Compilation

0368-3133

Lecture 13:

Course summary: Putting it all together

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Course Goals

- What is a compiler
- How does it work
- (Reusable) techniques & tools

Course Goals

- What is a compiler
- How does it work
- (Reusable) techniques & tools

- Programming language implementation
  - runtime systems
- Execution environments
  - Assembly, linkers, loaders, OS

What is a Compiler?

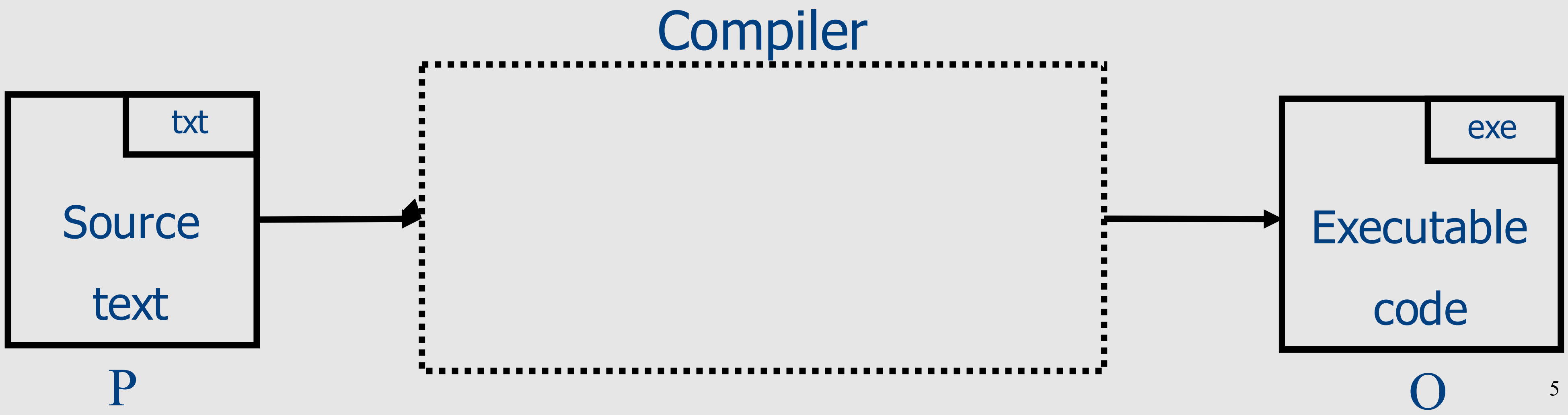
"A compiler is a computer program that transforms source code written in a programming language (source language) into another language (target language).

The most common reason for wanting to transform source code is to create an executable program."

--*Wikipedia*

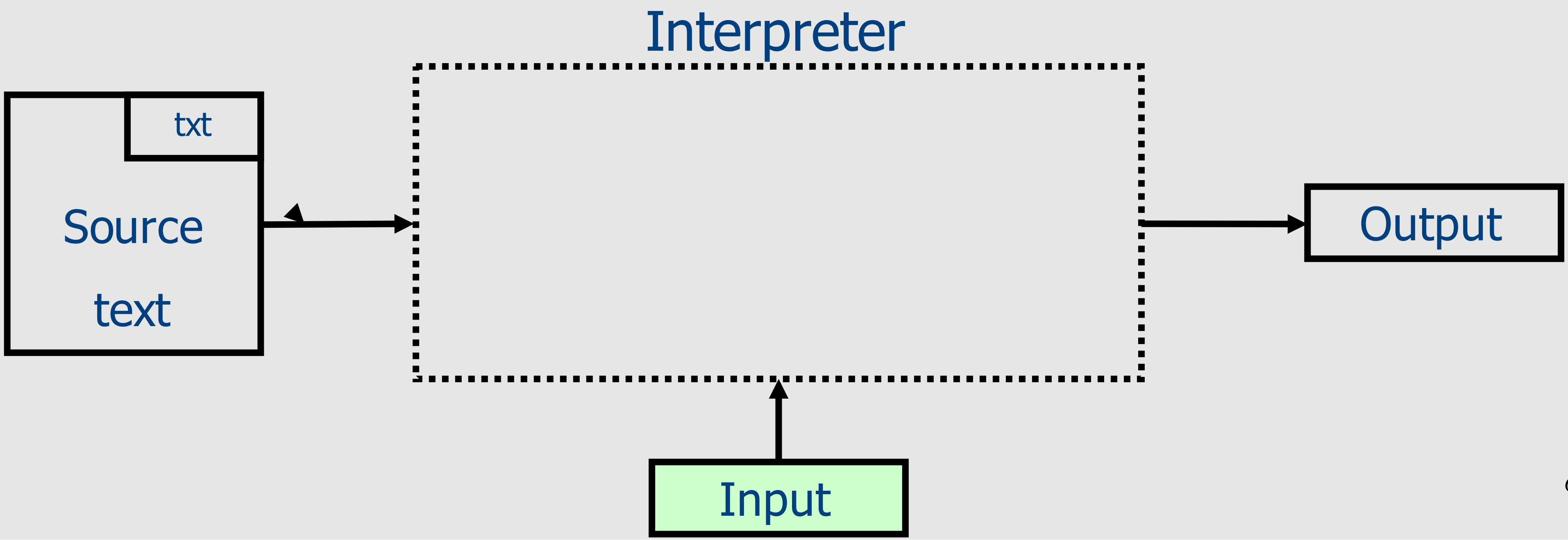
Compiler

- A program which transforms programs
- Input a program (P)
- Output an object program (O)
  - For any x, “O(x)” “=“ “P(x)”

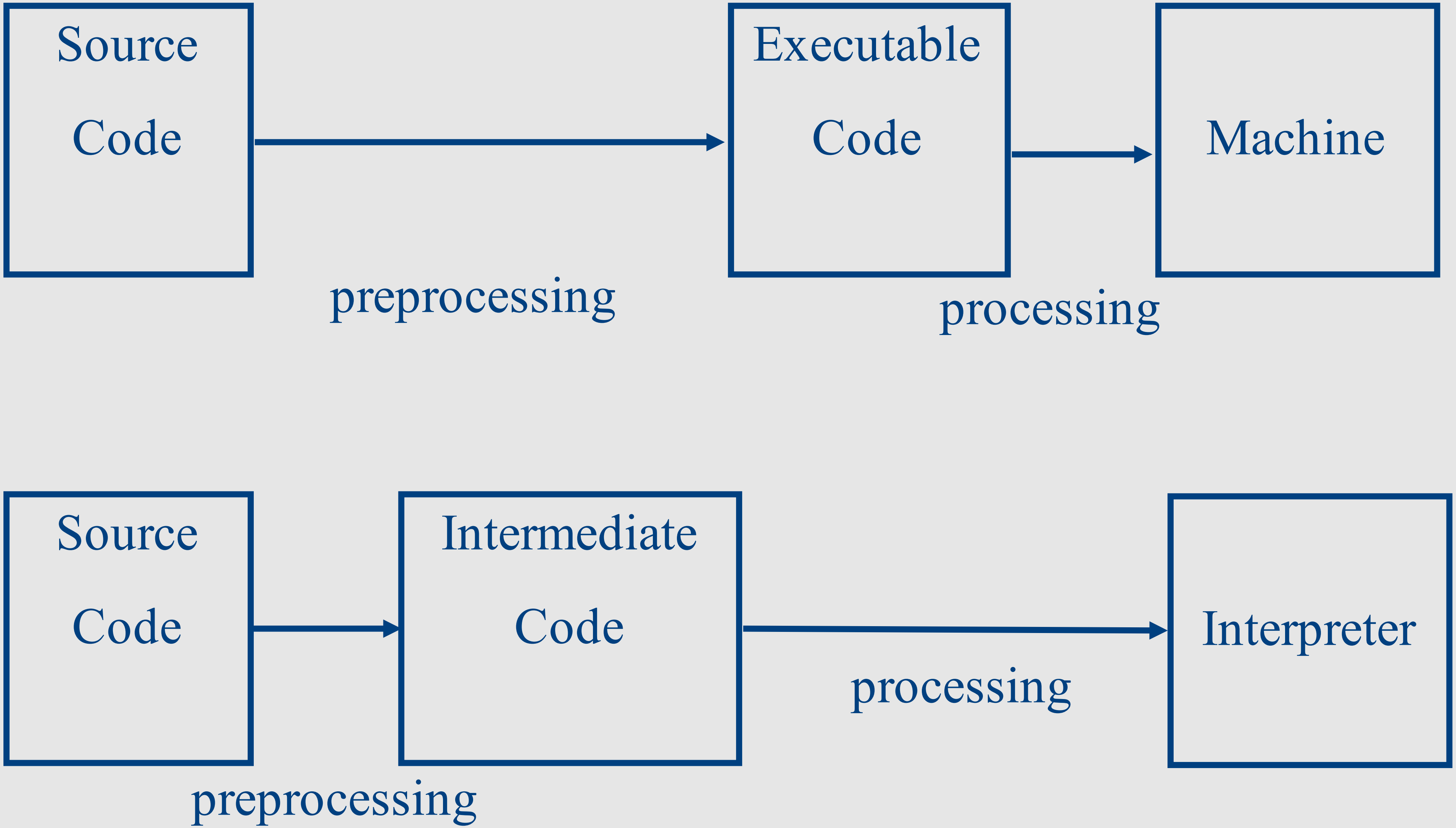


Interpreter

- A program which executes a program
- Input a program (P) + its input (x)
- Output the computed output (P(x))



