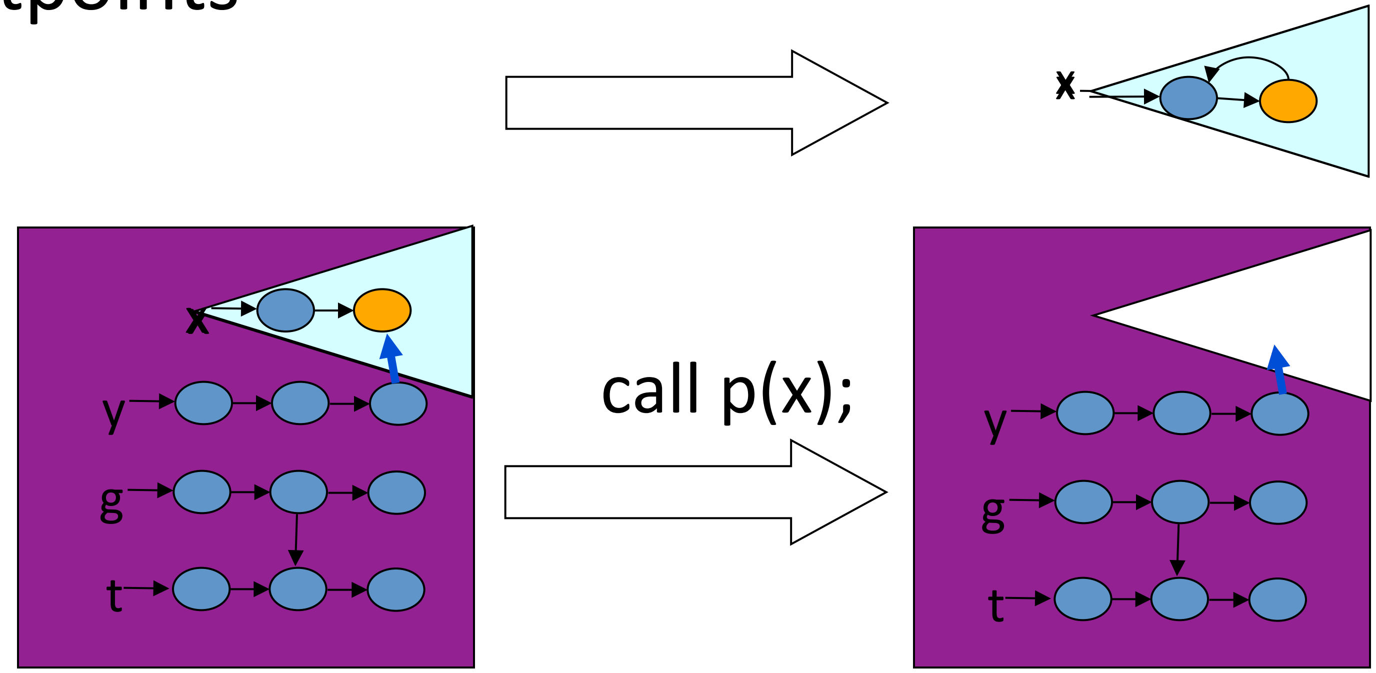
# Main idea

- Local heaps



# Main idea

- Local heaps
- Cutpoints



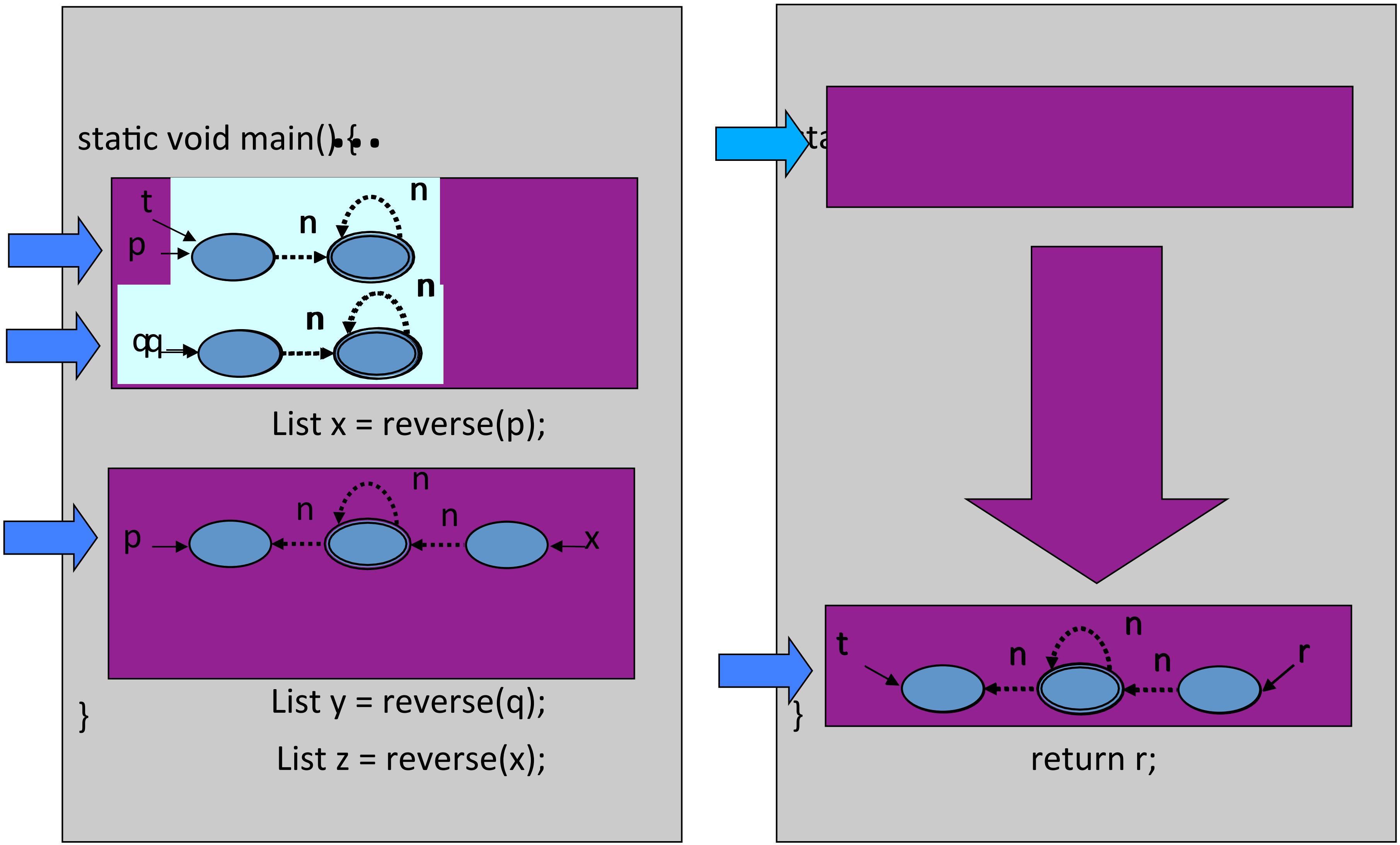
# Main Results

- Concrete operational semantics
  - Large step
    - Functional analysis
  - Storeless
    - Shape abstractions
  - Local heap
  - Observationally equivalent to "standard" semantics
    - Java and "clean" C
- Abstractions
  - Shape analysis [Sagiv, Reps, Wilhelm, TOPLAS '02]
  - May-alias [Deutsch, PLDI '94]
  - …

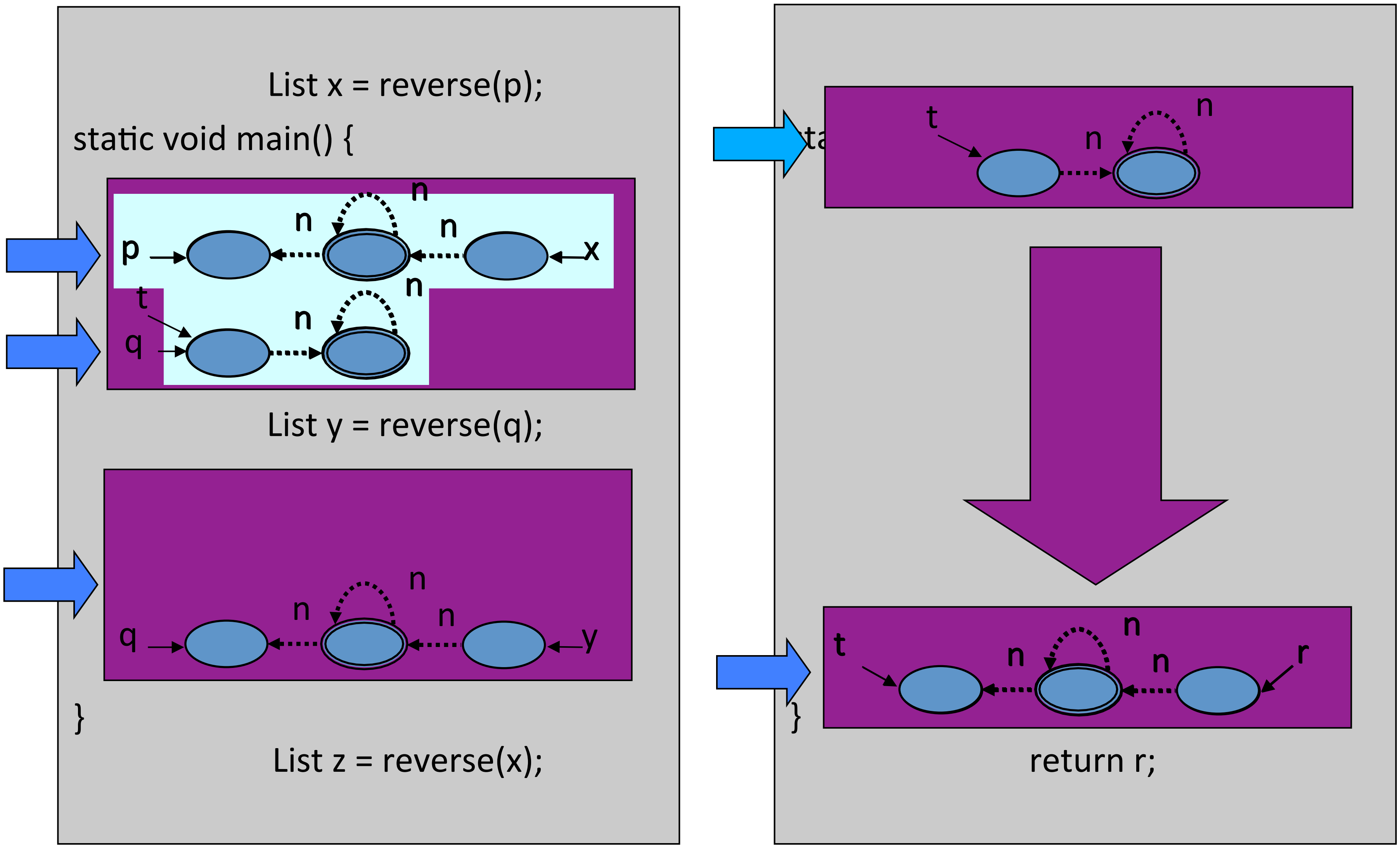
# Outline

- Motivating example
  - Local heaps
  - Cutpoints
- Why semantics
- Local heap storeless semantics
- Shape abstraction

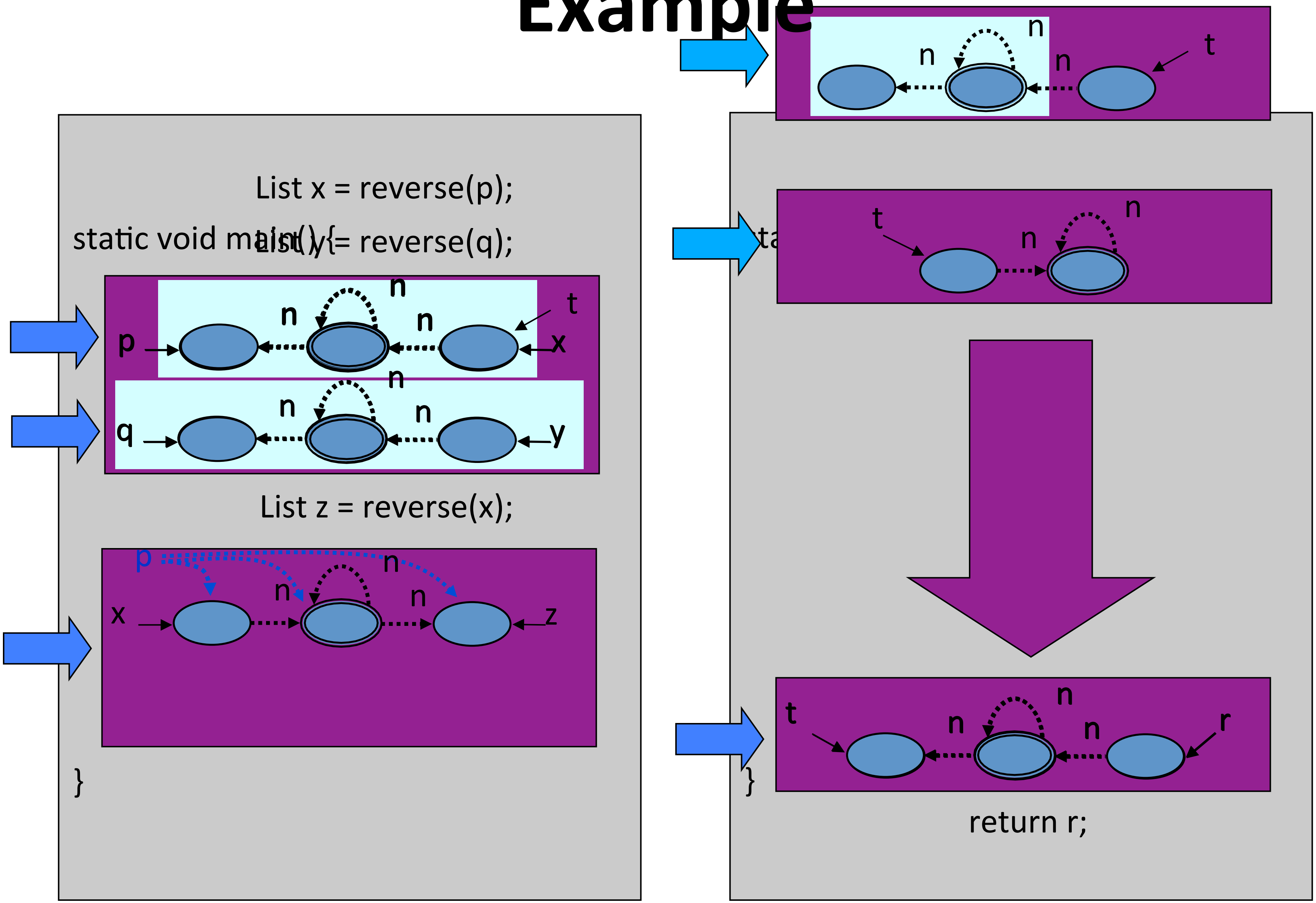
# Example


static void main() {
t
p
n
q
List x = reverse(p);
p
n
x
List y = reverse(q);
List z = reverse(x);
}
t
n
r
}
return r;

# Example


List x = reverse(p);
static void main() {
p
n
n
n
x
t
q
n
n
List y = reverse(q);
q
n
n
n
y
}
List z = reverse(x);
t
n
n
ta
t
n
n
n
r
}
return r;

# Example


List x = reverse(p);
static void main() {
List y = reverse(q);
p
q
n
t
x
y
List z = reverse(x);
z
}
return r;
r

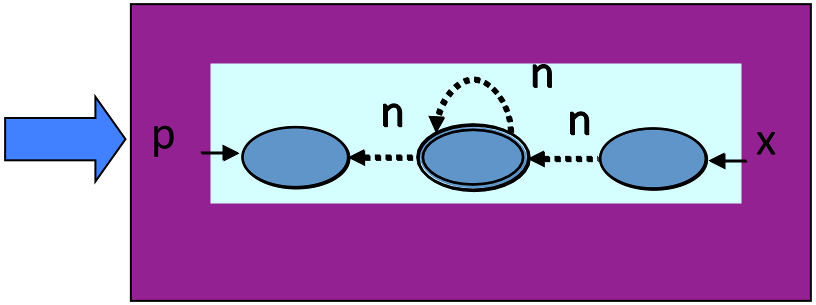
# Cutpoints

- Separating objects
  - Not pointed-to by a parameter

# Cutpoints

- Separating objects
  - Not pointed-to by a parameter

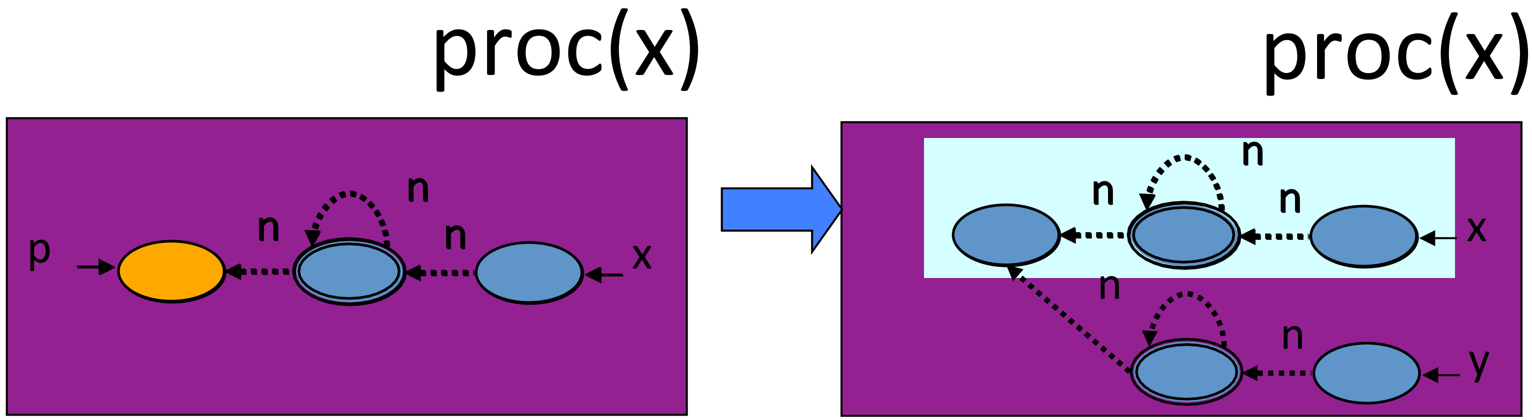
proc(x)




Stack sharing

# Cutpoints

- Separating objects
  - Not pointed-to by a parameter

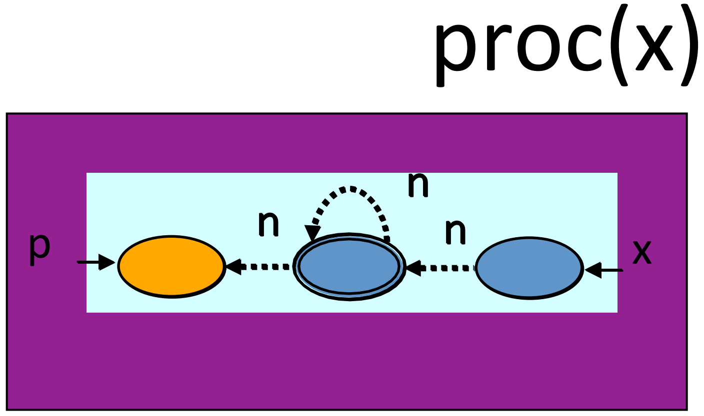



Stack sharing

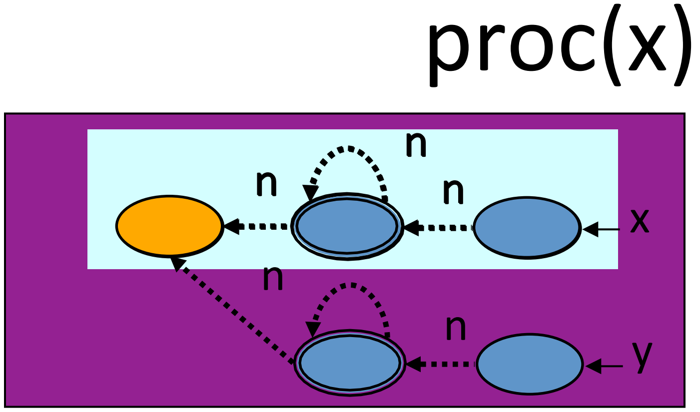
Heap sharing

# Cutpoints

- Separating objects
  - Not pointed-to by a parameter
- Capture external sharing patterns