Compiler vs. Interpreter



Interpreter vs. Compiler

Interpreter

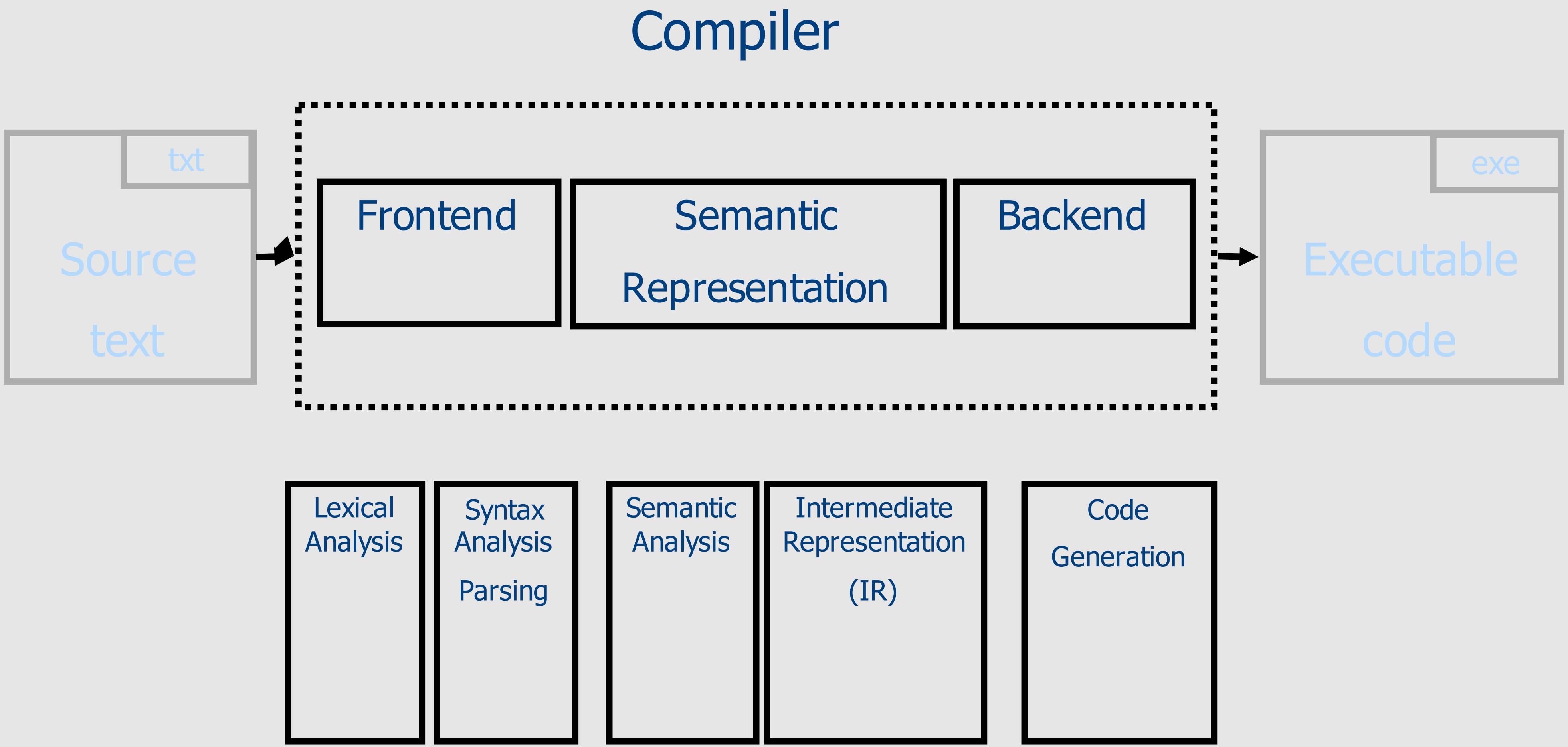
- Conceptually simpler
  - "define" the prog. lang.
- Can provide more specific error report
- Easier to port
- Faster response time
- [More secure]

Compiler

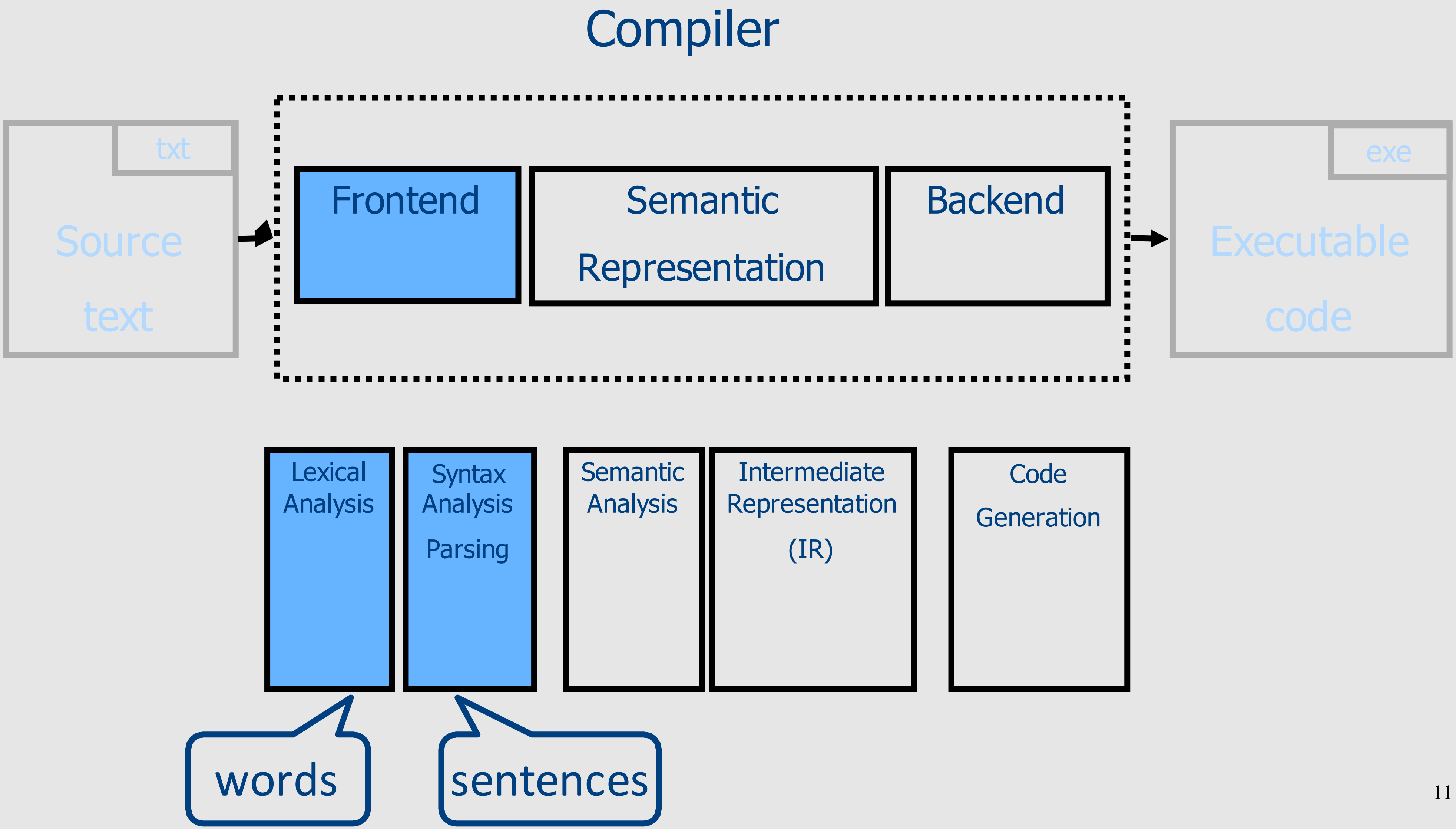
- How do we know the translation is correct?
- Can report errors before input is given
- More efficient code
  - Compilation can be expensive
  - move computations to compile-time
- *compile-time + execution-time < interpretation-time* is possible

Lexical Analysis

Conceptual Structure of a Compiler



Conceptual Structure of a Compiler



What does Lexical Analysis do?

- Partitions the input into stream of tokens
  - Numbers
  - Identifiers
  - Keywords
  - Punctuation

> - "word" in the source language
> - "meaningful" to the syntactical analysis

- Usually represented as (kind, value) pairs
  - (Num, 23)
  - (Op, '*')

Some basic terminology

- Lexeme (aka symbol) - a series of letters separated from the rest of the program according to a convention (space, semi-column, comma, etc.)

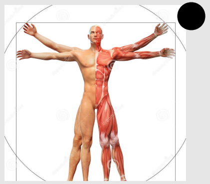
- Pattern - a rule specifying a set of strings. Example: “an identifier is a string that starts with a letter and continues with letters and digits”
  - (Usually) a regular expression

- Token - a pair of (pattern, attributes)

Regular languages

- Formal languages
  - Σ = finite set of letters
  - Word = sequence of letter
  - Language = set of words
- Regular languages defined equivalently by
  - Regular expressions
  - Finite-state automata

From regular expressions to NFA



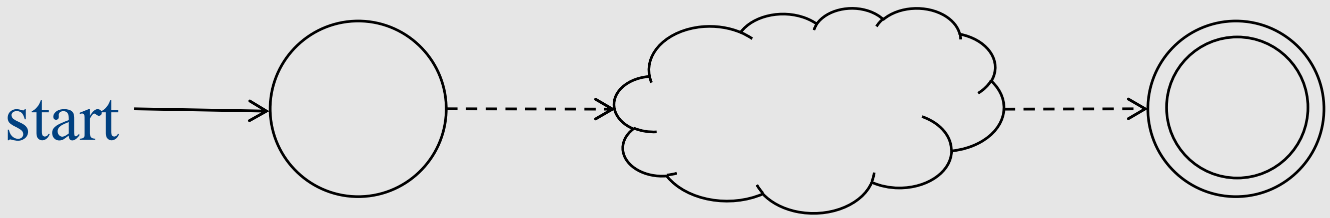
- Step 1: assign expression names and obtain pure regular expressions $R_1 \ldots R_m$

- Step 2: construct an NFA M_i for each regular expression R_i
- Step 3: combine all M_i into a single NFA

- *Ambiguity resolution: prefer longest accepting word*

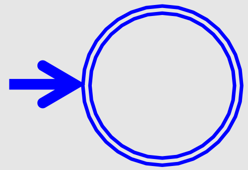
From reg. exp. to automata

- Theorem: *there is an algorithm to build an NFA+Є automaton for any regular expression*
- Proof: *by induction on the structure of the regular expression*

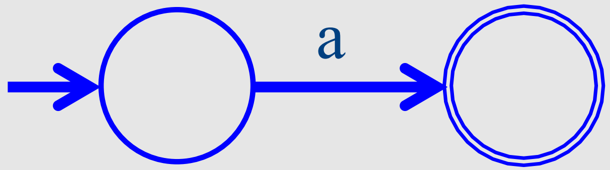


Basic constructs

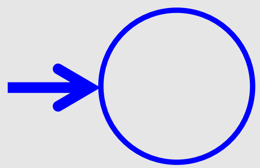
$R = \varepsilon$



$R = a$

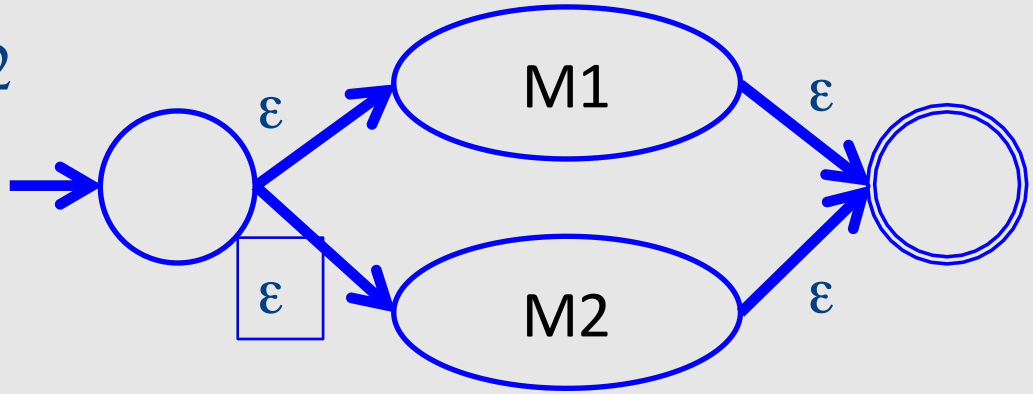


$R = \phi$

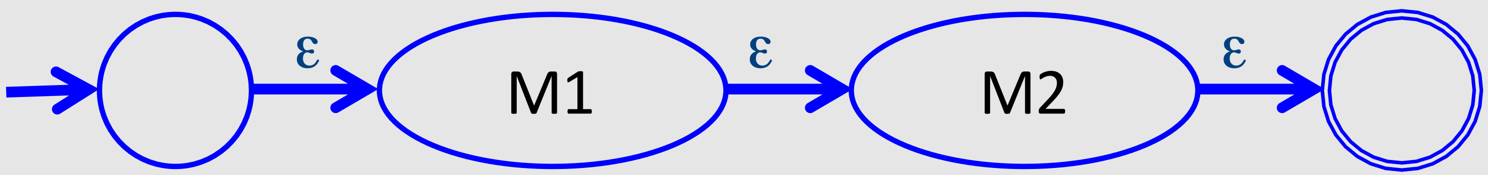


Composition

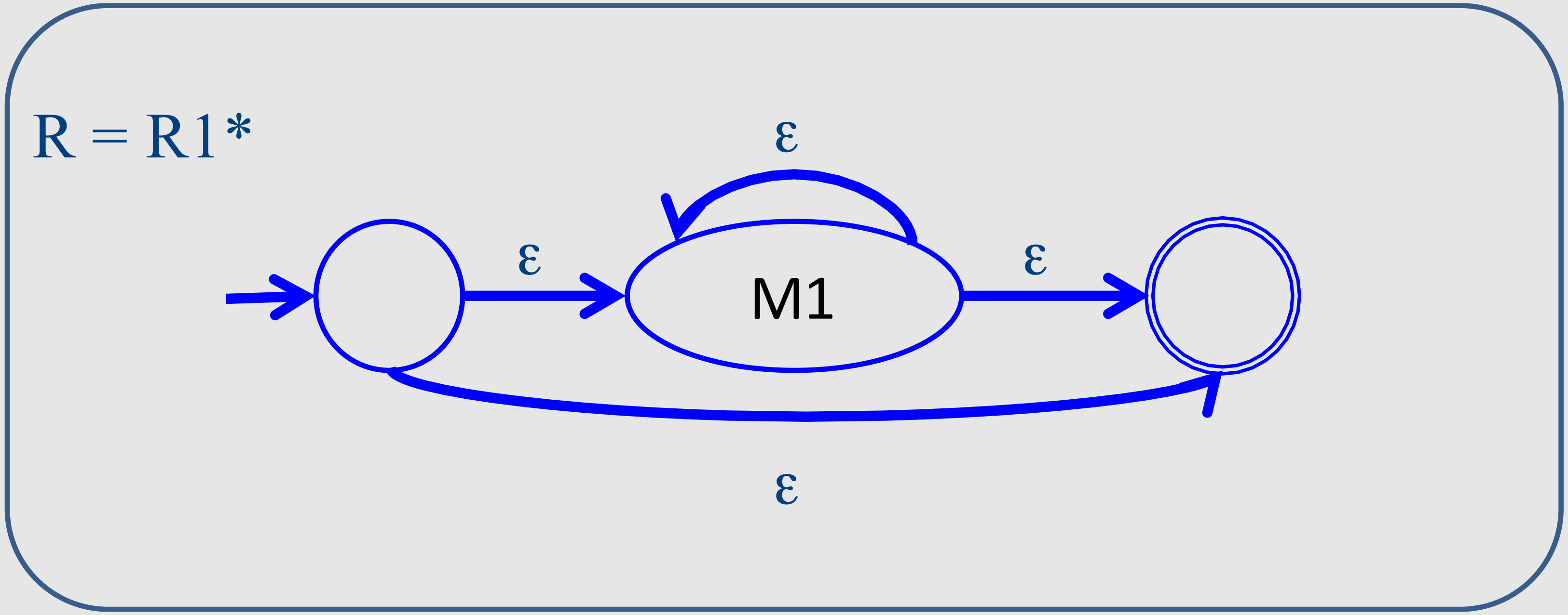
R = R1 | R2



R = R1R2



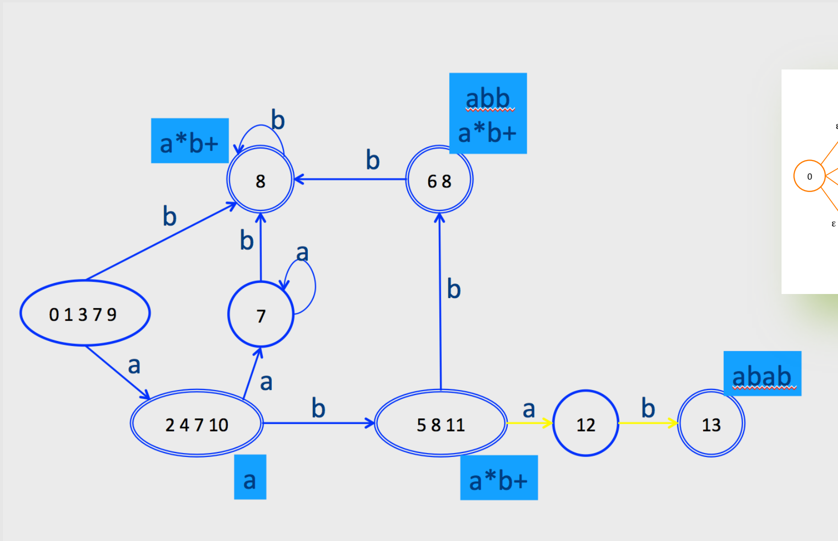
Repetition

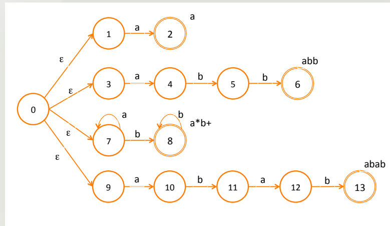


Scanning with DFA

- Run until stuck
  - **Remember last accepting state**
- Go back to accepting state
- Return token