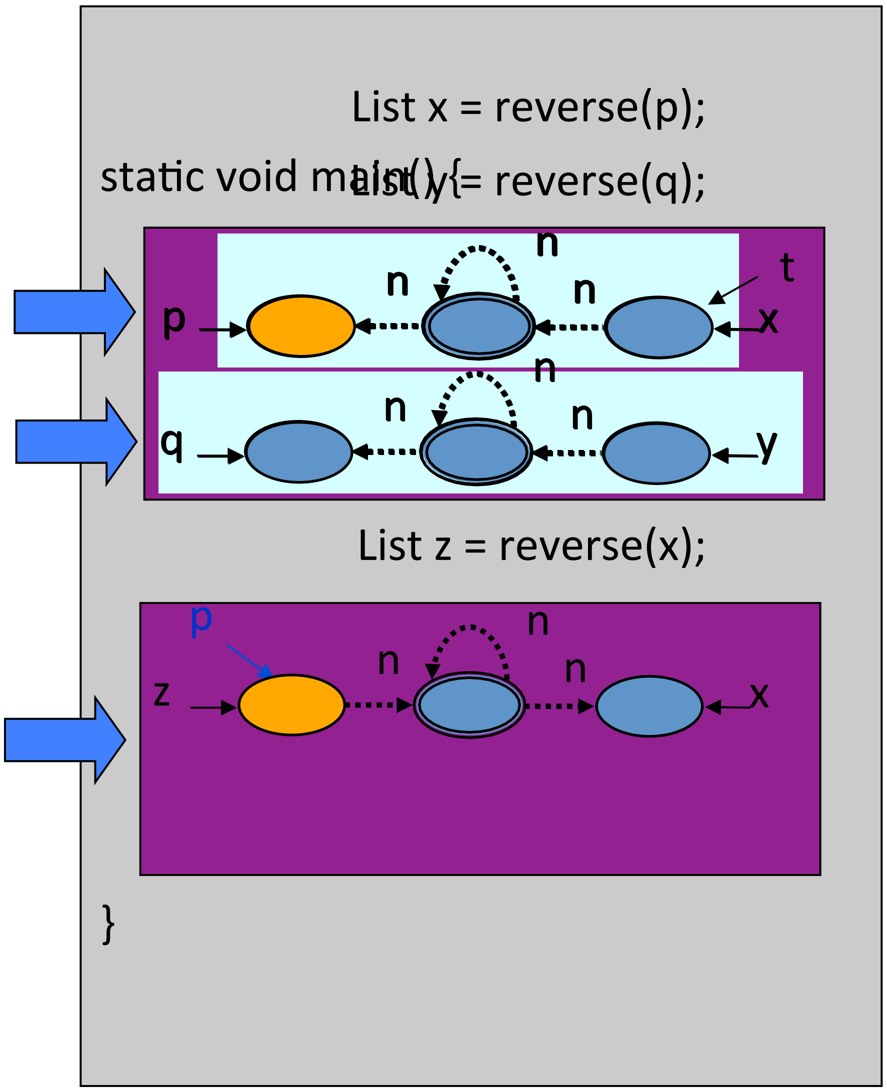


Stack sharing

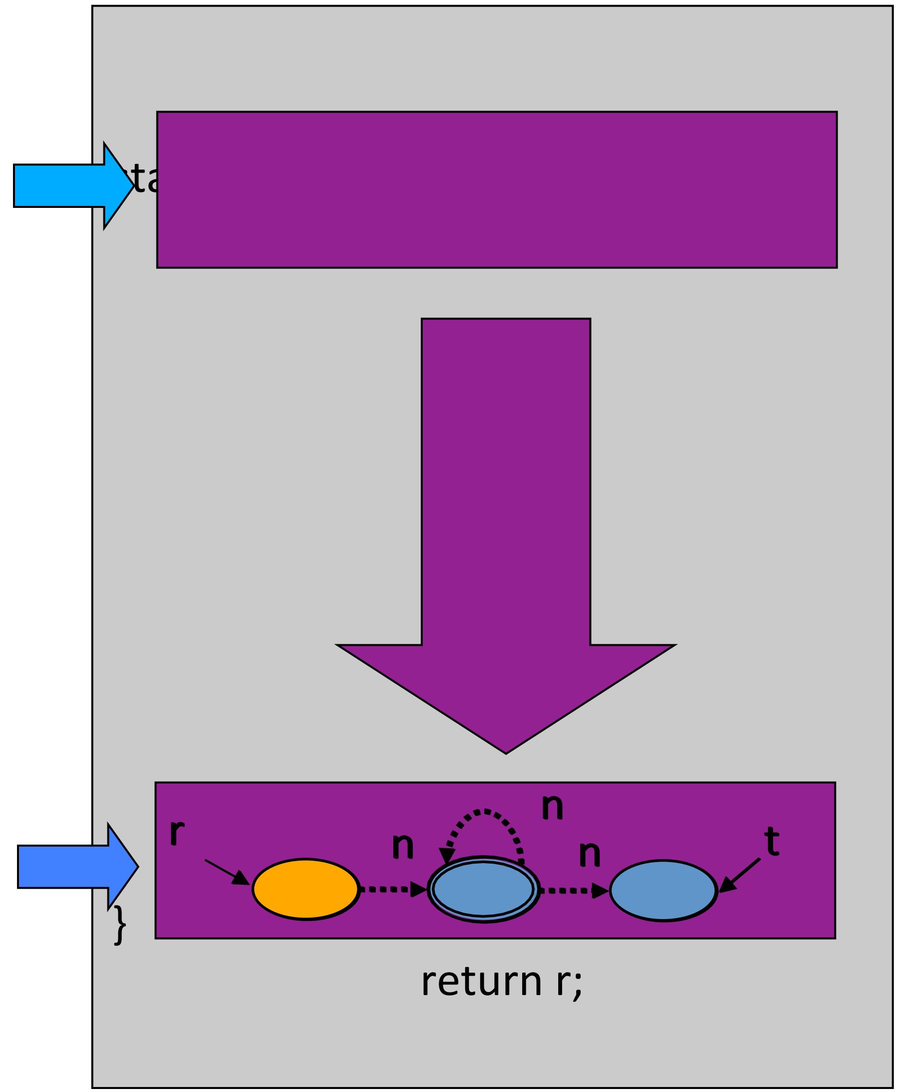


Heap sharing

# Example



List x = reverse(p);
static void main() {
List y = reverse(q);
p
n
n
n
t
x
q
n
n
n
y
List z = reverse(x);
p
z
n
n
n
x
}

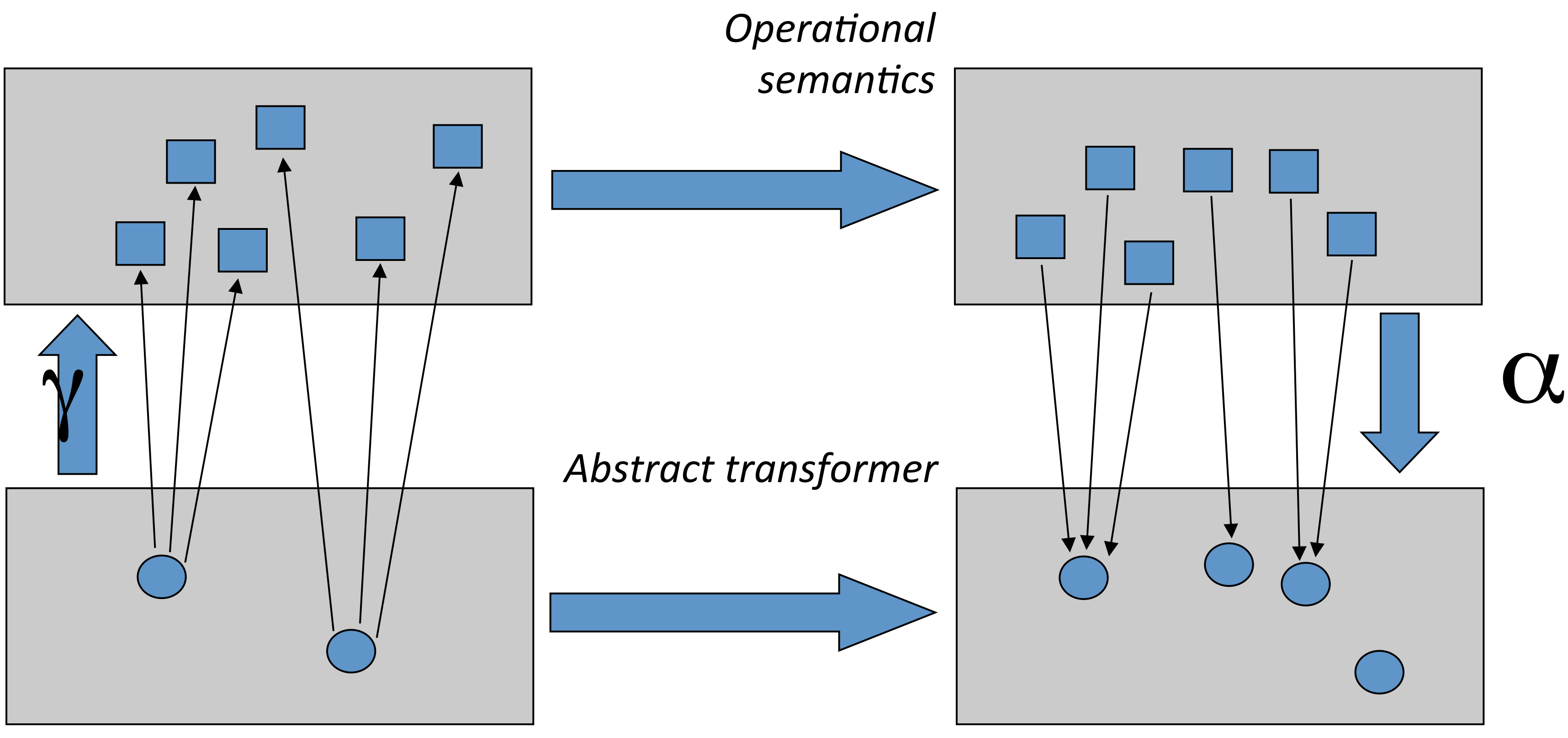



r
n
n
n
t
}
return r;

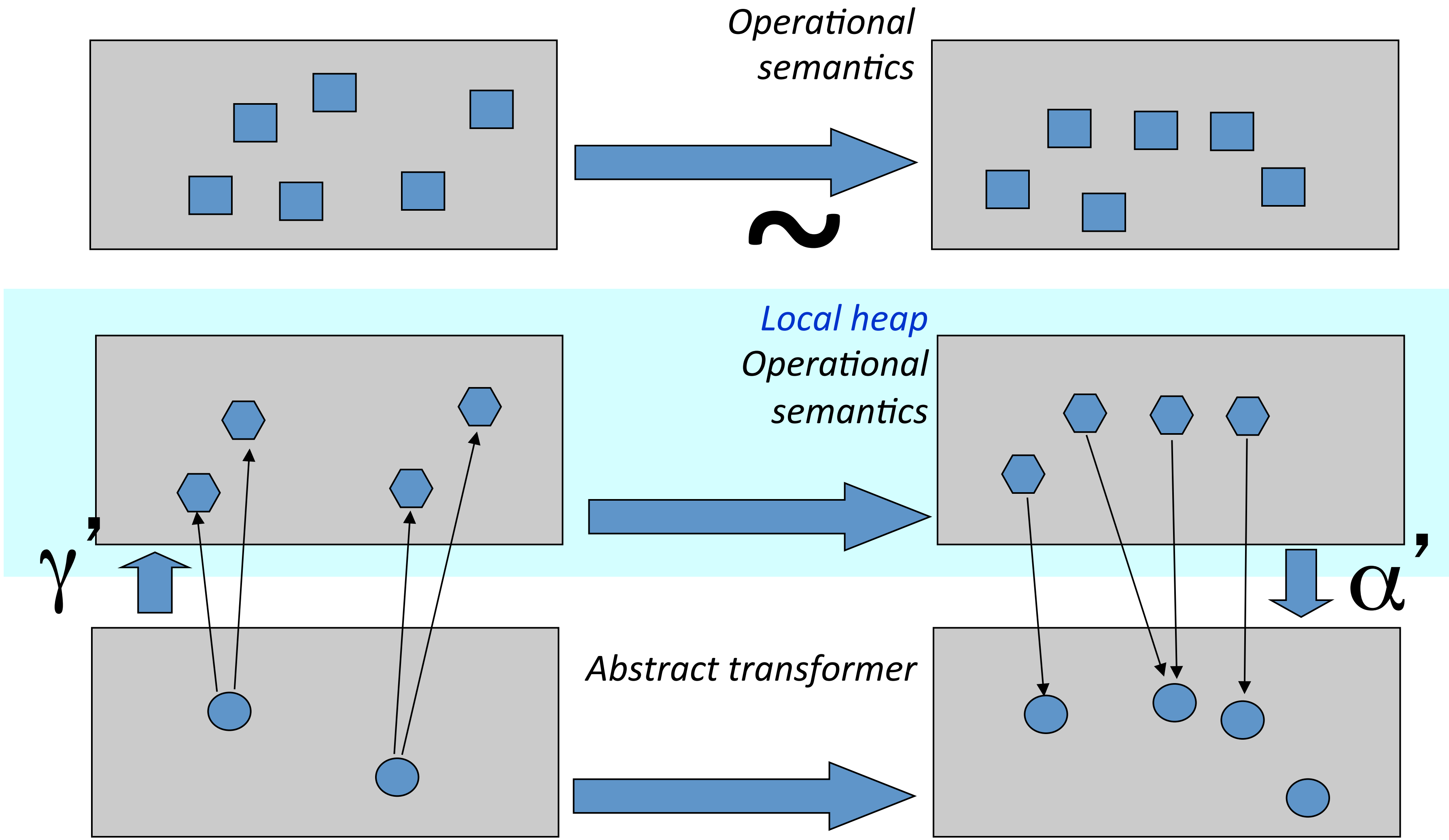
# Outline

✓ Motivating example
- Why semantics
- Local heap storeless semantics
- Shape abstraction

# Abstract Interpretation
## [Cousot and Cousot, POPL’77]



# Introducing local heap semantics



# Outline

- ✓ Motivating example
- ✓ Why semantics
- Local heap storeless semantics
- Shape abstraction

# Programming model

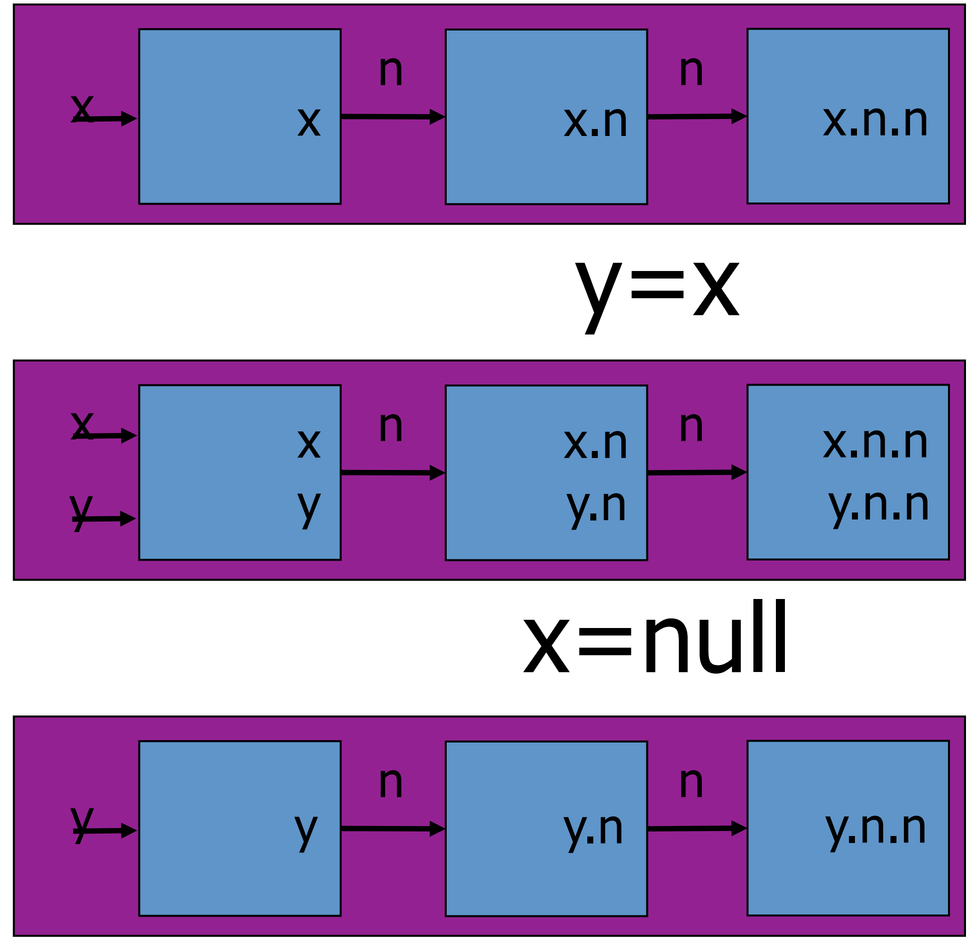
- Single threaded
- Procedures
  - ✓ Value parameters
  - ✓ Recursion
- Heap
  - ✓ Recursive data structures
  - ✓ Destructive update
  - ✘ No explicit addressing (&)
  - ✘ No pointer arithmetic

# Simplifying assumptions

- No primitive values (only references)
- No globals
- Formals not modified

# Storeless semantics

- No addresses
- Memory state:
  - Object: $2^{\text{Access paths}}$
  - Heap: $2^{\text{Object}}$
- Alias analysis

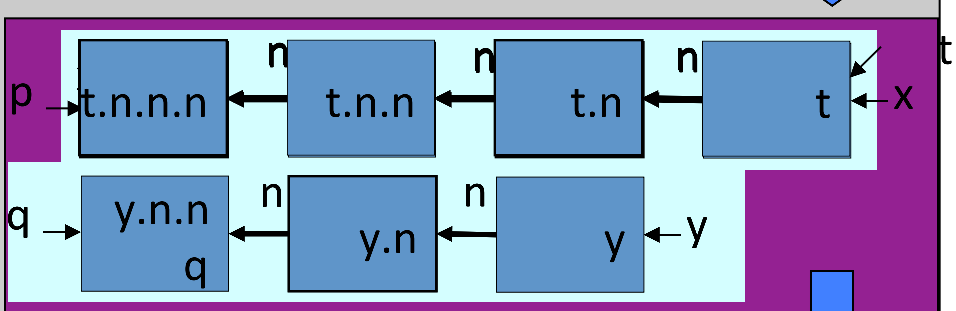


# Example

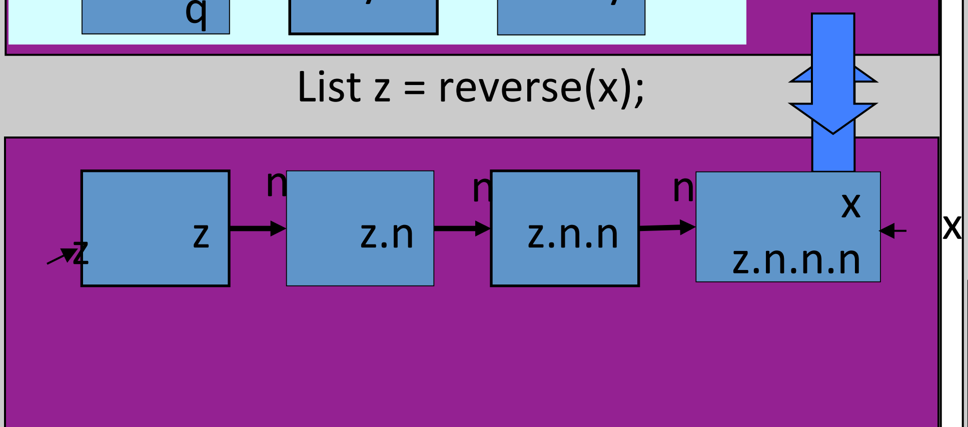
static void main() {

List x = reverse(p);