Ambiguity resolution





- Longest word
- Tie-breaker based on **order of rules** when words have same length

Creating a Scanner using Flex

```
int num_lines = 0;
%%
\n       ++num_lines;
.        ;
%%
main() {
  yylex();
  printf( "# of lines = %d\n", num_lines);
}
```

Syntax Analysis

Frontend: Scanning & Parsing

program text **((23 + 7) * x)**

↓ Lexical Analyzer ↓

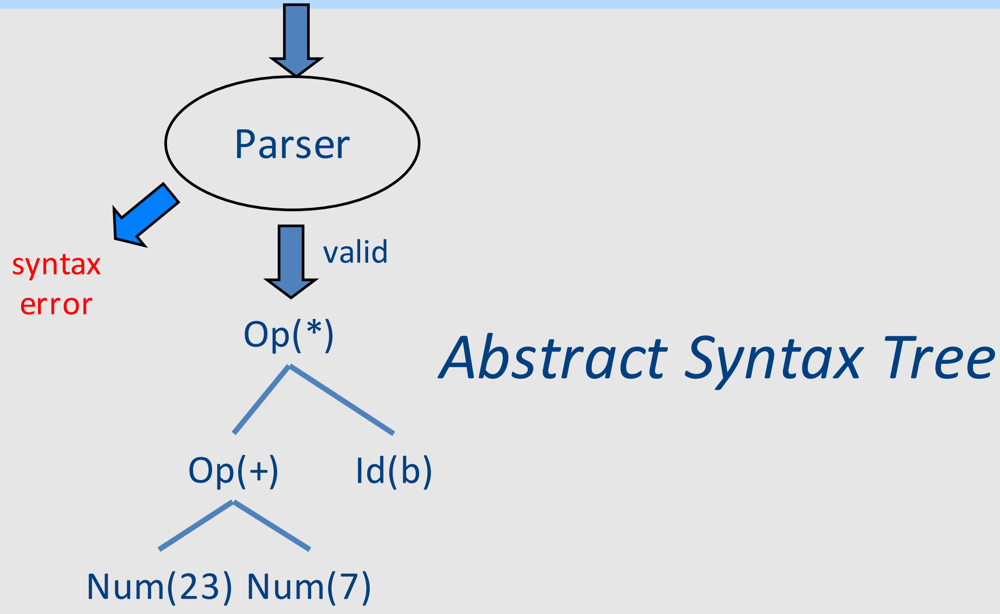
token stream

(	(	23	+	7	)	*	x	)
LP	LP	Num	OP	Num	RP	OP	Id	RP

Grammar:

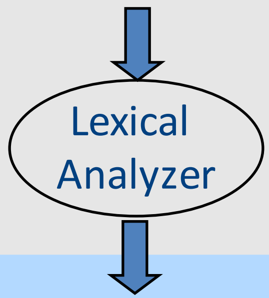
E → … | Id

Id → ‘a’ | … | ‘z’



From scanning to parsing

program text **((23 + 7) * x)**



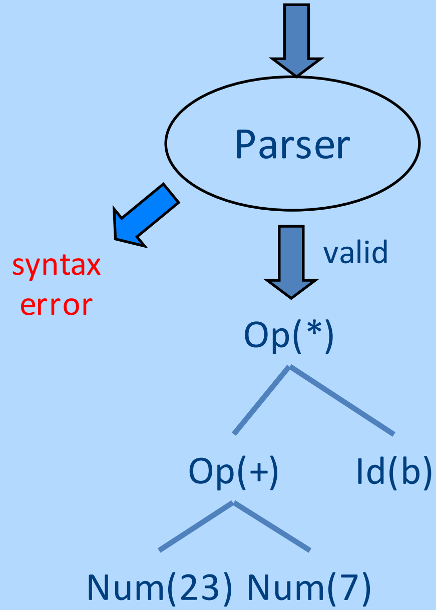
token stream

(	(	23	+	7	)	*	x	)
LP	**LP**	**Num**	**OP**	**Num**	**RP**	**OP**	**Id**	**RP**

Grammar:

E → ... | Id

Id → 'a' | ... | 'z'



Abstract Syntax Tree

Context free grammars (CFG)

$G = (V,T,P,S)$

- **V** – non terminals (syntactic variables)
- **T** – terminals (tokens)
- **P** – derivation rules
  - Each rule of the form $V \longrightarrow (T \cup V)^*$
- **S** – start symbol

Pushdown Automata (PDA)

- Nondeterministic PDAs define all CFLs
- Deterministic PDAs model parsers.
  - Most programming languages have a deterministic PDA
  - Efficient implementation

CFG terminology

- Derivation - a sequence of replacements of non-terminals using the derivation rules
- Language - the set of strings of terminals derivable from the start symbol
- Sentential form - the result of a partial derivation
  - May contain non-terminals