List y = reverse(q);

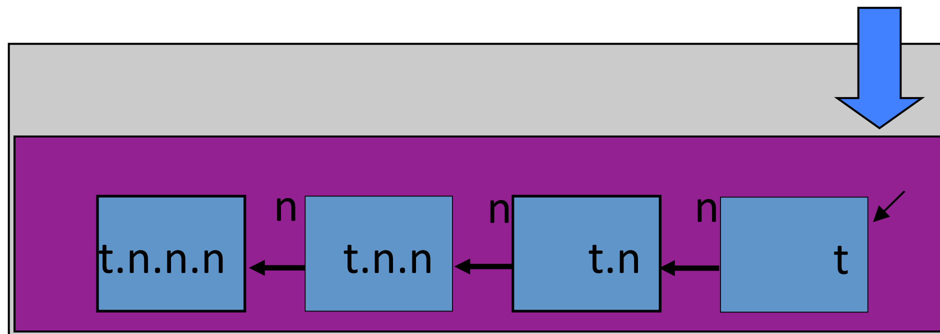


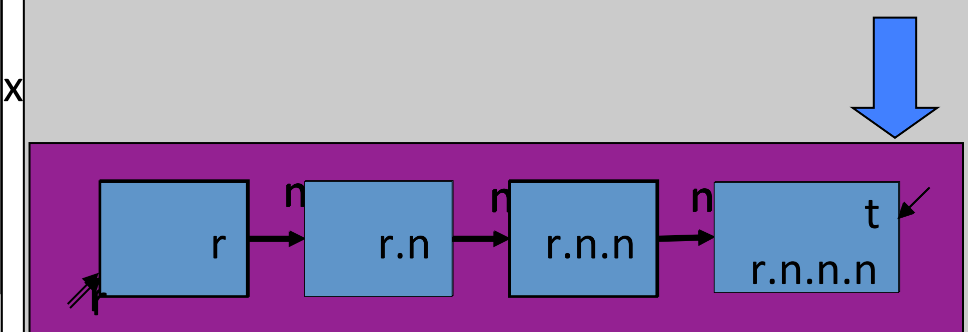

List z = reverse(x);




}

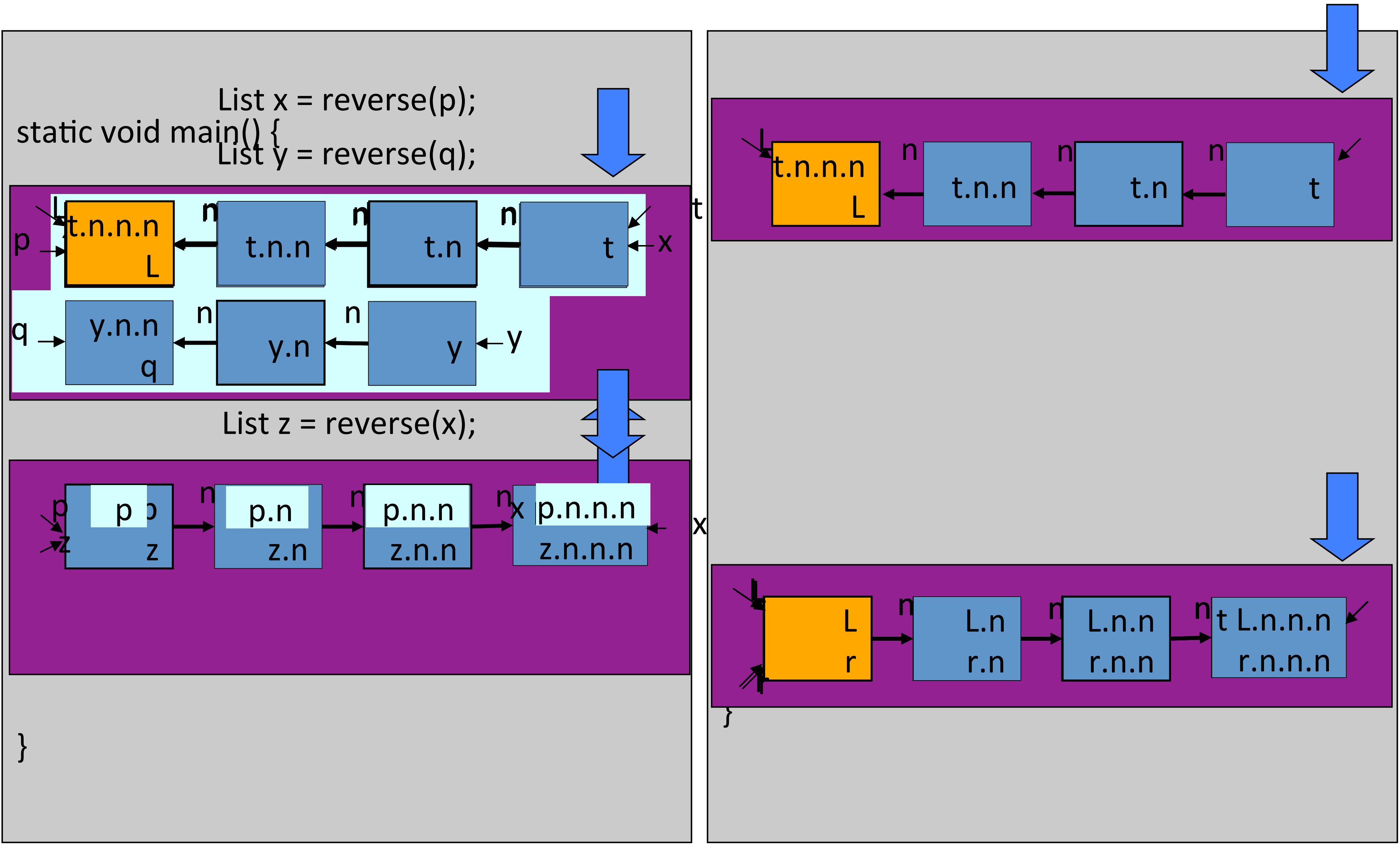
p?




}

# Example



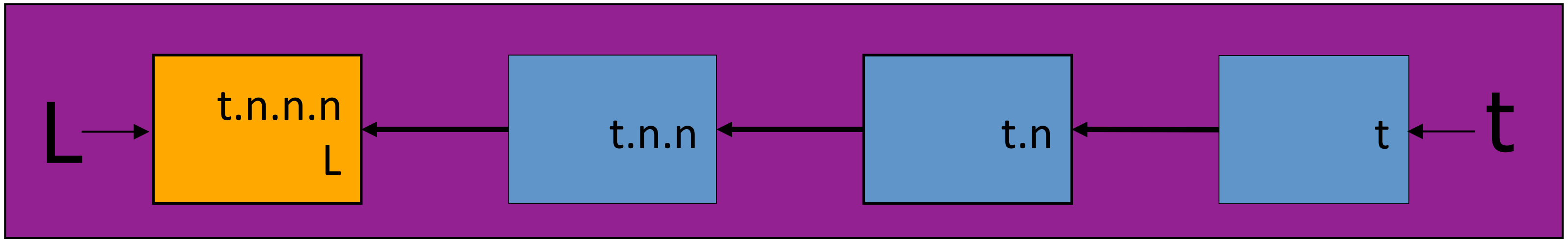
List x = reverse(p);
static void main() {
List y = reverse(q);
t.n.n.n
L
t.n.n
t.n
t
n
p
x
t
y.n.n
q
y.n
y
q
y
List z = reverse(x);
p
z
p.n
z.n
p.n.n
z.n.n
p.n.n.n
z.n.n.n
x
}
t.n.n.n
L
t.n.n
t.n
t
L
r
L.n
r.n
L.n.n
r.n.n
t L.n.n.n
r.n.n.n
}

# Cutpoint labels

- Relate pre-state with post-state
- Additional roots
- Mark cutpoints at and throughout an invocation

# Cutpoint labels

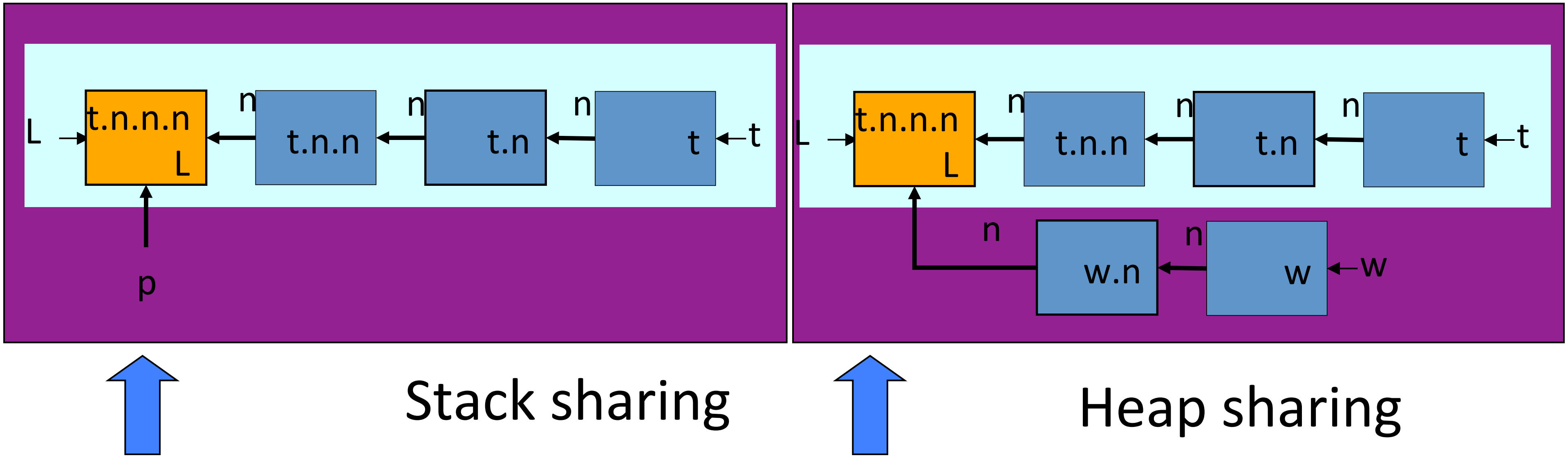
- **Cutpoint label**: the set of access paths that point to a cutpoint
  - when the invoked procedure starts




$$L \equiv \{t.n.n.n\}$$

# Sharing patterns

- Cutpoint labels encode sharing patterns




$L \equiv \{t.n.n.n\}$

# Observational equivalence

- $\sigma_L \in \Sigma_L$ (Local-heap Storeless Semantics)
- $\sigma_G \in \Sigma_G$ (Global-heap Store-based Semantics)

$\sigma_L$ and $\sigma_G$ observationally equivalent

when for every access paths $AP_1$, $AP_2$

$$[\![ AP_1 = AP_2 ]\!](\sigma_L) \Leftrightarrow [\![ AP_1 = AP_2 ]\!](\sigma_G)$$

# Main theorem: semantic equivalence

- $\sigma_L \in \Sigma_L$ (Local-heap Storeless Semantics)
- $\sigma_G \in \Sigma_G$ (Global-heap Store-based Semantics)
- $\sigma_L$ and $\sigma_G$ observationally equivalent

$$\langle st, \sigma_L \rangle \overset{LSL}{\rightsquigarrow} \sigma'_L \Leftrightarrow \langle st, \sigma_G \rangle \overset{GSB}{\rightsquigarrow} \sigma'_G$$

$\sigma'_L$ and $\sigma'_G$ are observationally equivalent

# Corollaries

- Preservation of invariants
  - Assertions: $AP_1 = AP_2$
- Detection of memory leaks

# Applications

- Develop new static analyses
  - Shape analysis
- Justify soundness of existing analyses

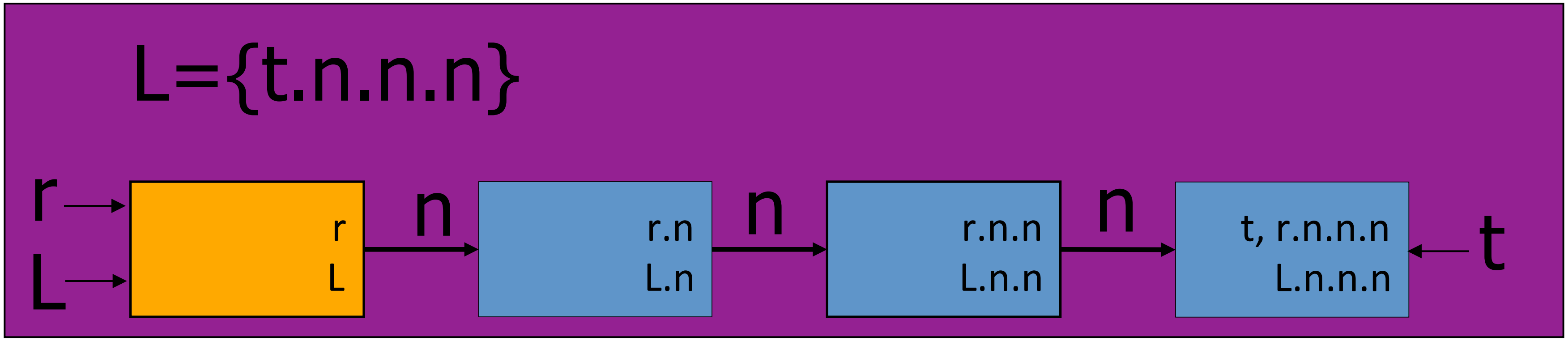
# Related work

- **Storeless semantics**
  - Jonkers, Algorithmic Languages ‘81
  - Deutsch, ICCL ‘92

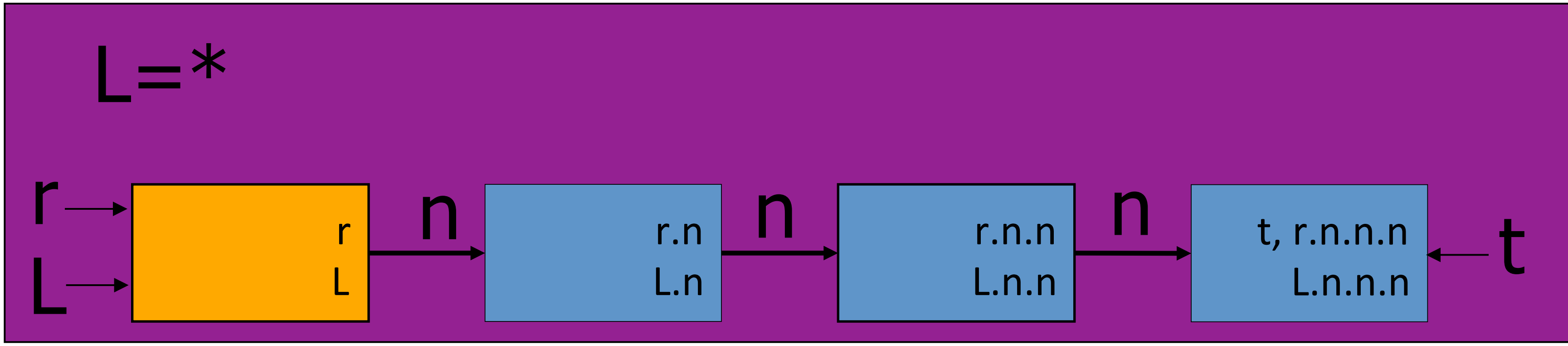
# Shape abstraction

- Shape descriptors represent unbounded memory states
  - Conservatively
  - In a bounded way
- Two dimensions
  - Local heap (objects)
  - Sharing pattern (cutpoint labels)