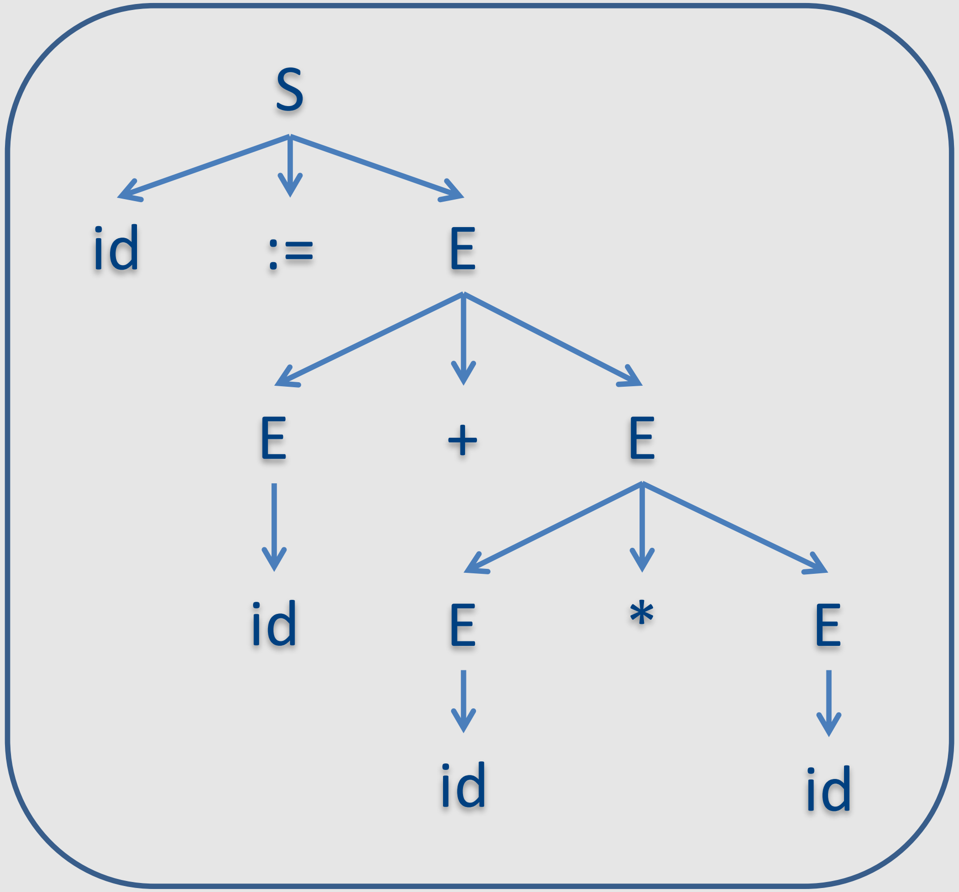
Derivations

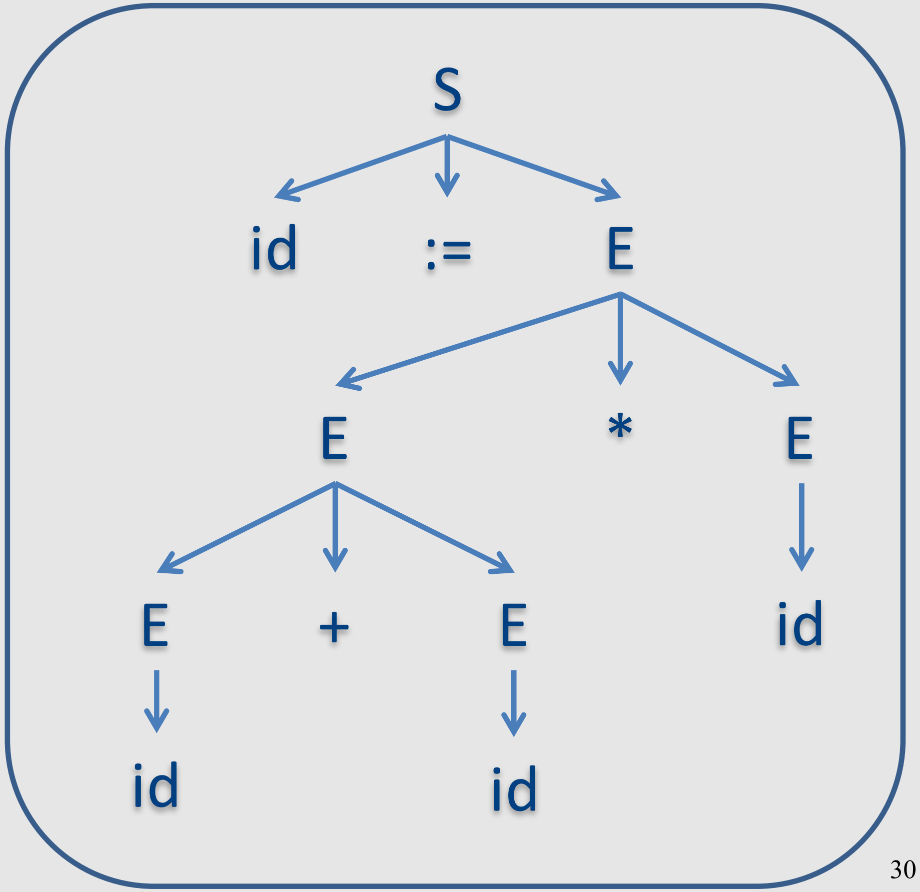
- Show that a sentence ω is in a grammar G
  - Start with the start symbol
  - Repeatedly replace one of the non-terminals by a right-hand side of a production
  - Stop when the sentence contains only terminals
- Given a sentence $\alpha N \beta$ and rule $N \rightarrow \mu$
  $\alpha N \beta \Rightarrow \alpha \mu \beta$
- ω is in L(G) if $S \Rightarrow^* \omega$

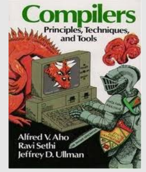
Ambiguity

x := y+z*w

```
S → S ; S
S → id := E | …
E → id | E + E | E * E | …
```







p. 174

“dangling-else” example

Ambiguous grammar

S → **if** E **then** S
S | **if** E **then** S **else** S
| other

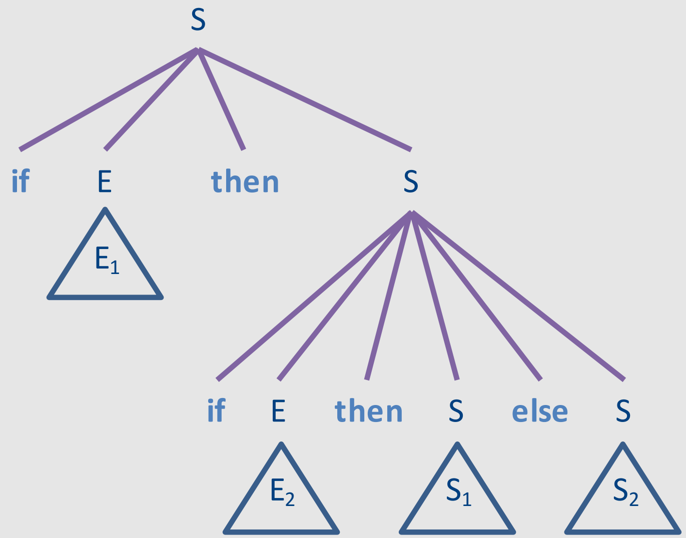
This is what we usually want: match **else** to closest unmatched **then**

Unambiguous grammar

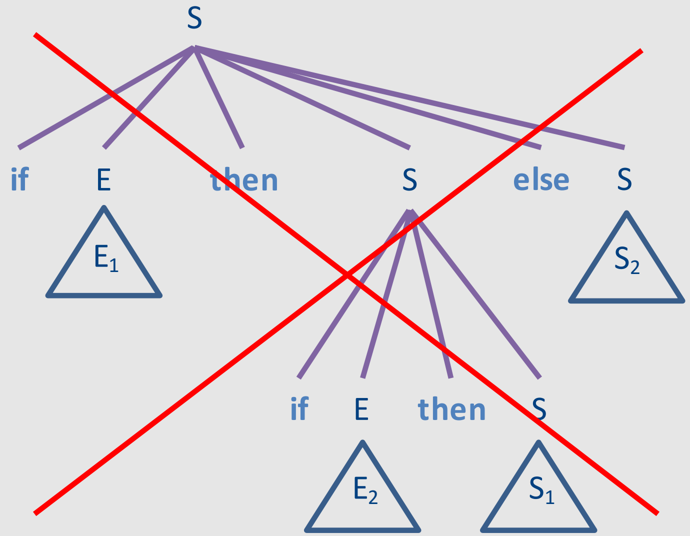
?

if E_1 **then if** E_2 **then** S_1 **else** S_2

if E_1 **then (if** E_2 **then** S_1 **else** S_2**)**



if E_1 **then (if** E_2 **then** S_1**) else** S_2



Broad kinds of parsers

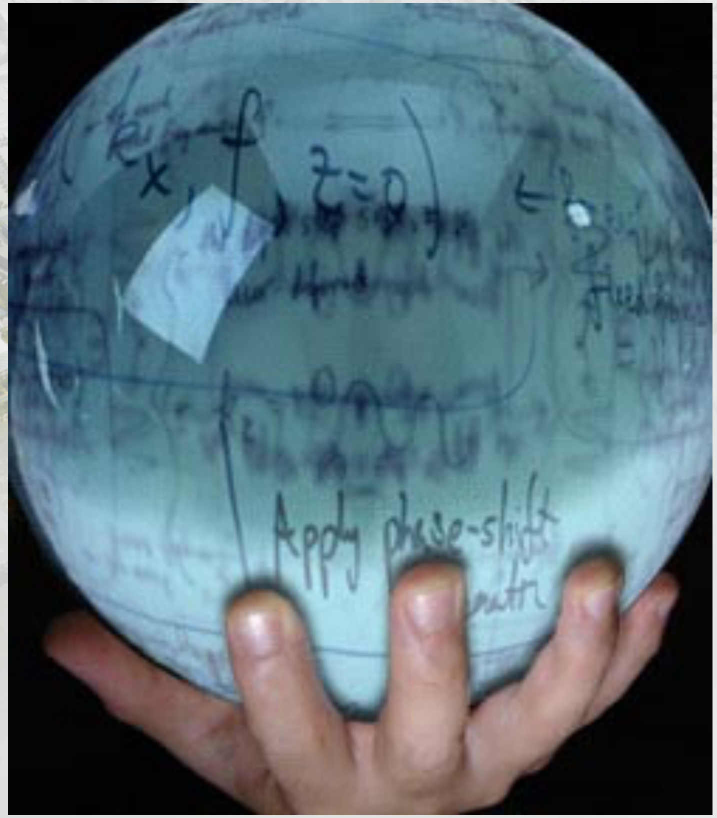
- Parsers for arbitrary grammars
  - Earley's method, CYK method
  - Usually, not used in practice (though might change)
- Top-down parsers
  - Construct parse tree in a top-down matter
  - Find the leftmost derivation
- Bottom-up parsers
  - Construct parse tree in a bottom-up manner
  - Find the rightmost derivation in a reverse order

Predictive parsing

- Given a grammar G and a word w attempt to derive w using G
- Idea
  - Apply production to leftmost nonterminal
  - Pick production rule based on next input token
- General grammar
  - More than one option for choosing the next production based on a token
- Restricted grammars (LL)
  - Know exactly which single rule to apply
  - May require some lookahead to decide

Top-Down Parsing: Predictive parsing

- Recursive descent
- LL(k) grammars



Recursive descent parsing

- Define a function for every nonterminal
- Every function work as follows
  - Find applicable production rule
  - Terminal function checks match with next input token
  - Nonterminal function calls (recursively) other functions
- If there are several applicable productions for a nonterminal, use lookahead

LL(k) grammars

- A grammar is in the class LL(K) when it can be derived via:
  - Top-down derivation
  - Scanning the input from left to right (L)
  - Producing the leftmost derivation (L)
  - With lookahead of k tokens (k)
- A language is said to be LL(k) when it has an LL(k) grammar

FIRST sets

- FIRST(X) = { t | X $\rightarrow^*$ t β} ∪ {Ɛ | X $\rightarrow^*$ Ɛ}
  - FIRST(X) = all terminals that α can appear as first in some derivation for X
    - + Ɛ if can be derived from X

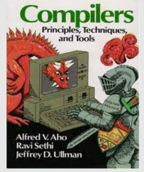
- Example:
  - FIRST(LIT) = { true, false }
  - FIRST((E OP E)) = { '(' }
  - FIRST(not E) = { not }

FIRST sets

- No intersection between FIRST sets => can always pick a single rule
- If the FIRST sets intersect, may need longer lookahead
  - LL(k) = class of grammars in which production rule can be determined using a lookahead of k tokens
  - LL(1) is an important and useful class

LL(1) grammars

- A grammar is in the class LL(K) iff
  - For every two productions $A \rightarrow \alpha$ and $A \rightarrow \beta$ we have
    - FIRST(α) $\cap$ FIRST(β) = {} // including ε
    - If $\varepsilon \in$ FIRST(α) then FIRST(β) $\cap$ FOLLOW(A) = {}
    - If $\varepsilon \in$ FIRST(β) then FIRST(α) $\cap$ FOLLOW(A) = {}



p. 189

FOLLOW sets

- What do we do with nullable (ε) productions?
  - A → B C D   B → ε C → ε
  - Use what comes afterwards to predict the right production
- For every production rule A → α
  - FOLLOW(A) = set of tokens that can immediately follow A
- Can predict the alternative A_k for a non-terminal N when the lookahead token is in the set
  - FIRST(A_k) → (if A_k is nullable then FOLLOW(N))

FOLLOW sets: Constraints

- $ $\in$ FOLLOW(S)

- FIRST(β) – {ε} $\subseteq$ FOLLOW(X)
  - For each A → α X β

- FOLLOW(A) $\subseteq$ FOLLOW(X)
  - For each A → α X β and ε $\in$ FIRST(β)

Prediction Table

- $A \rightarrow \alpha$

- $T[A,t] = \alpha$ if $t \in FIRST(\alpha)$
- $T[A,t] = \alpha$ if $\varepsilon \in FIRST(\alpha)$ and $t \in FOLLOW(A)$
  - t can also be $

- T is not well defined ➔ the grammar is not LL(1)

Problem 1: productions with common prefix

term → ID | indexed_elem
indexed_elem → ID [expr]

- FIRST(term) = { ID }
- FIRST(indexed_elem) = { ID }

- FIRST/FIRST conflict

Solution: left factoring

- Rewrite the grammar to be in LL(1)

term → ID | indexed_elem
indexed_elem → ID [expr]

⇩

term → ID after_ID
After_ID → [expr] | ε

Intuition: just like factoring x*y + x*z into x*(y+z)

Problem 2: null productions

S → A a b
A → a | ε

- FIRST(S) = { a } FOLLOW(S) = { }
- FIRST(A) = { a , ε } FOLLOW(A) = { a }

- FIRST/FOLLOW conflict

Solution: substitution

$S \rightarrow A\ a\ b$
$A \rightarrow a \mid \varepsilon$

Substitute A in S

$S \rightarrow a\ a\ b \mid a\ b$

Left factoring

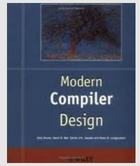
$S \rightarrow a\ after\_A$
$after\_A \rightarrow a\ b \mid b$

Problem 3: left recursion

$E \rightarrow E$ - term | term

- Left recursion cannot be handled with a bounded lookahead

- What can we do?



p. 130

Left recursion removal

$N \rightarrow N\alpha \mid \beta$

G_1

$N \rightarrow \beta N'$
$N' \rightarrow \alpha N' \mid \varepsilon$

G_2

Can be done algorithmically.
Problem: grammar becomes mangled beyond recognition

- $L(G_1) = \beta, \beta\alpha, \beta\alpha\alpha, \beta\alpha\alpha\alpha, \ldots$
- $L(G_2)$ = same

- For our 3rd example:

E → E - term | term

E → term TE | term
TE → - term TE | ε

Bottom-up parsing

Bottom-up parsing: LR(k) Grammars

- A grammar is in the class LR(K) when it can be derived via:
  - Bottom-up derivation
  - Scanning the input from left to right (L)
  - Producing the rightmost derivation (R)
  - With lookahead of k tokens (k)
- A language is said to be LR(k) if it has an LR(k) grammar
- The simplest case is LR(0), which we will discuss

Terminology: Reductions & Handles

- The opposite of derivation is called *reduction*
  - Let A ➔ α be a production rule
  - Derivation: βAμ ➔ βαμ
  - Reduction: βαμ ➔ βAμ

- A *handle* is the reduced substring
  - α is the handles for βαμ

How does the parser know what to do?

- A state will keep the info gathered on handle(s)
  - A state in the "control" of the PDA
  - Also (part of) the stack alpha bet

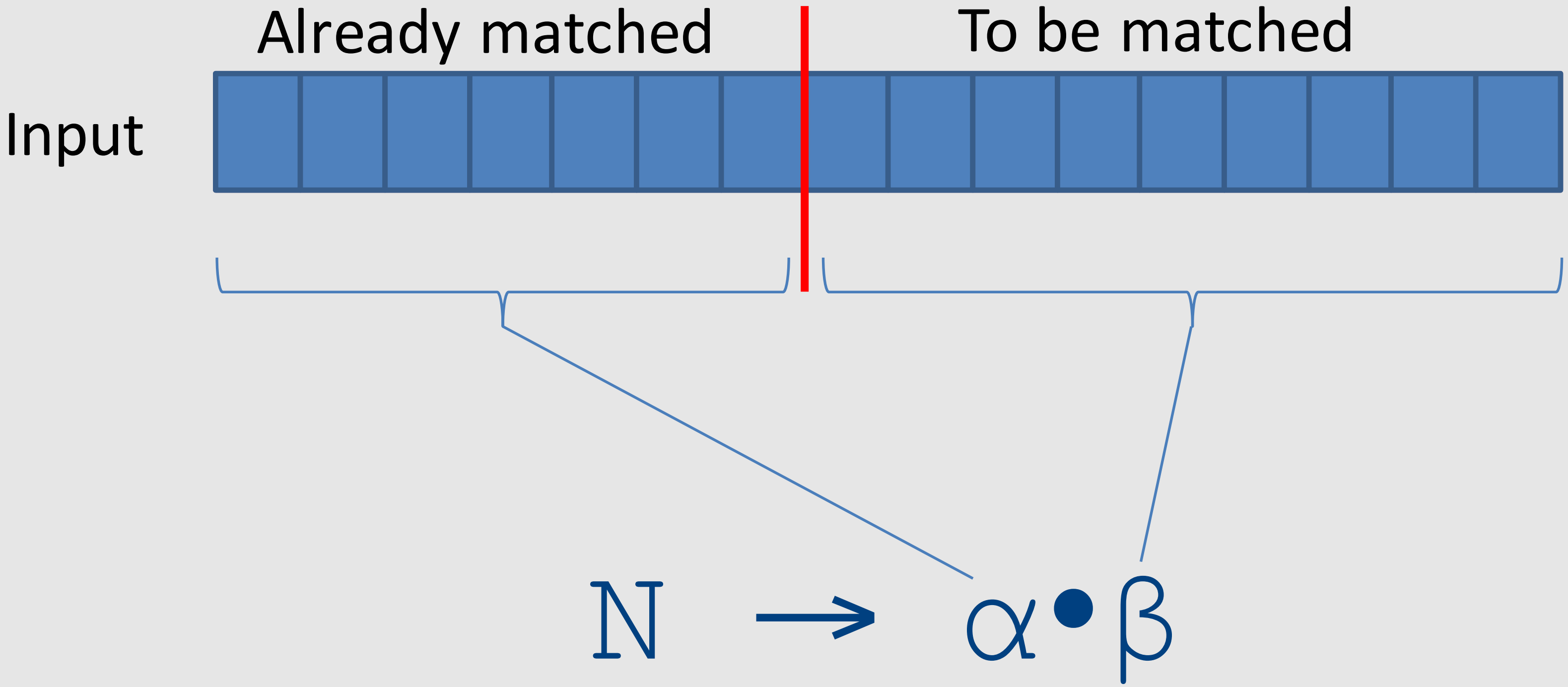
Set of LR(0) items

- A table will tell it "what to do" based on current state and next token
  - The transition function of the PDA
- A stack will records the "nesting level"
  - Prefixes of handles