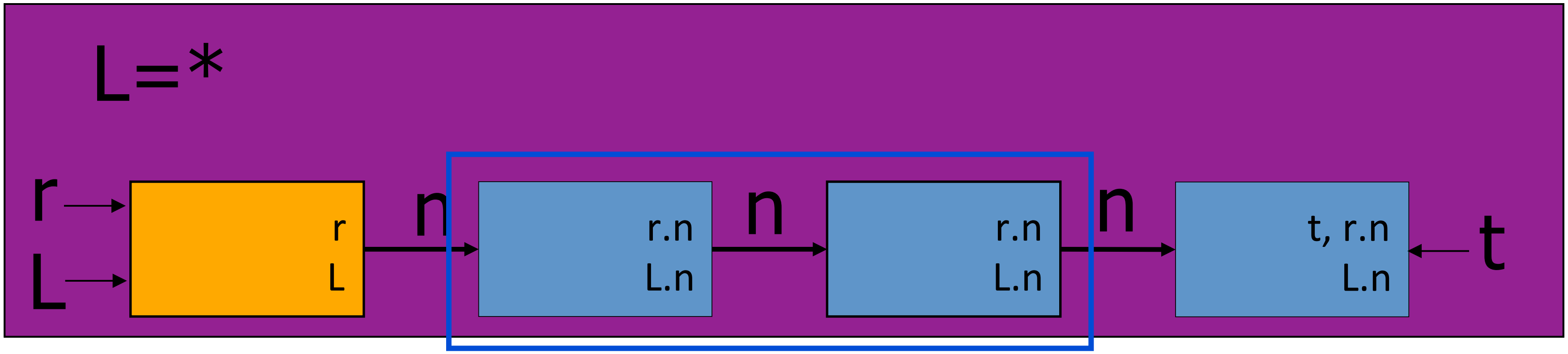
# A Shape abstraction



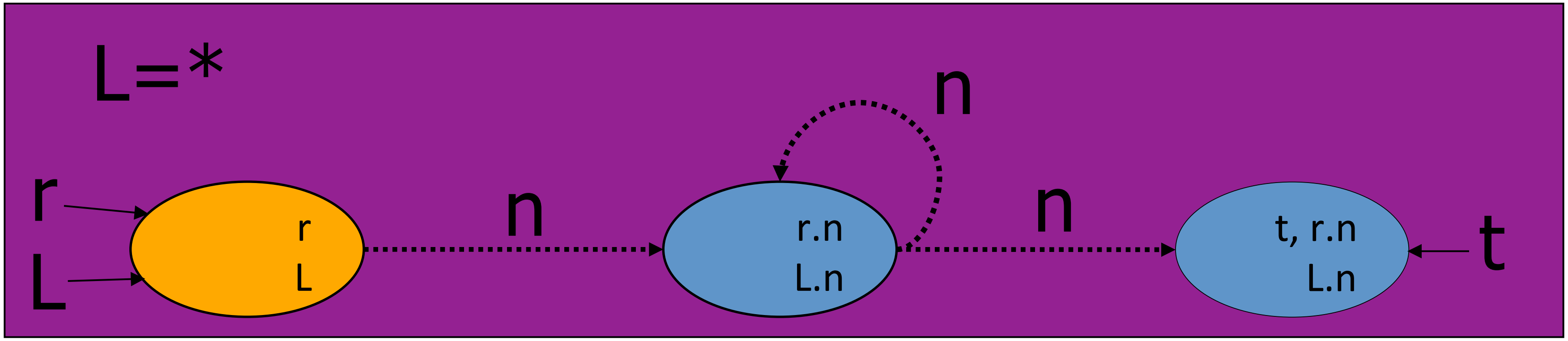
# A Shape abstraction



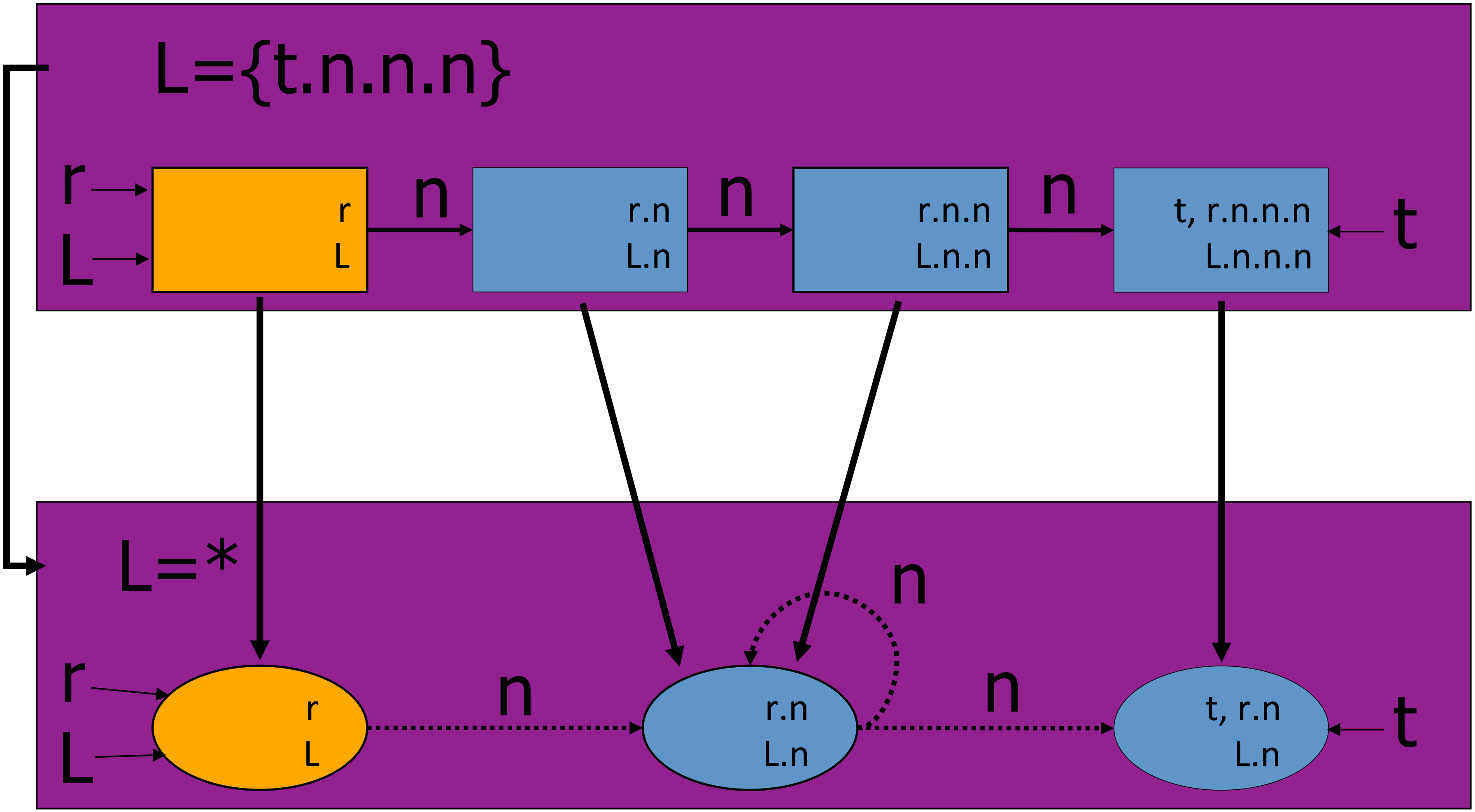
# A Shape abstraction



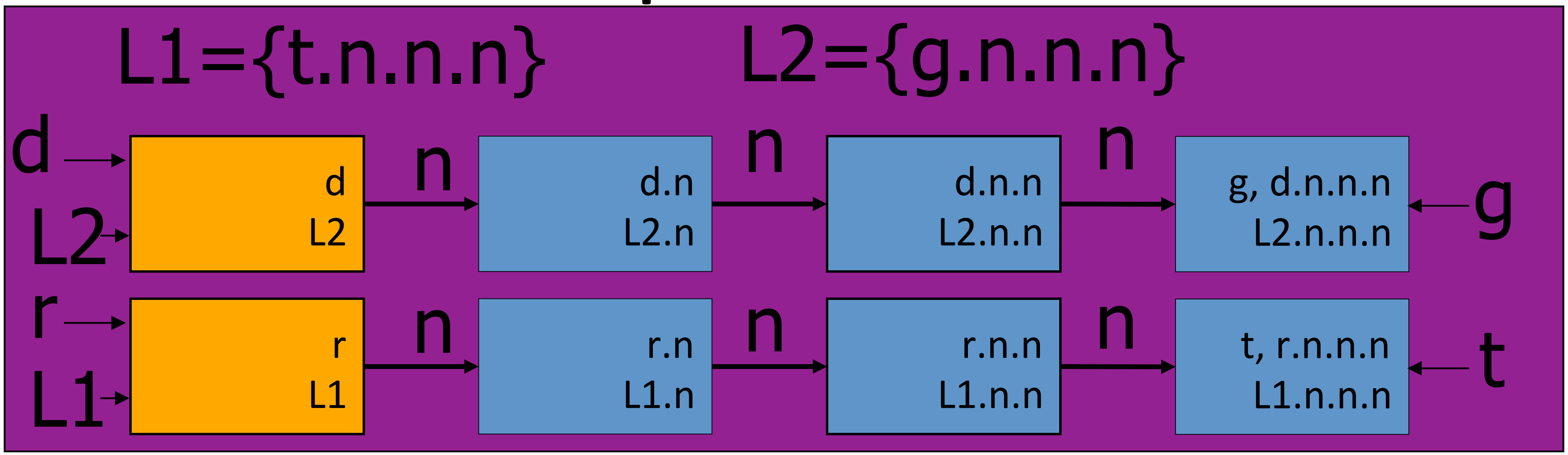
# A Shape abstraction

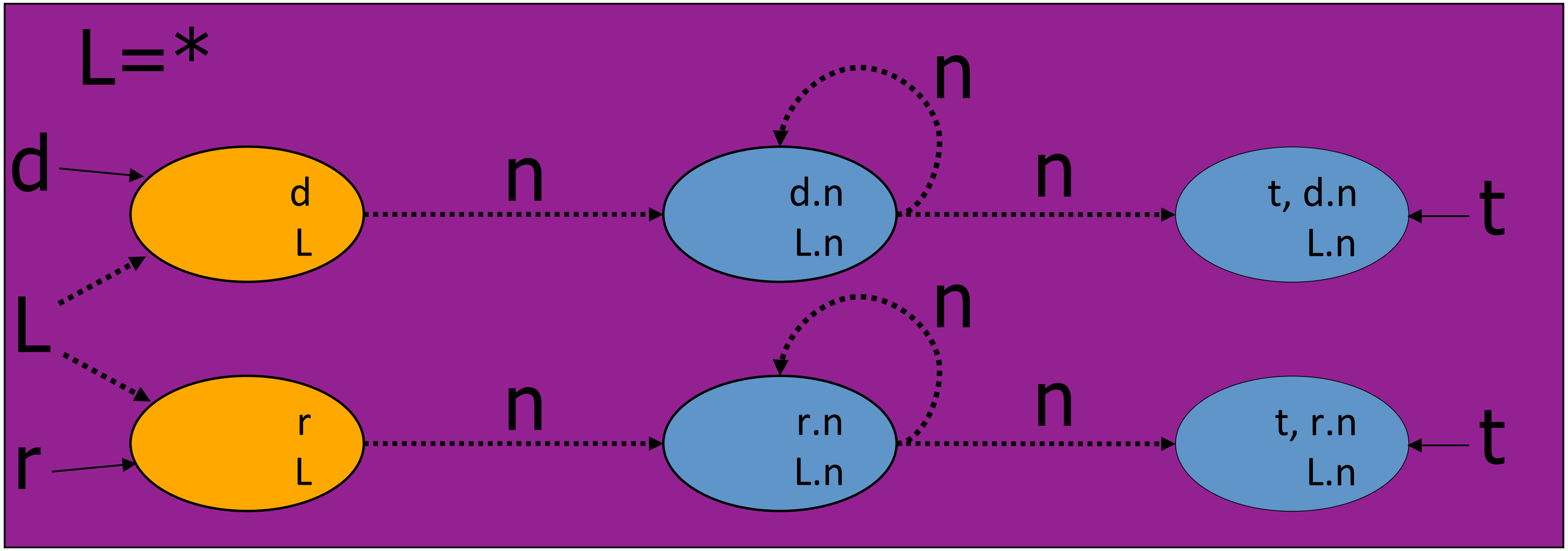


# A Shape abstraction



# A Shape abstraction

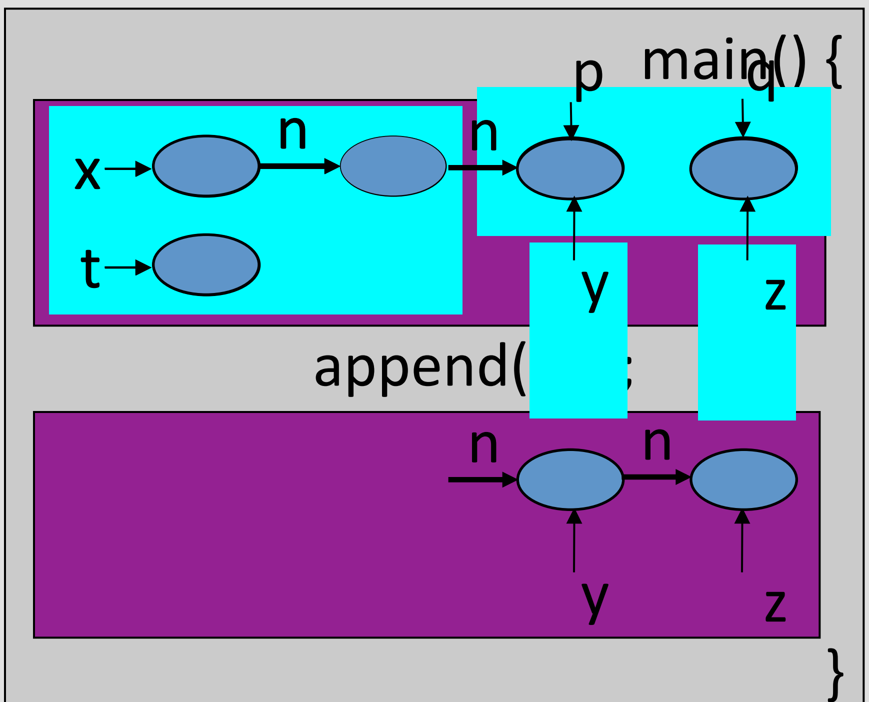


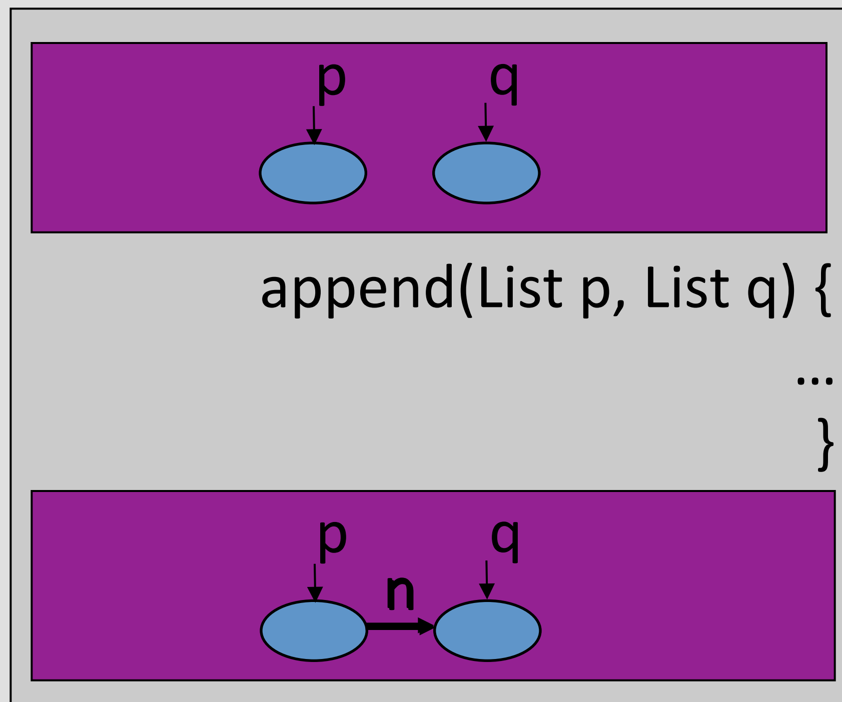



# Cutpoint-Freedom

# How to tabulate procedures?

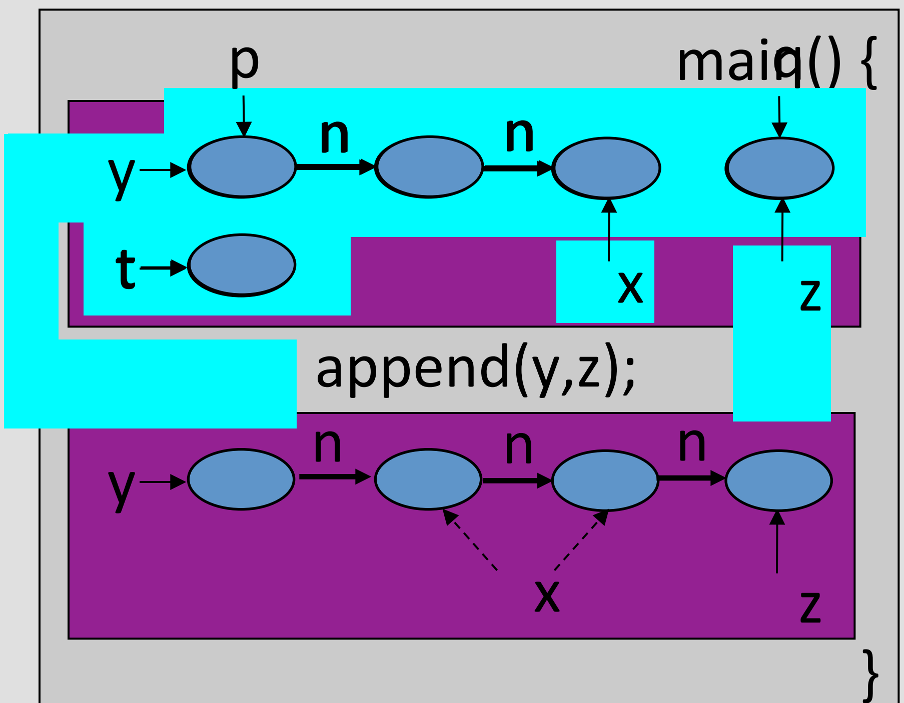
- Procedure ≡ input/output relation
  - Not reachable ➔ Not effected
  - proc: local (≡reachable) heap ➔ local heap

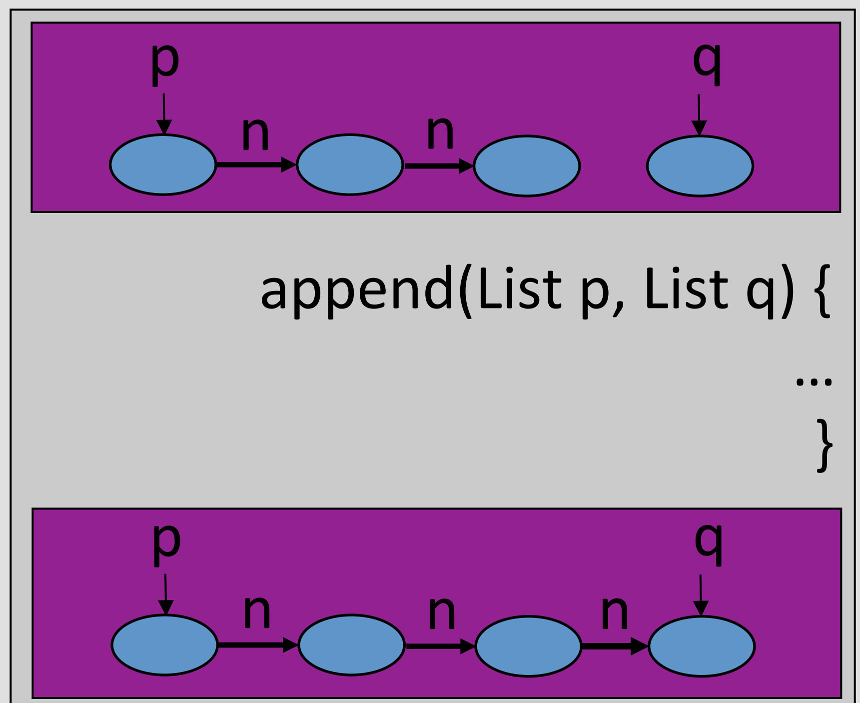


# How to handle sharing?

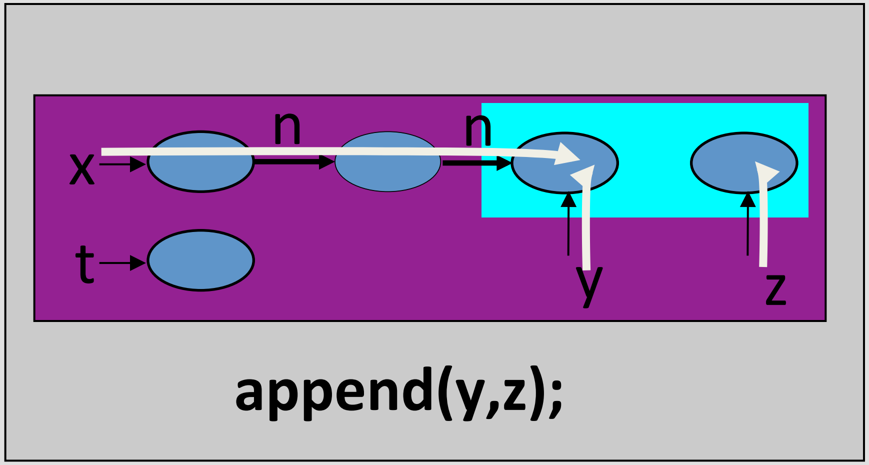
- External sharing may break the functional view



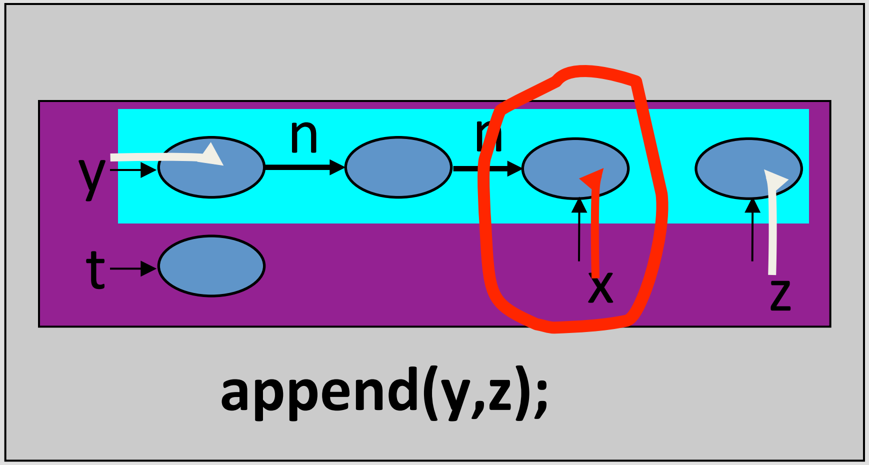



# What's the difference?

**1st Example**



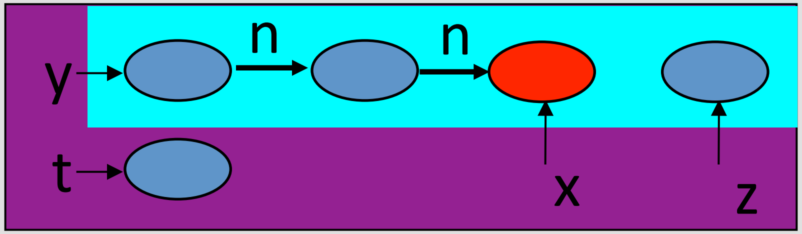

**2nd Example**



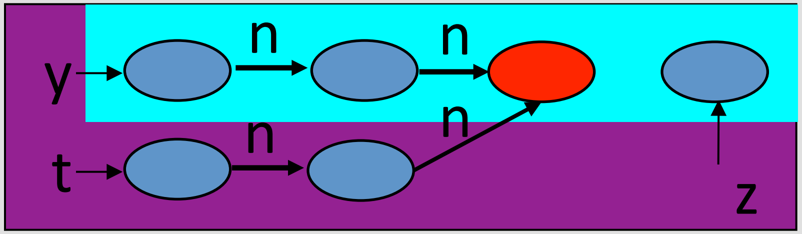
# Cutpoints

- An object is a **cutpoint** for an invocation
  - Reachable from actual parameters
  - Not pointed to by an actual parameter
  - Reachable without going through a parameter

append(y,z)



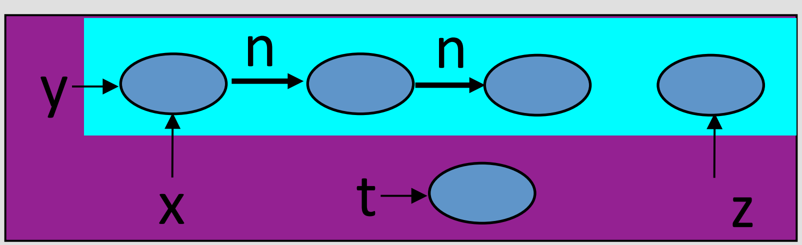

append(y,z)



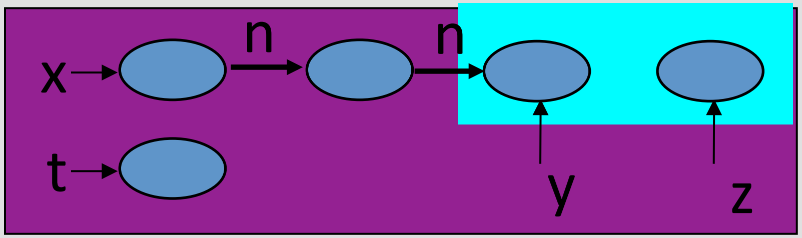
# Cutpoint freedom

- **Cutpoint-free**
  - Invocation: has no cutpoints
  - Execution: every invocation is cutpoint-free
  - Program: every execution is cutpoint-free

append(y,z)




append(y,z)

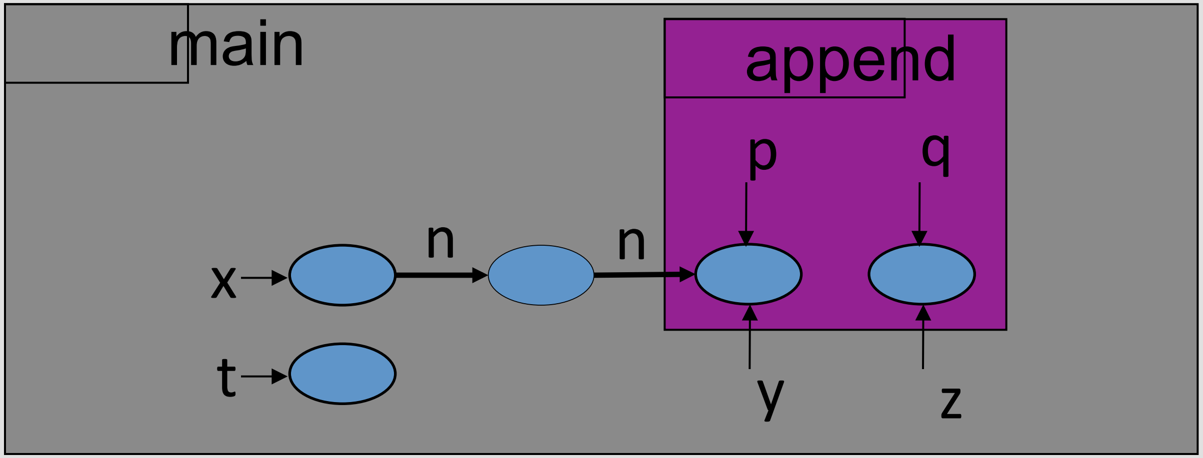


# Interprocedural shape analysis for cutpoint-free programs

# using 3-Valued Shape Analysis

# Memory states: 2-Valued Logical Structure

- A memory state encodes a **local heap**
  - Local variables of the **current procedure invocation**
  - Relevant part of the heap
    - Relevant ≡ Reachable



# Memory states

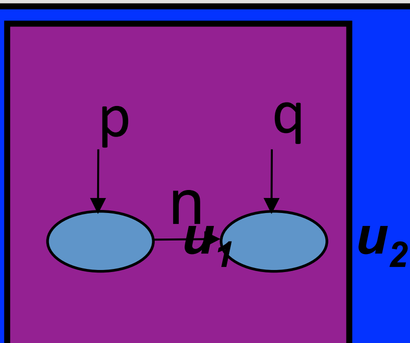
- Represented by first-order logical structures

| Predicate | Meaning |
|---|---|
| **x**(*v*) | Variable **x** points to *v* |
| **n**($v_1$,$v_2$) | Field **n** of object $v_1$ points to $v_2$ |

# Memory states

- Represented by first-order logical structures

| Predicate | Meaning |
|---|---|
| **x**(*v*) | Variable **x** points to *v* |
| **n**($v_1$,$v_2$) | Field **n** of object $v_1$ points to $v_2$ |




| | **p** |
|---|---|
| $u_1$ | 1 |
| $u_2$ | 0 |

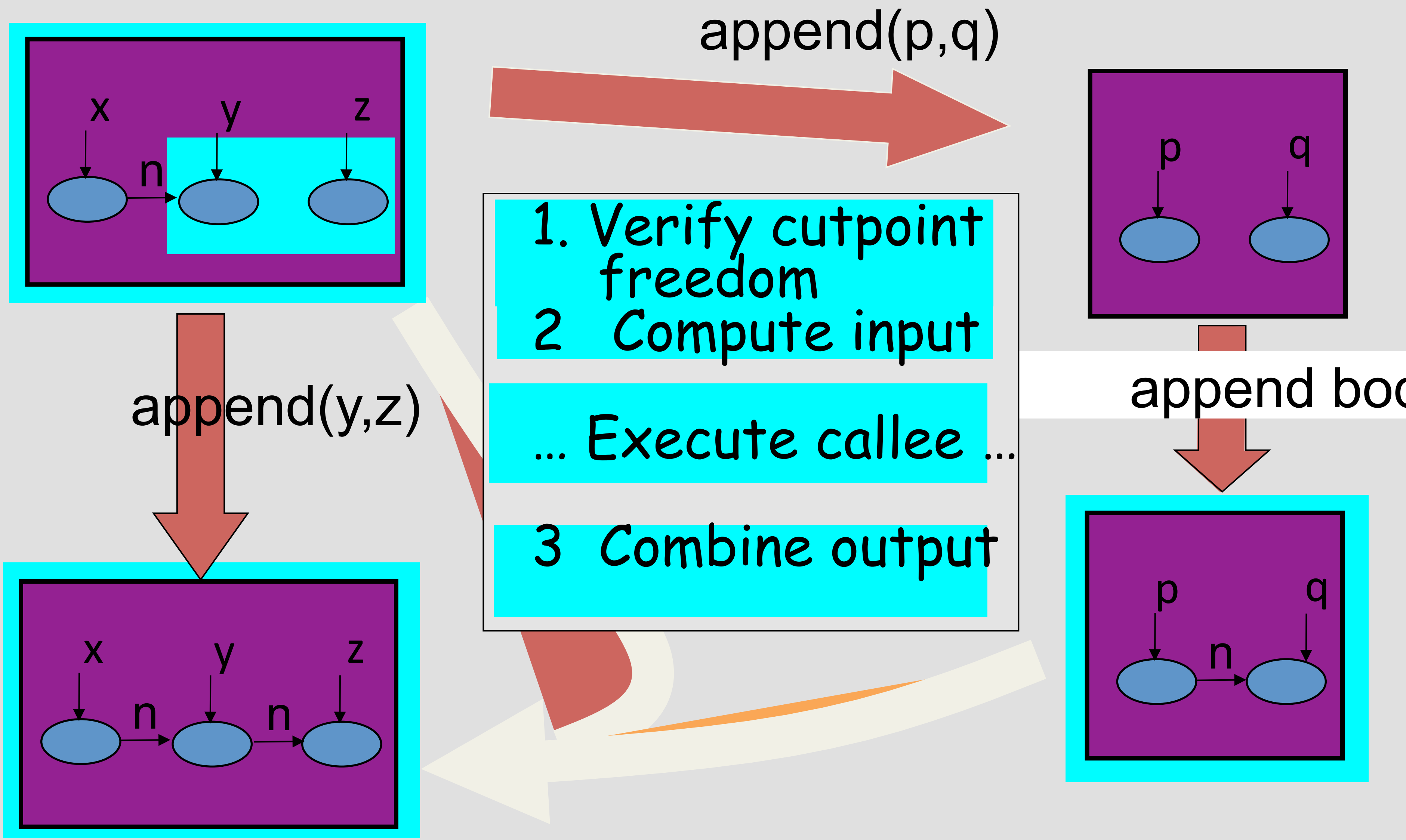
| | **q** |
|---|---|
| $u_1$ | 0 |
| $u_2$ | 1 |

| **n** | $u_1$ | $u_2$ |
|---|---|---|
| $u_1$ | 0 | 1 |
| $u_2$ | 0 | 0 |

# Operational semantics

- Statements modify values of predicates
- Specified by predicate-update formulae
  - Formulae in FO-TC

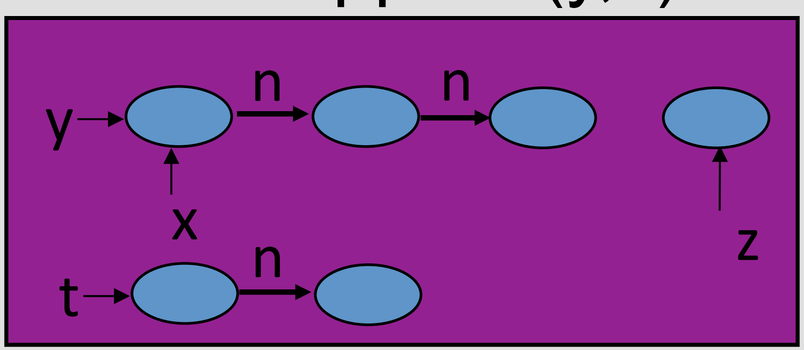
# Procedure calls



# Procedure call:
# 1. Verifying cutpoint-freedom

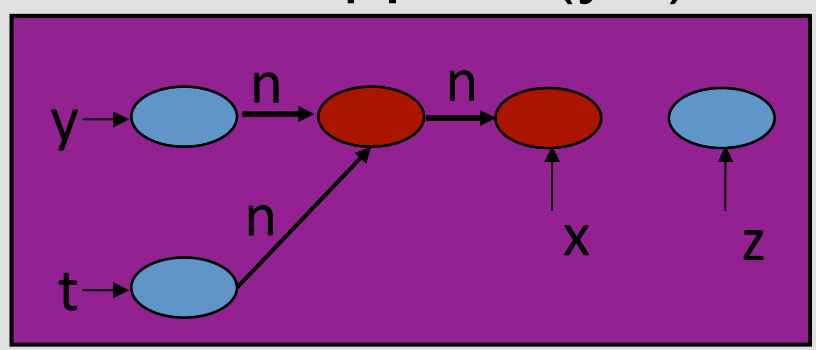
- An object is a **cutpoint** for an invocation
  - Reachable from actual parameters
  - Not pointed to by an actual parameter
  - Reachable without going through a parameter

append(y,z)




**Cutpoint free**

append(y,z)

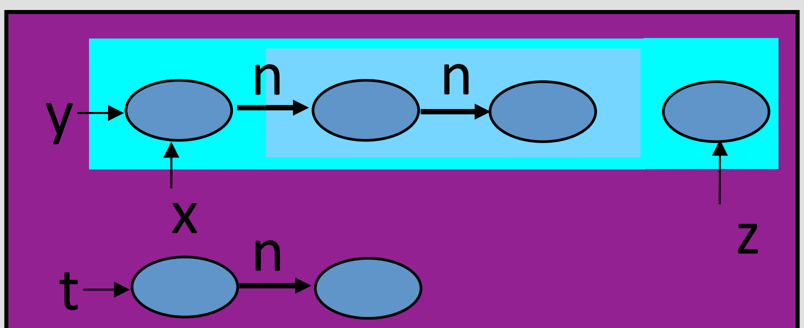



**Not Cutpoint free**

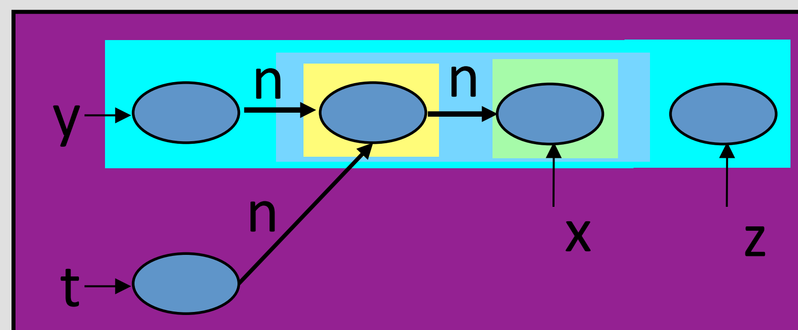
# Procedure call:
# 1. Verifying cutpoint-freedom

- Invoking append(y,z) in main
  - $R_{\{y,z\}}(v)=\exists v_1:y(v_1)\wedge n^*(v_1,v) \vee \exists v_1:z(v_1)\wedge n^*(v_1,v)$
  - $isCP_{main,\{y,z\}}(v)= R_{\{y,z\}}(v) \wedge (\neg y(v)\wedge\neg z(v_1)) \wedge$
    $( x(v) \vee t(v) \vee \exists v_1: \neg R_{\{y,z\}}(v_1)\wedge n(v_1,v))$

(main's locals: x,y,z,t)



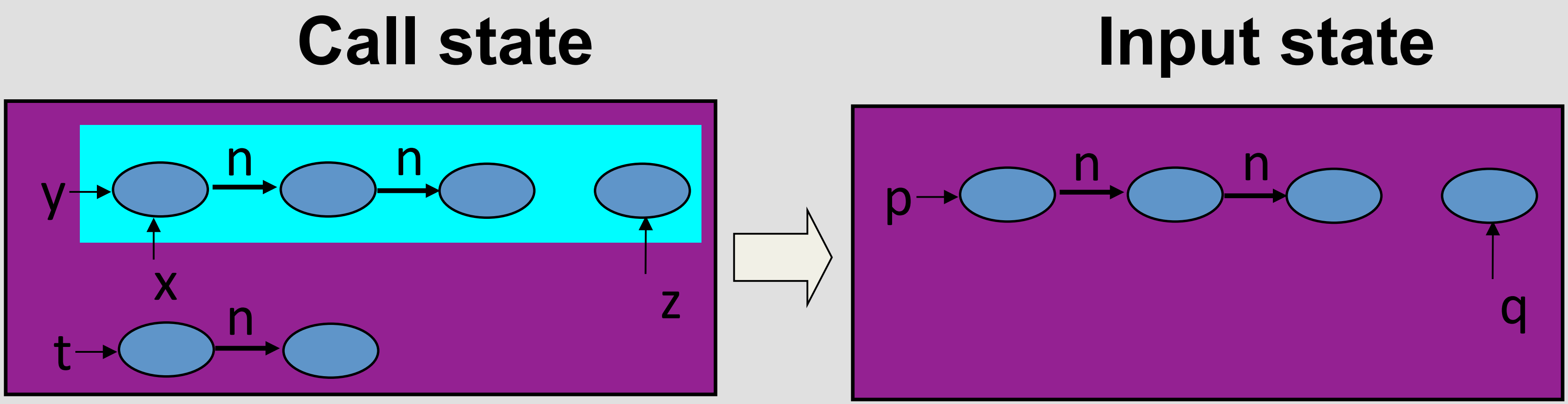

**Cutpoint free**



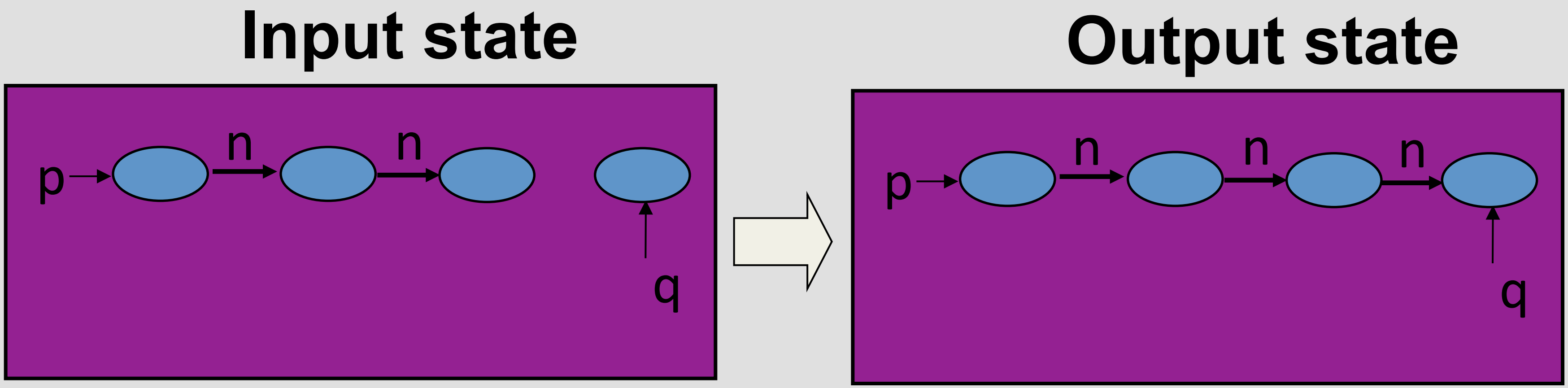

**Not Cutpoint free**

# Procedure call:
# 2. Computing the input local heap

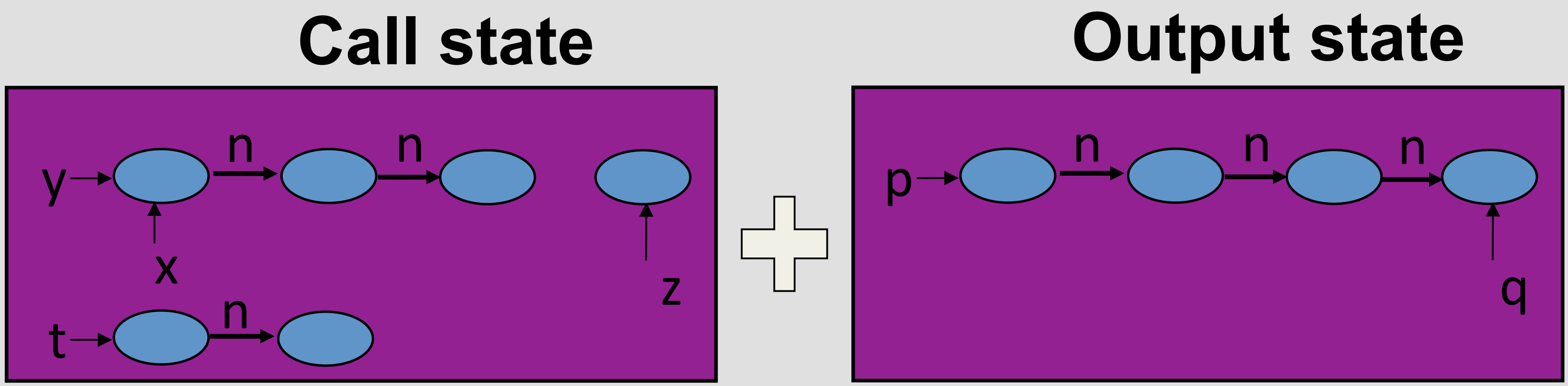
- Retain only reachable objects
- Bind formal parameters