Constructing an LR parsing table

- Construct a (determinized) transition diagram from LR items
- If there are conflicts – stop
- Fill table entries from diagram

LR item



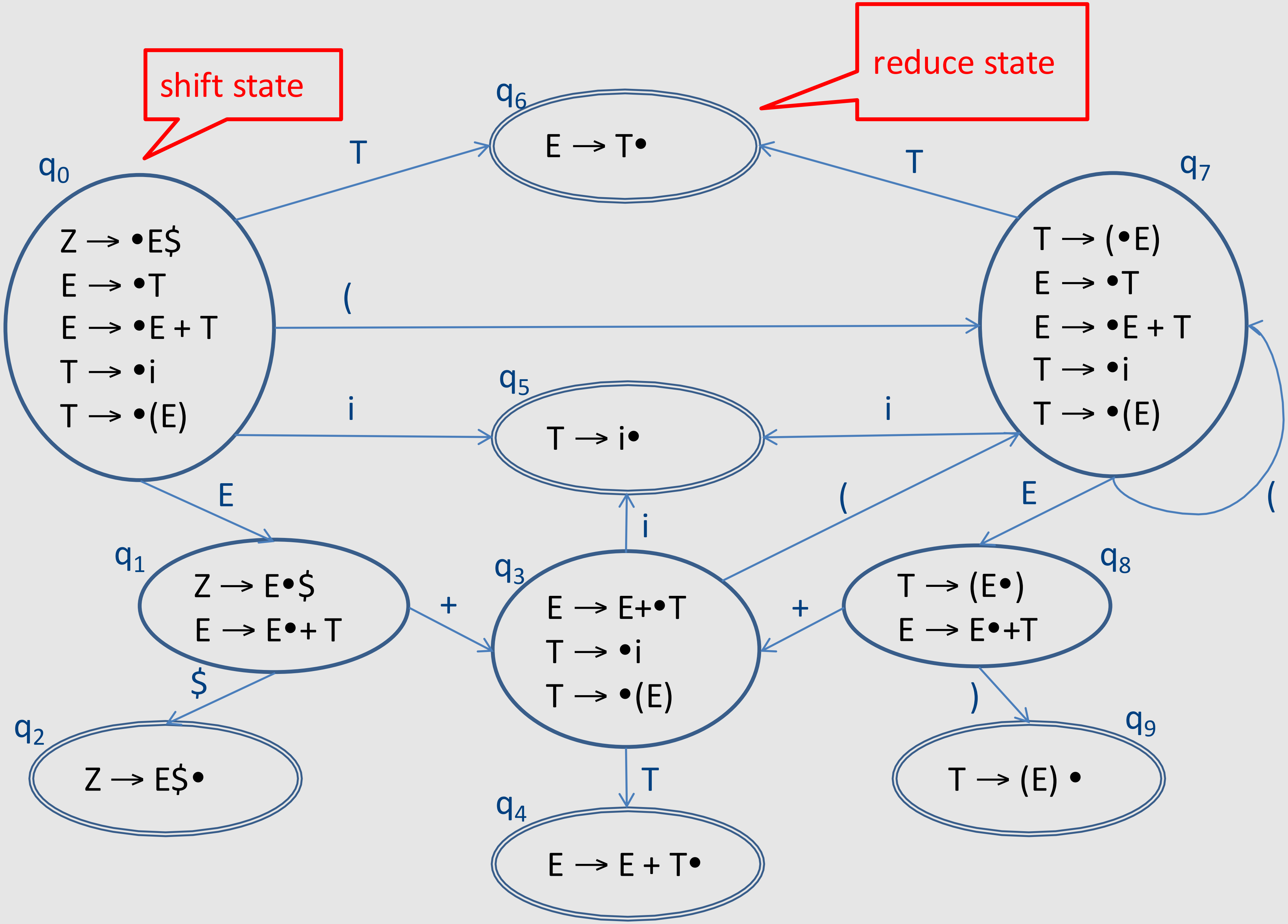
Hypothesis about αβ being a possible handle, so far we've matched α, expecting to see β

Types of LR(0) items

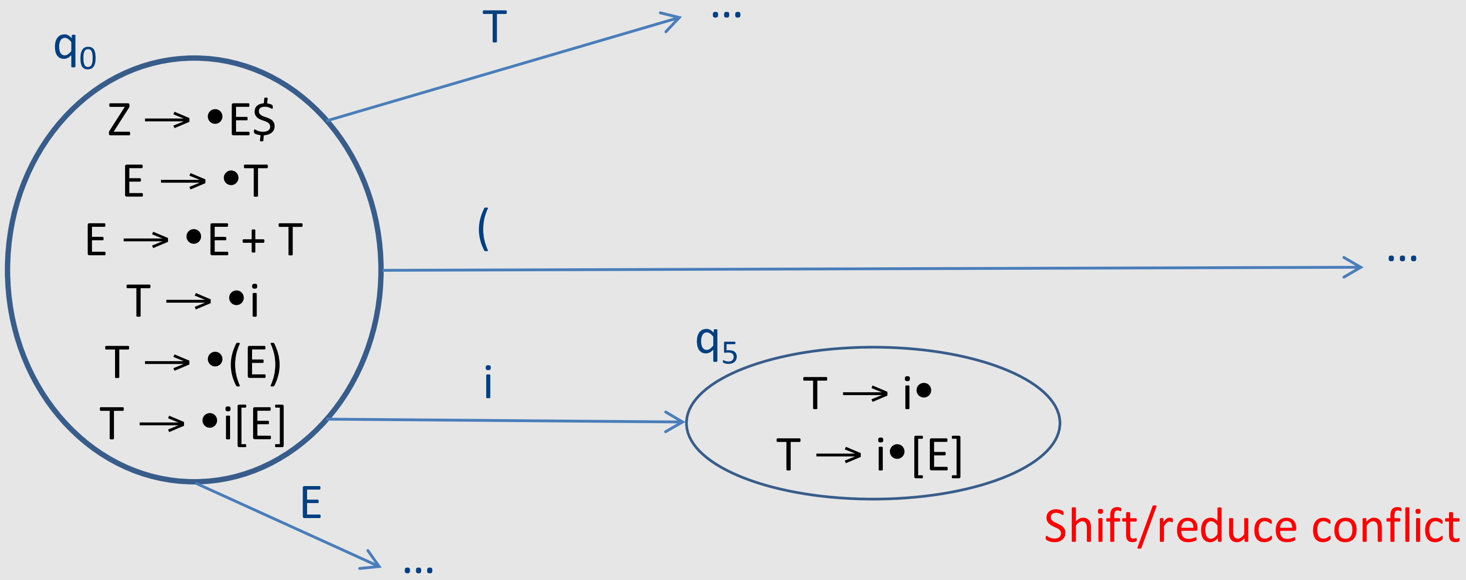
$N \rightarrow \alpha \bullet \beta$ Shift Item

$N \rightarrow \alpha \beta \bullet$ Reduce Item

LR(0) automaton example

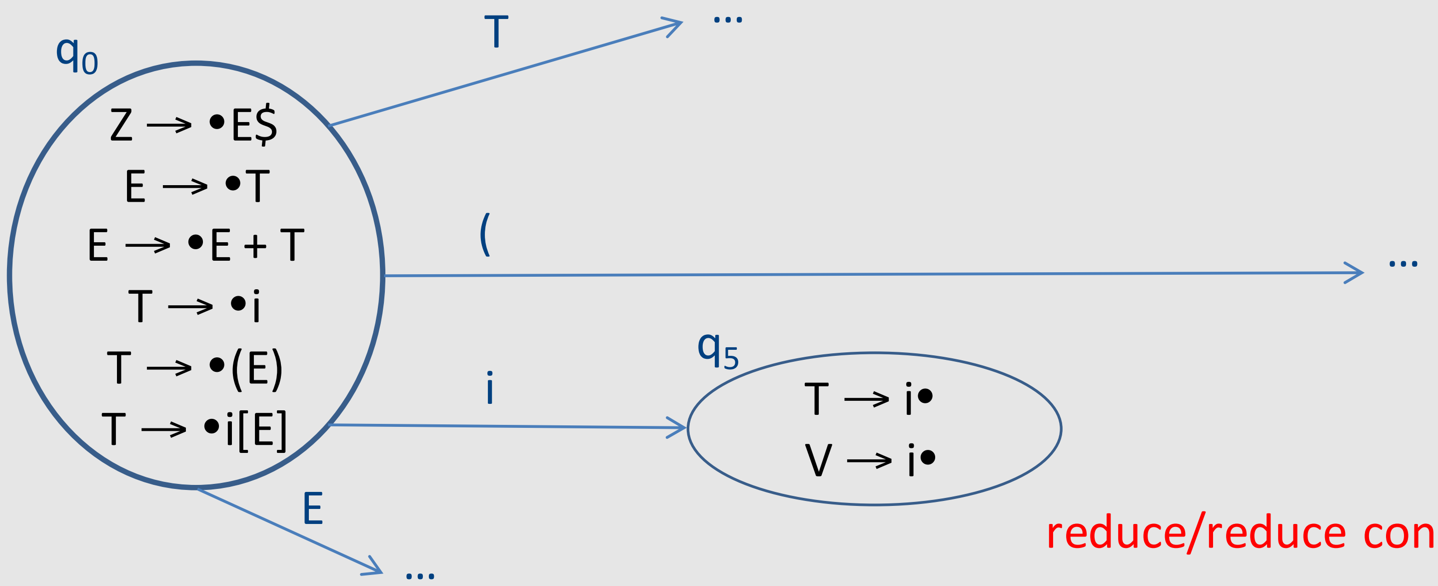


LR(0) conflicts



Z → E $
E → T
E → E + T
T → i
T → (E)
T → i[E]