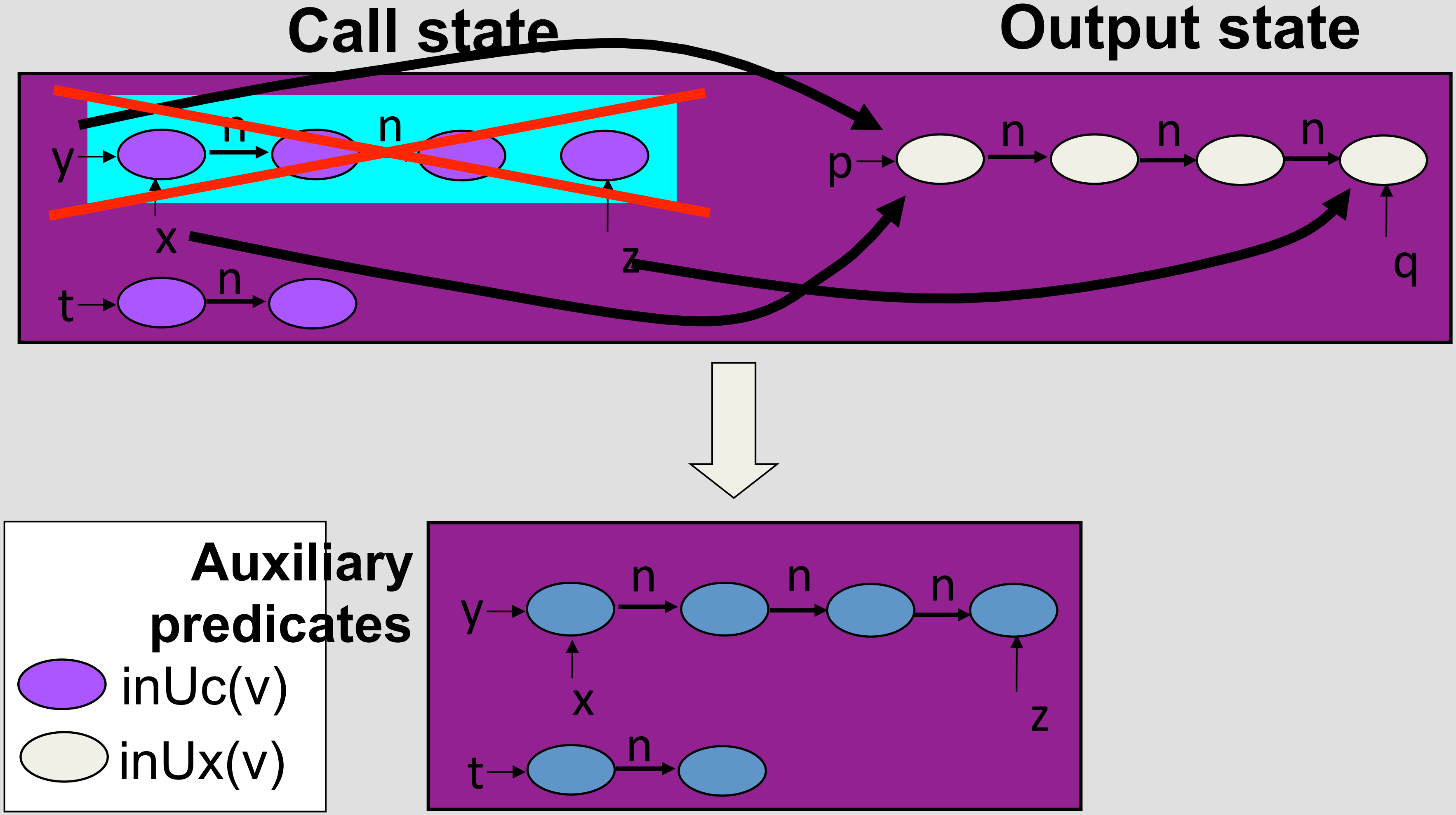
# Procedure body: append(p,q)



# Procedure call:
# 3. Combine output



# Procedure call:
# 3. Combine output



# Observational equivalence

- $\sigma_{CPF} \in \Sigma_{CPF}$ (Cutpoint free semantics)
- $\sigma_{GSB} \in \Sigma_{GSB}$ (Standard semantics)

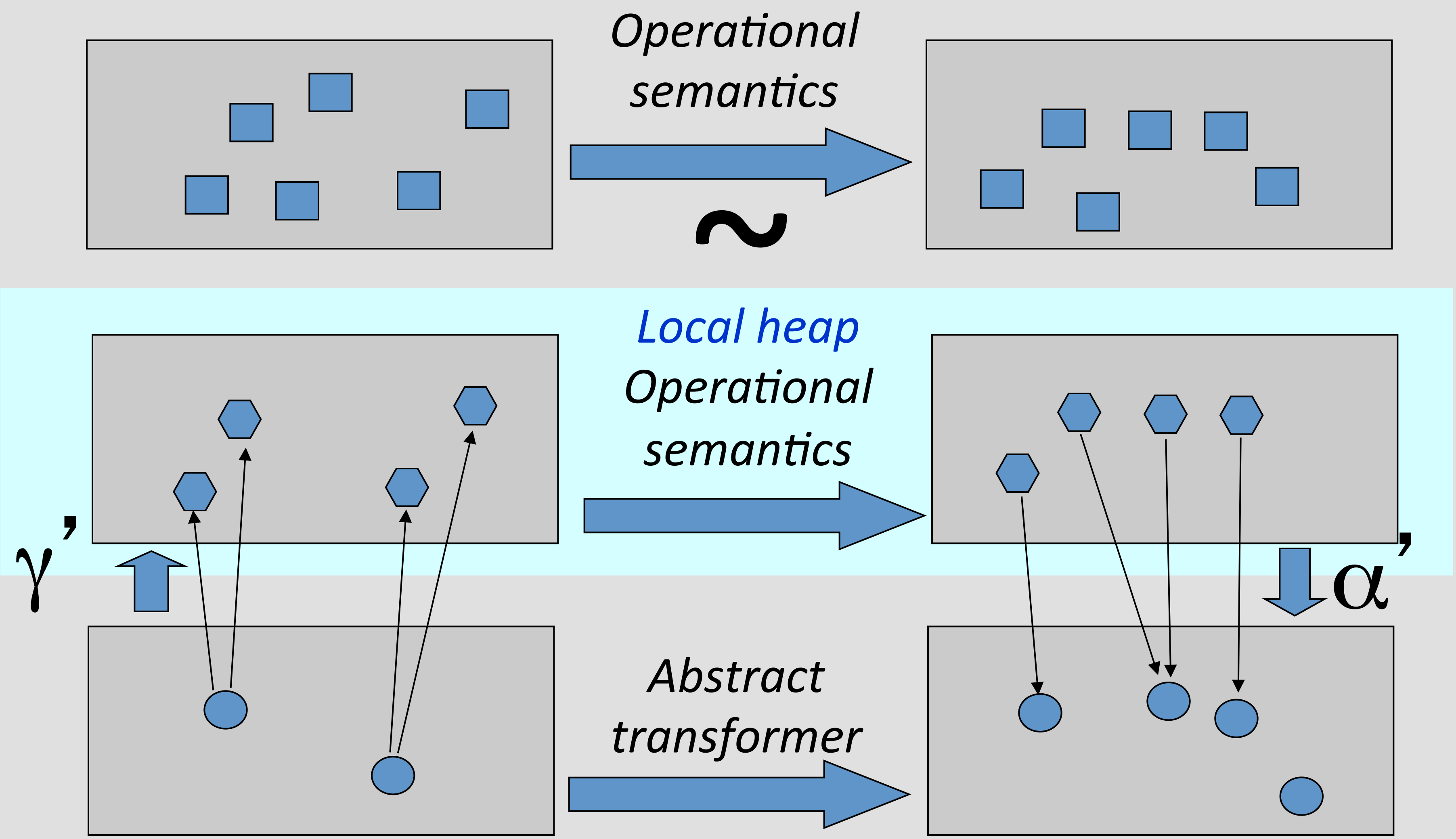
$\sigma_{CPF}$ and $\sigma_{GSB}$ observationally equivalent when for every access paths $AP_1$, $AP_2$

$$[\![ AP_1 = AP_2 ]\!](\sigma_{CPF}) \Leftrightarrow [\![ AP_1 = AP_2 ]\!](\sigma_{GSB})$$

# Observational equivalence

- For cutpoint free programs:
  - $\sigma_{CPF} \in \Sigma_{CPF}$ (Cutpoint free semantics)
  - $\sigma_{GSB} \in \Sigma_{GSB}$ (Standard semantics)
  - $\sigma_{CPF}$ and $\sigma_{GSB}$ observationally equivalent
- It holds that
  - $\langle st, \sigma_{CPF} \rangle \rightsquigarrow \sigma'_{CPF} \Leftrightarrow \langle st, \sigma_{GSB} \rangle \rightsquigarrow \sigma'_{GSB}$
  - $\sigma'_{CPF}$ and $\sigma'_{GSB}$ are observationally equivalent

# Introducing local heap semantics



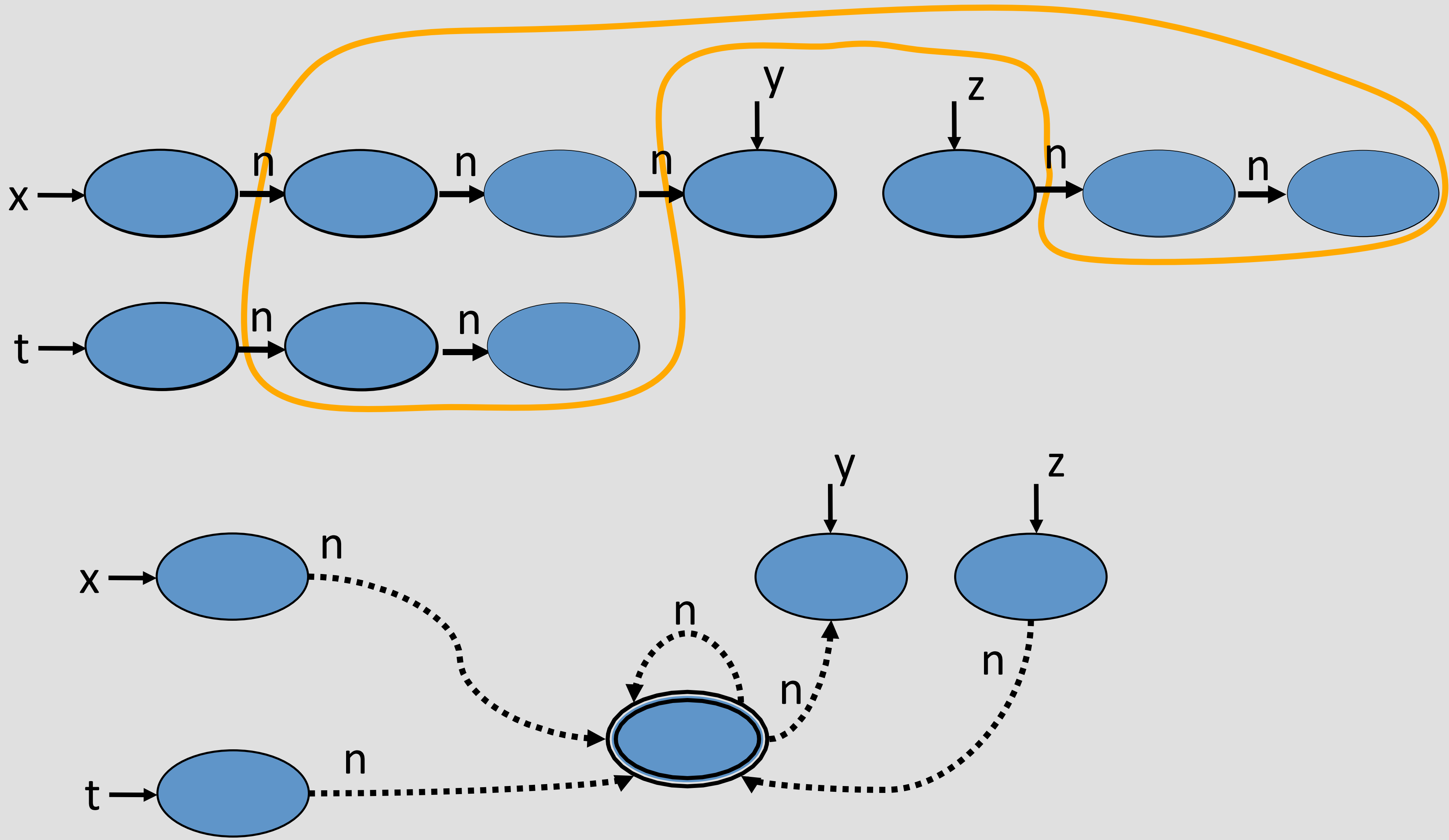
# Shape abstraction

- Abstract memory states represent unbounded concrete memory states
  - Conservatively
  - In a bounded way
  - Using 3-valued logical structures

# 3-Valued logic

- 1 = true
- 0 = false
- 1/2 = unknown
- A join semi-lattice, $0 \sqcup 1 = 1/2$

# Canonical abstraction

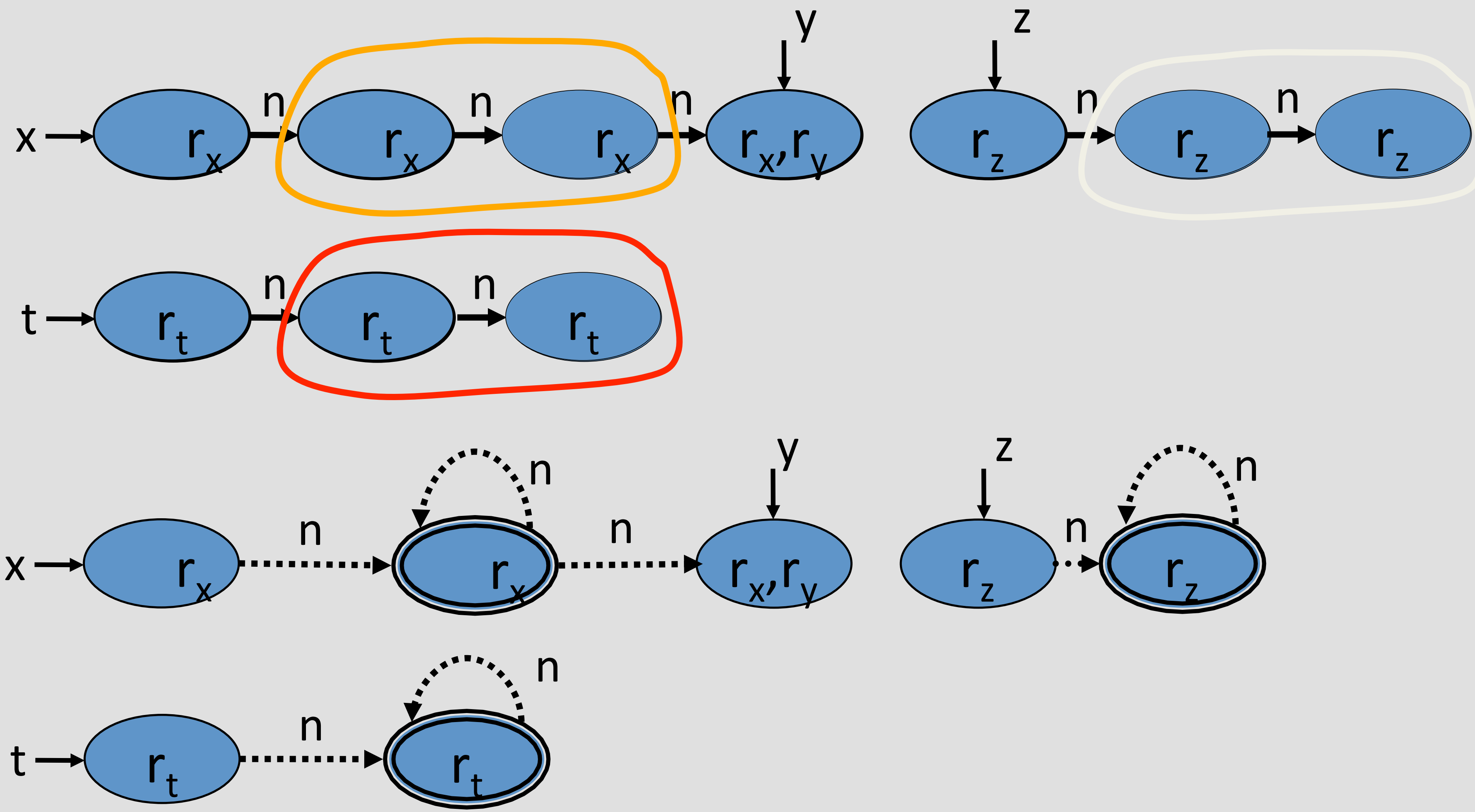


# Instrumentation predicates

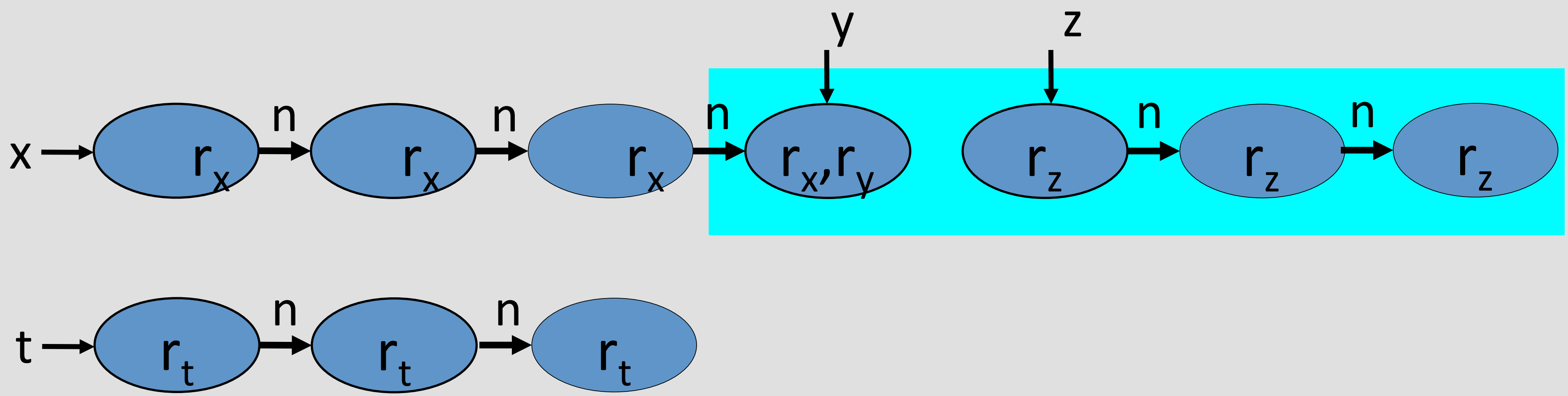
- Record derived properties
- Refine the abstraction
  - Instrumentation principle [SRW, TOPLAS'02]
- Reachability is central!

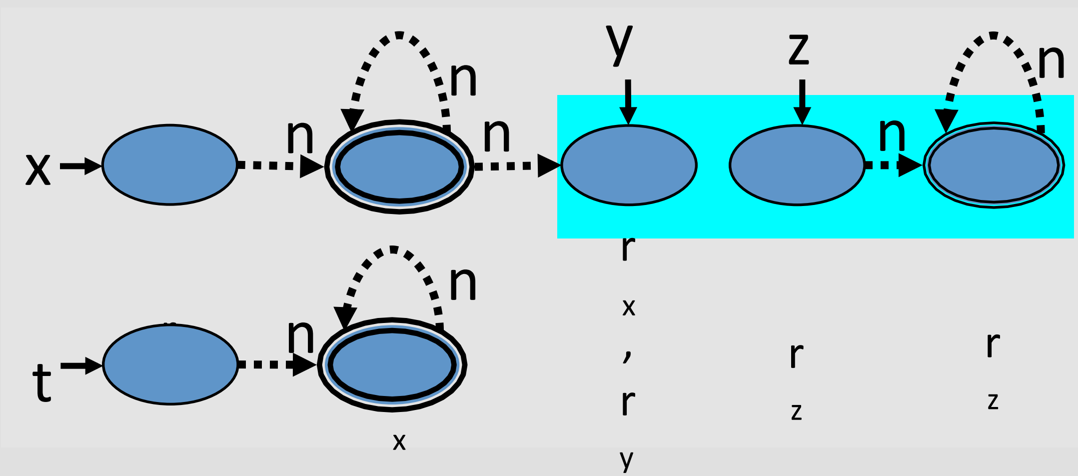
| Predicate | Meaning |
|---|---|
| $r_x(v)$ | $v$ is reachable from variable x |
| $r_{obj}(v_1,v_2)$ | $v_2$ is reachable from $v_1$ |
| ils($v$) | $v$ is heap-shared |
| c($v$) | $v$ resides on a cycle |

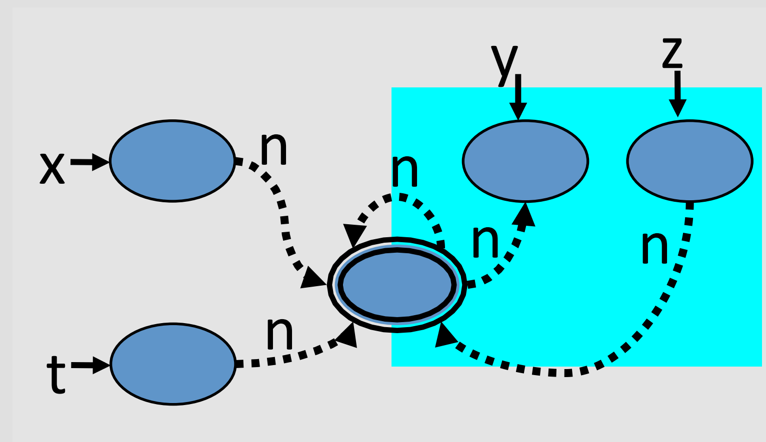
# Abstract memory states (with reachability)



# The importance of reachability: Call append(y,z)



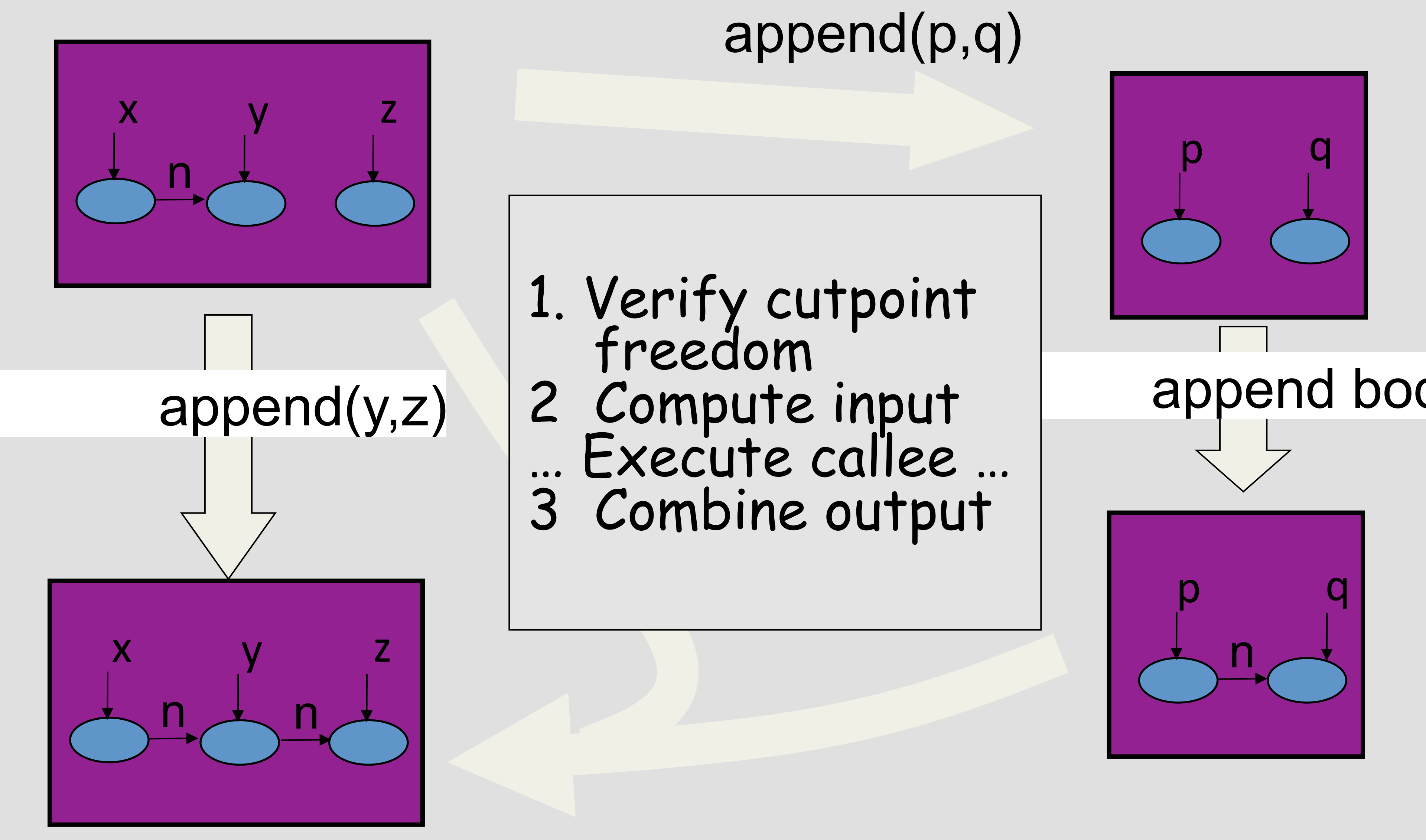



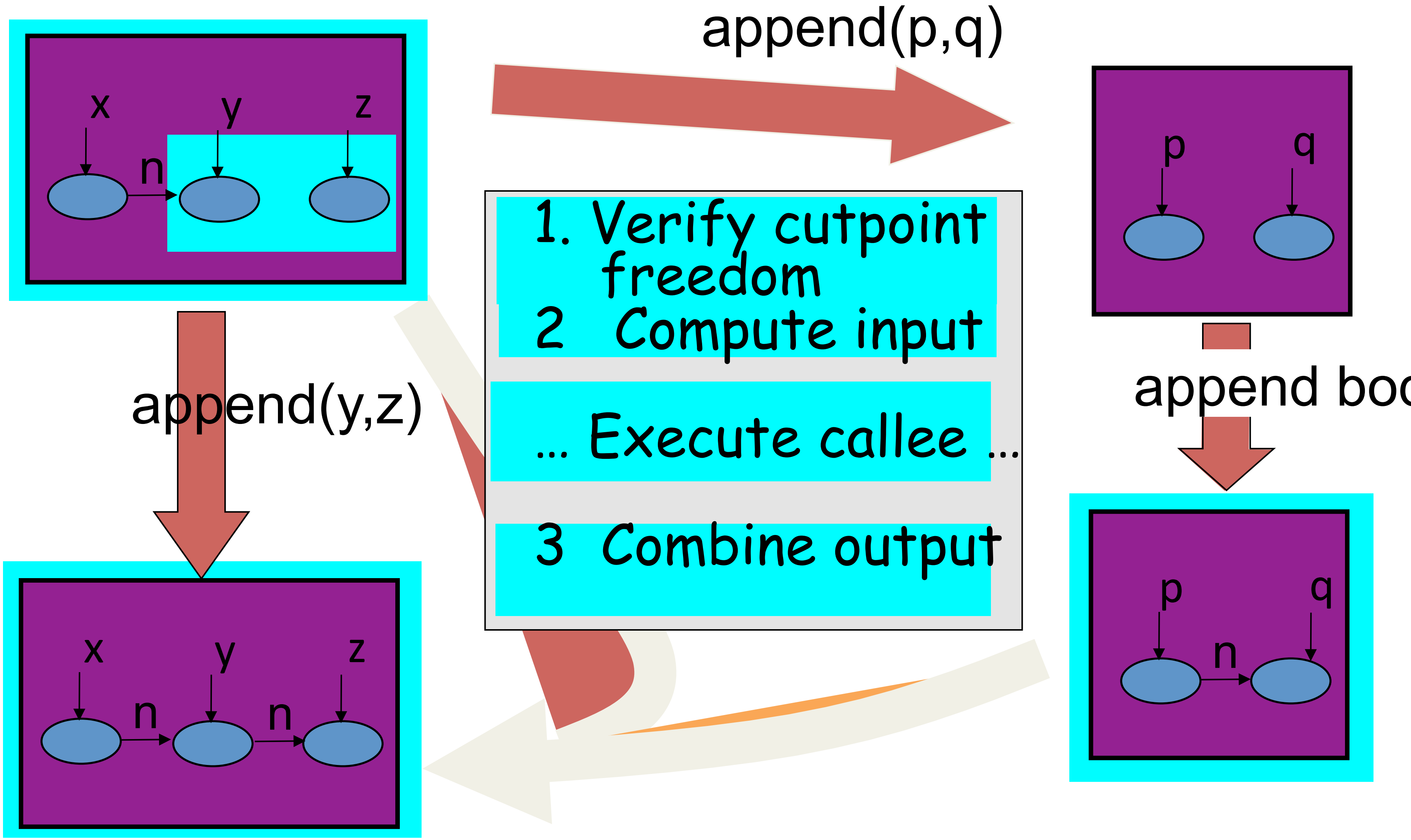
# Abstract semantics

- Conservatively apply statements on abstract memory states
  - Same formulae as in concrete semantics
  - Soundness guaranteed [SRW, TOPLAS'02]

# Procedure calls

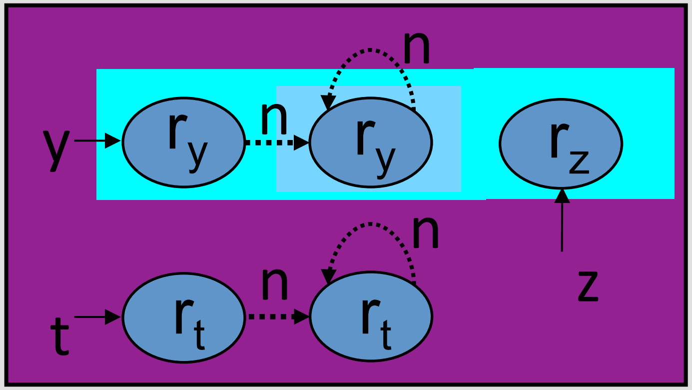


# Procedure calls

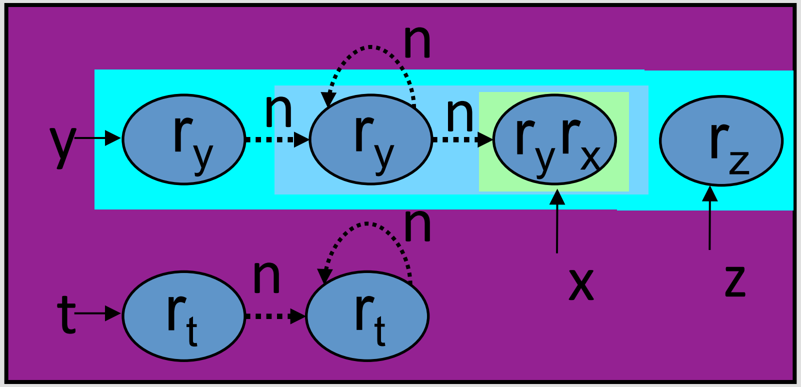


# Conservative verification of cutpoint-freedom

- Invoking append(y,z) in main
  - $R_{\{y,z\}}(v)=\exists v_1{:}y(v_1)\wedge n^*(v_1,v) \vee \exists v_1{:}z(v_1)\wedge n^*(v_1,v)$
  - $isCP_{main,\{y,z\}}(v)= R_{\{y,z\}}(v) \wedge (\neg y(v)\wedge\neg z(v_1)) \wedge$ $( x(v) \vee t(v) \vee \exists v_1{:} \neg R_{\{y,z\}}(v_1)\wedge n(v_1,v))$

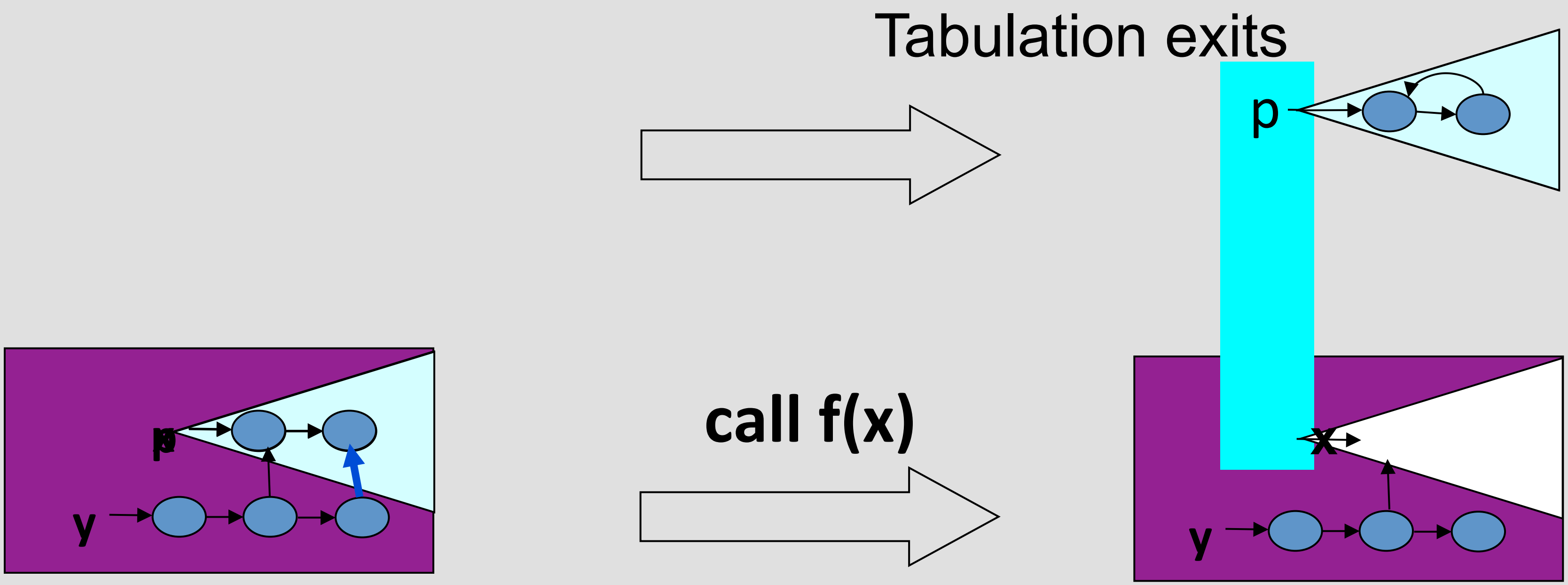



**Cutpoint free**

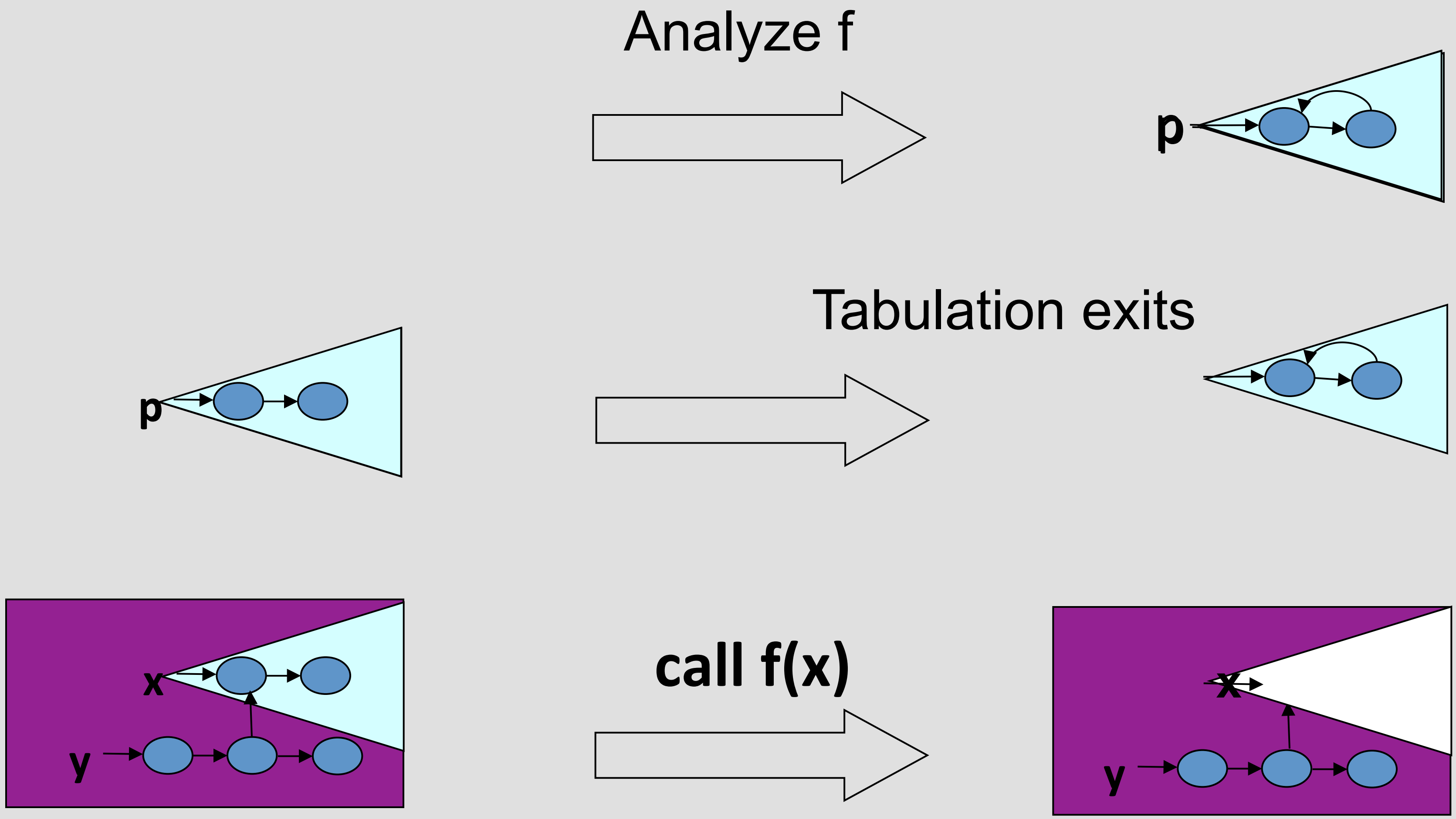



**Not Cutpoint free**

# Interprocedural shape analysis

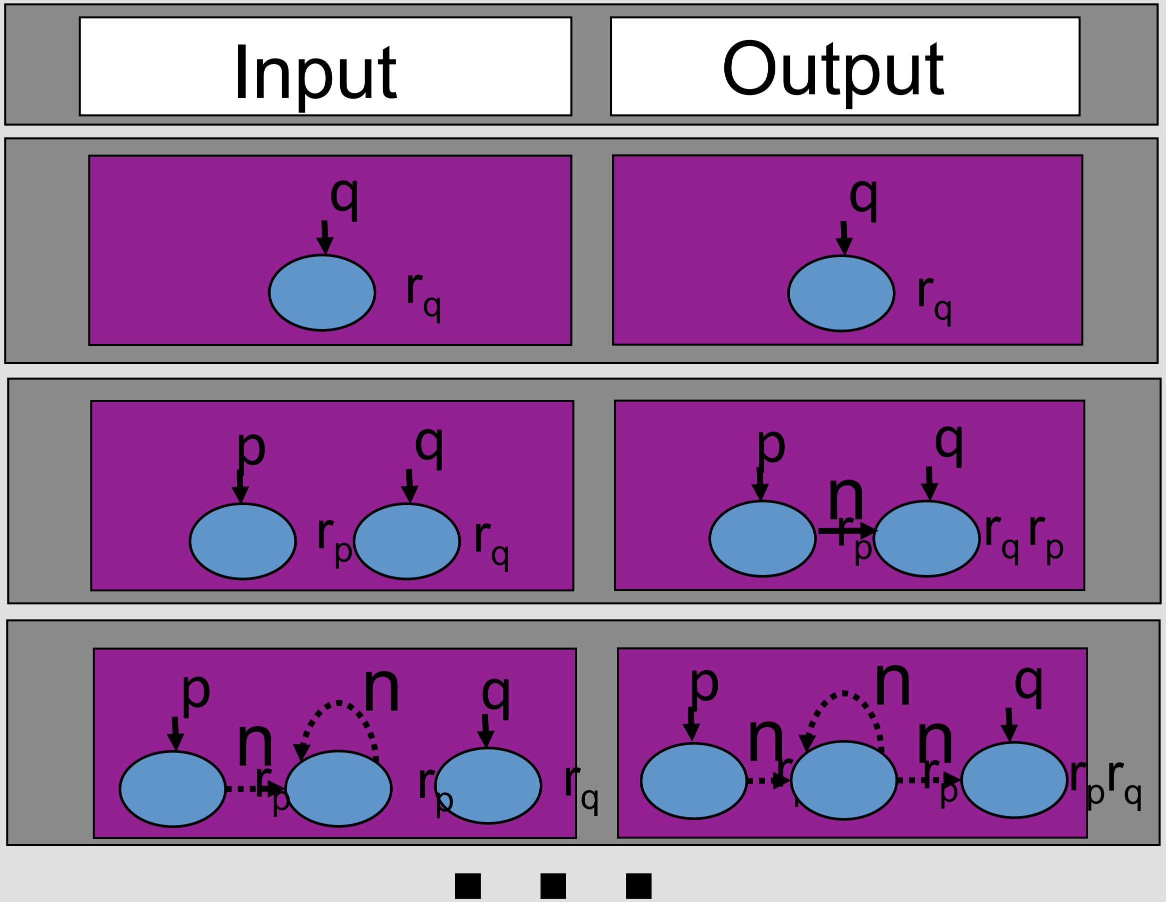


# Interprocedural shape analysis



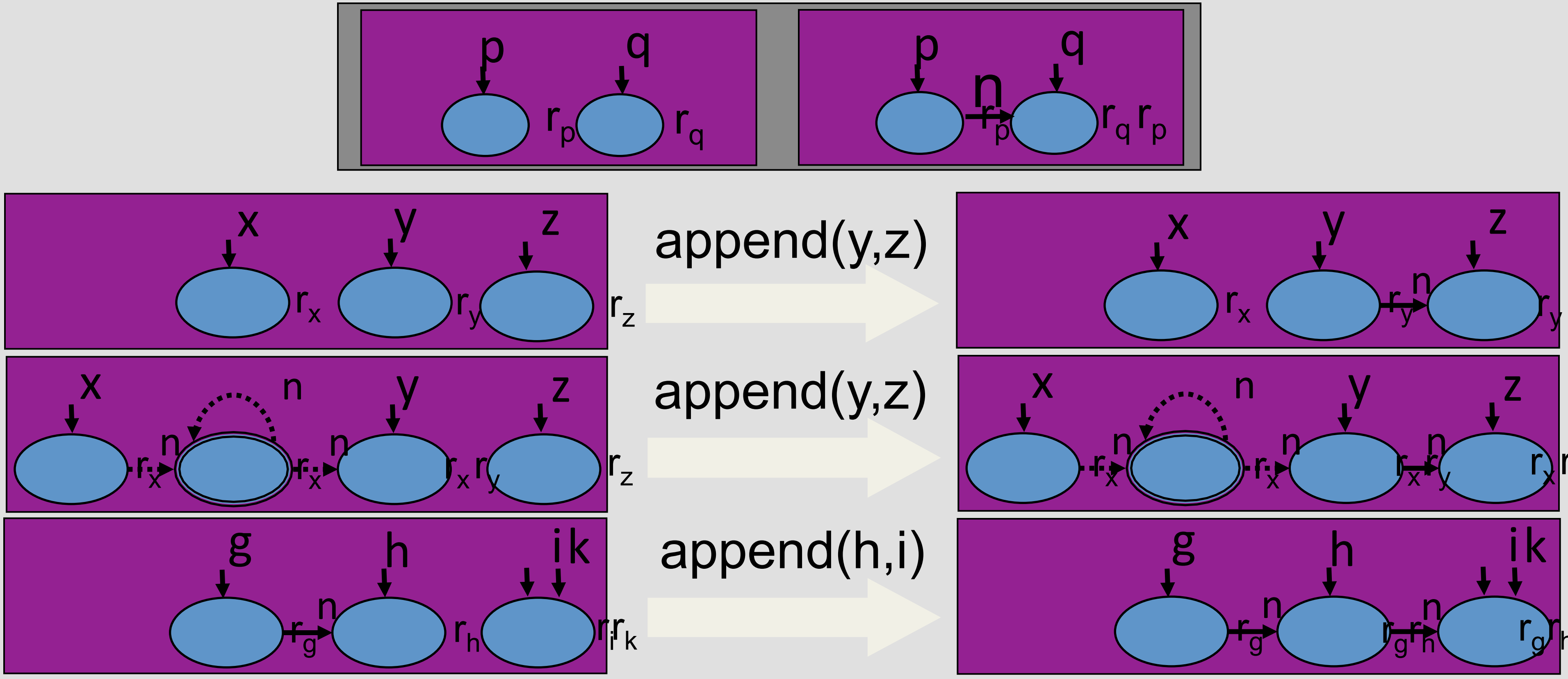
# Interprocedural shape analysis

- Procedure ≡ input/output relation



# Interprocedural shape analysis

- Reusable procedure summaries
  - Heap modularity



# Plan

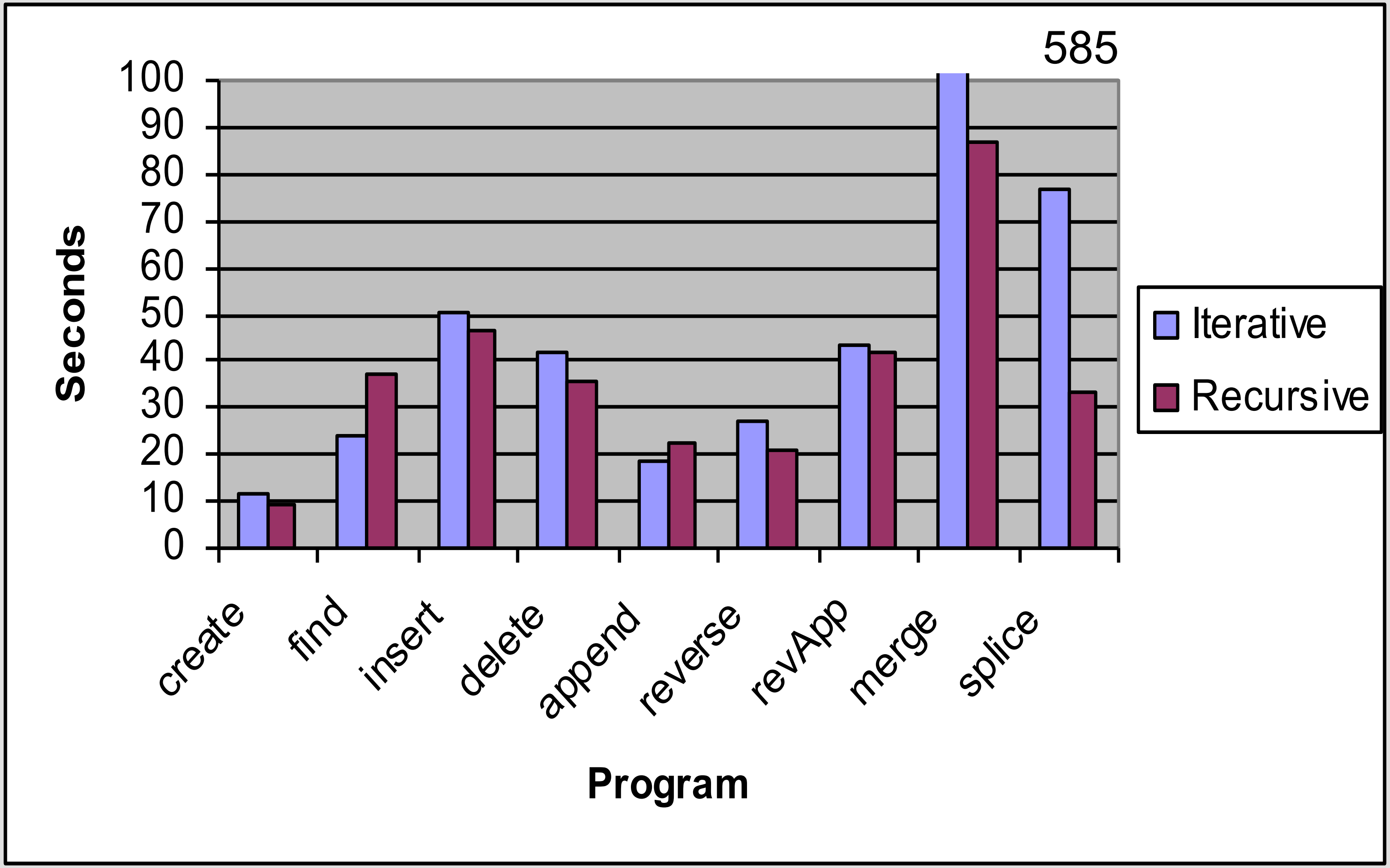
- ✓ Cutpoint freedom
- ✓ Non-standard concrete semantics
- ✓ Interprocedural shape analysis
- Prototype implementation

# Prototype implementation

- TVLA based analyzer
- Soot-based Java front-end
- Parametric abstraction

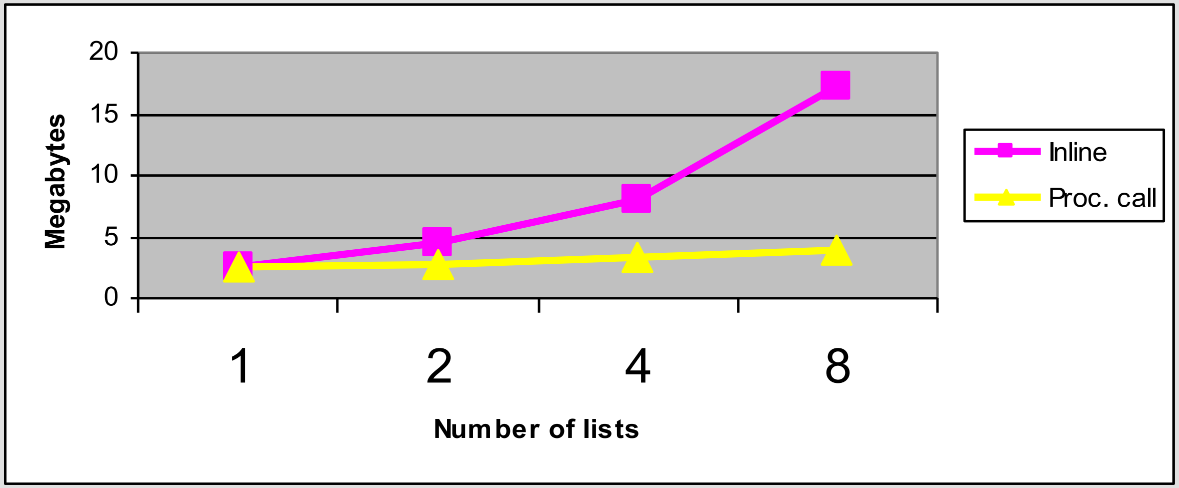
| Data structure | Verified properties |
|---|---|
| Singly linked list | Cleanness, acyclicity |
| Sorting (of SLL) | + Sortedness |
| Unshared binary trees | Cleaness, tree-ness |

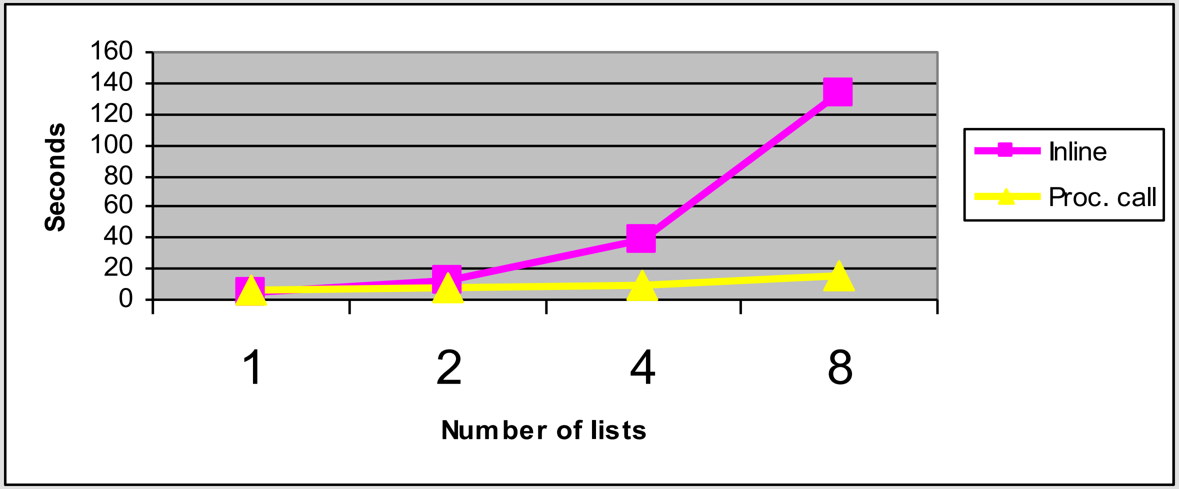
# Iterative vs. Recursive (SLL)



# Inline vs. Procedural abstraction

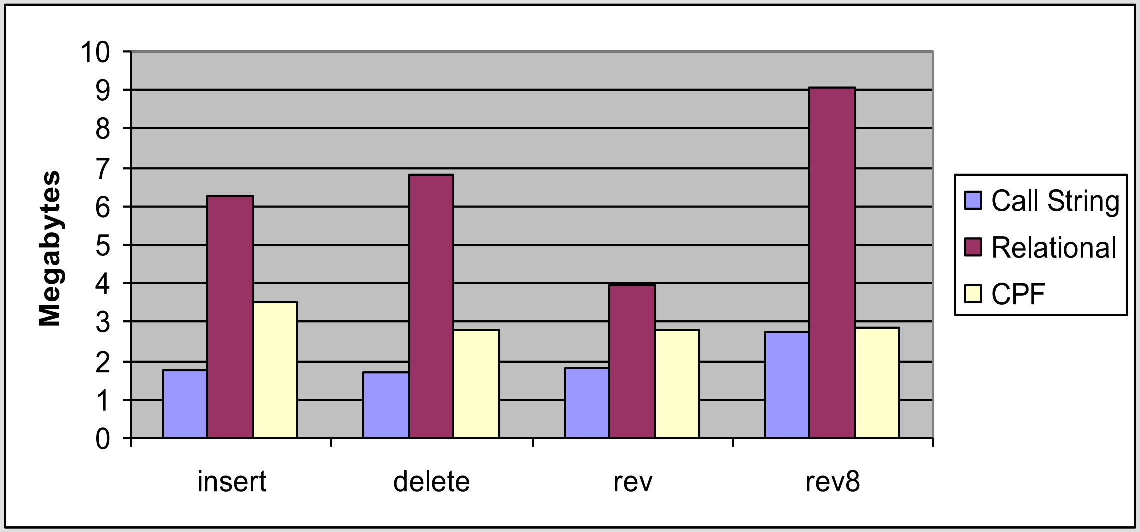
```
// Allocates a list of
// length 3
List create3(){
…
}

main() {
List x1 = create3();
List x2 = create3();
List x3 = create3();
List x4 = create3();
…
}
```

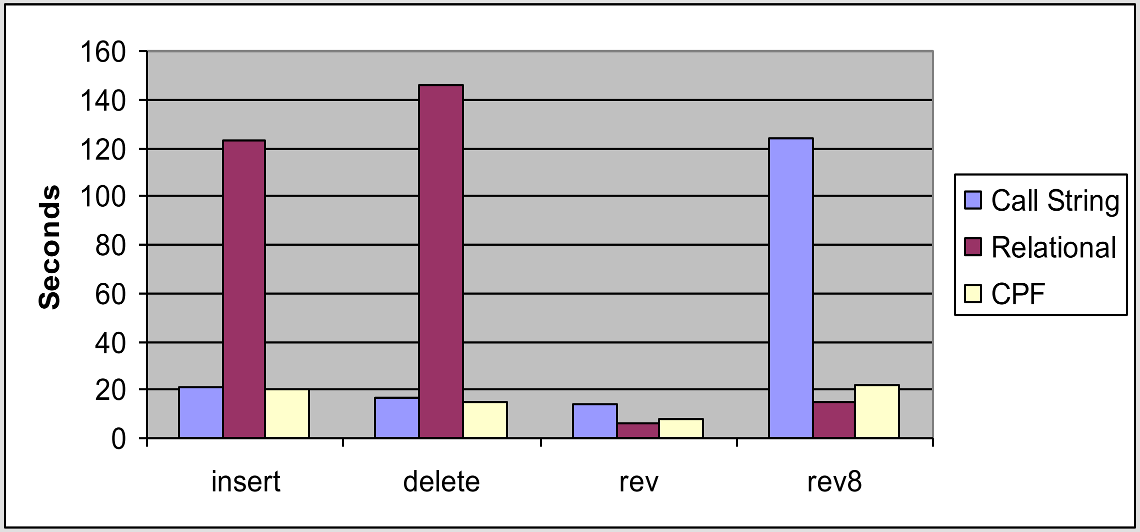


# Call string vs. Relational vs. CPF

[Rinetzky and Sagiv, CC' 01] [Jeannet et al., SAS' 04]



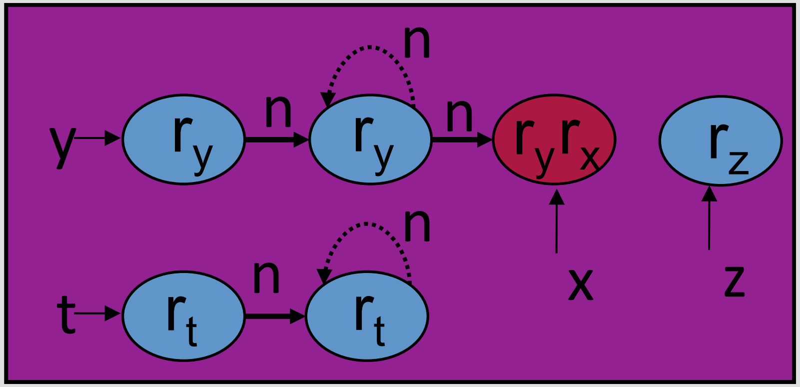
# Related Work

- **Interprocedural shape analysis**
  - Rinetzky and Sagiv, CC ’01
  - Chong and Rugina, SAS ’03
  - Jeannet et al., SAS ’04
  - Hackett and Rugina, POPL ’05
  - Rinetzky et al., POPL ‘05
- **Local Reasoning**
  - Ishtiaq and O’Hearn, POPL ‘01
  - Reynolds, LICS ’02
- **Encapsulation**
  - Noble et al. IWACO ’03
  - ...

# Future work

- Bounded number of cutpoints
- False cutpoints
  - Liveness analysis

```
append(y,z);
x = null;
```



# Summary

- Cutpoint freedom
- Non-standard operational semantics
- Interprocedural shape analysis
  - Partial correctness of quicksort
- Prototype implementation

# Application

- Properties proved
  - Absence of null dereferences
  - Listness preservation
  - API conformance
- Recursive ≈ Iterative
- Procedural abstraction

# Related work

- **Interprocedural shape analysis**
  - Rinetzky, Bauer, Reps, Sagiv, Wilhelm POPL'05
    - Cutpoints
  - Rinetzky and Sagiv, CC ' 01
    - Global heap
  - Jeannet et al., SAS ' 04
    - Local heap, relational
  - Chong and Rugina, SAS ' 03
    - Local heap
- **Local reasoning**
  - Ishtiaq and O' Hearn, POPL '01
  - Reynolds, LICS ' 02

# Summary

- Operational semantics
  - Storeless
  - Local heap
  - Cutpoints
  - Equivalence theorem
- Applications
  - Shape analysis
  - May-alias analysis

# Project

- 1-2 Students in a group
- Theoretical + Practical
- Your choice of topic
  - Contact me in 2 weeks
- Submission – 15/Sep
  - Code + Examples
  - Document
  - 20 minutes presentation

# Past projects

- JavaScript Dominator Analysis
- Attribute Analysis for JavaScript
- Simple Pointer Analysis for C
- Adding program counters to Past Abstraction
- Verification of Asynchronous programs
- Verifying SDNs using TVLA
- Verifying independent accesses to arrays in GO

# Past projects

- Detecting index out of bound errors in C programs
- Lattice-Based Semantics for Combinatorial Models Evolution