LR(0) conflicts



reduce/reduce conflict

Z → E $
E → T
E → E + T
T → i
V → i
T → (E)

LR(0) conflicts

- Any grammar with an ε-rule cannot be LR(0)
- Inherent shift/reduce conflict
  - $A \rightarrow \varepsilon\bullet$ – reduce item
  - $P \rightarrow \alpha\bullet A\beta$ – shift item
  - $A \rightarrow \varepsilon\bullet$ can always be predicted from $P \rightarrow \alpha\bullet A\beta$

LR variants

- LR(0) – what we've seen so far
- SLR(0)
  - Removes infeasible reduce actions via FOLLOW set reasoning
- LR(1)
  - LR(0) with one lookahead token in items
- LALR(0)
  - LR(1) with merging of states with same LR(0) component

Semantic Analysis

Abstract Syntax Tree

- AST is a simplification of the parse tree
- Can be built by traversing the parse tree
  - E.g., using visitors
- Can be built directly during parsing
  - Add an action to perform on each production rule
  - Similarly to the way a parse tree is constructed

Abstract Syntax Tree

- The interface between the parser and the rest of the compiler
  - Separation of concerns
  - Reusable, modular and extensible
- The AST is defined by a context free grammar
  - The grammar of the AST can be ambiguous!
    - E→ E + E
    - Is this a problem?
- Keep syntactic information
  - Why?

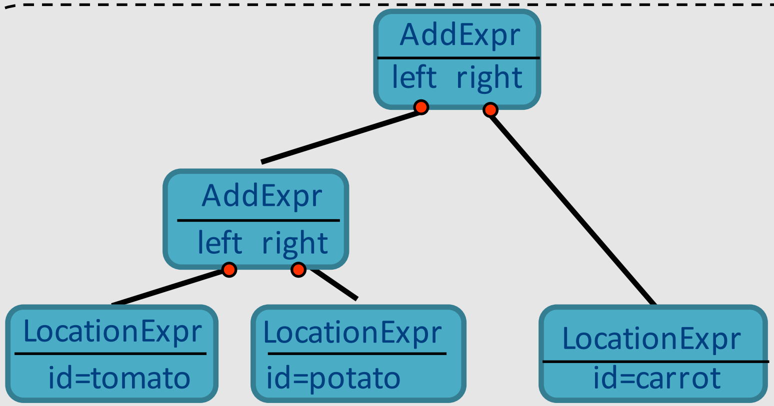
What we want

Potato potato;
Carrot carrot;
x = tomato + potato + carrot

Lexical analyzer

...<id,tomato>,<**PLUS**>,<id,potato>,<**PLUS**>,<id,carrot>,EOF

Parser



symbol	kind	type	properties
x	var	?	
tomato	var	?	
potato	var	Potato	
carrot	var	Carrot	

‘tomato’ is undefined

‘potato’ used before initialized

Cannot add Potato and Carrot

Context Analysis

- Check properties contexts of in which constructs occur
  - Properties that cannot be formulated via CFG
    - Type checking
    - Declare before use
      - Identifying the same word “w” re-appearing – wbw
    - Initialization
    - …
  - Properties that are hard to formulate via CFG
    - “break” only appears inside a loop
    - …
- Processing of the AST

Context Analysis

- Identification
  - Gather information about each named item in the program
  - e.g., what is the declaration for each usage
- Context checking
  - Type checking
  - e.g., the condition in an if-statement is a Boolean

Scopes

- Typically stack structured scopes

- Scope entry
  - push new empty scope element
- Scope exit
  - pop scope element and discard its content
- Identifier declaration
  - identifier created inside top scope
- Identifier Lookup
  - Search for identifier top-down in scope stack

Scope and symbol table

- Scope x Identifier -> properties
  - Expensive lookup
- A better solution
  - hash table over identifiers

Types

- What is a type?
  - Simplest answer: a set of values + allowed operations
  - Integers, real numbers, booleans, …

- Why do we care?
  - Code generation: $1 := $1 + $2
  - Safety
    - Guarantee that certain errors cannot occur at runtime
  - Abstraction
    - Hide implementation details
  - Documentation
  - Optimization

Typing Rules

If E1 has type int and E2 has type int,
then E1 + E2 has type int

$$\frac{E1 : int \qquad E2 : int}{E1 + E2 : int}$$

Syntax Directed Translation

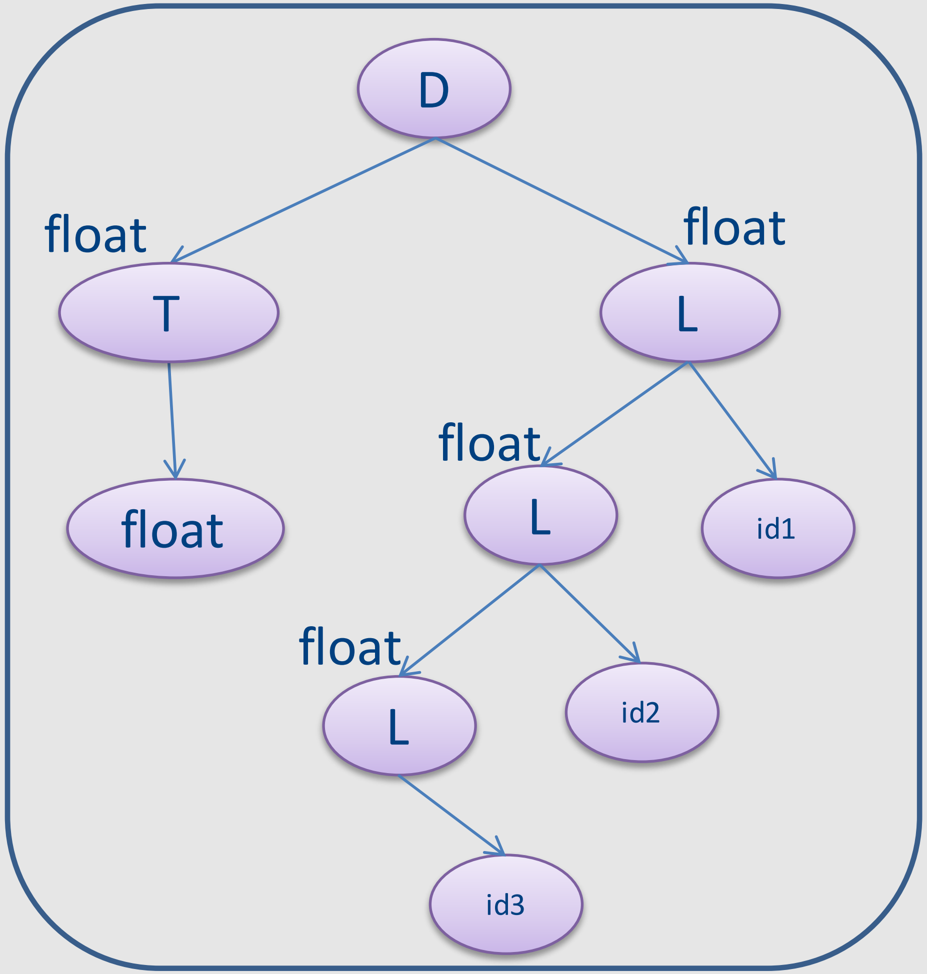
- Semantic attributes
  - Attributes attached to grammar symbols
- Semantic actions
  - How to update the attributes
- Attribute grammars

Attribute grammars

- Attributes
  - Every grammar symbol has attached attributes
    - Example: Expr.type
- Semantic actions
  - Every production rule can define how to assign values to attributes
    - Example:
      Expr → Expr + Term
      Expr.type = Expr1.type when (Expr1.type == Term.type)
      Error otherwise

Example

float x,y,z



Production	Semantic Rule
D → T L	L.in = T.type
T → int	T.type = integer
T → float	T.type = float
L → L1, id	L1.in = L.in addType(id.entry,L.in)
L → id	addType(id.entry,L.in)

Attribute Evaluation

- Build the AST
- Fill attributes of terminals with values derived from their representation
- Execute evaluation rules of the nodes to assign values until no new values can be assigned
  - In the right order such that
    - No attribute value is used before its available
    - Each attribute will get a value only once

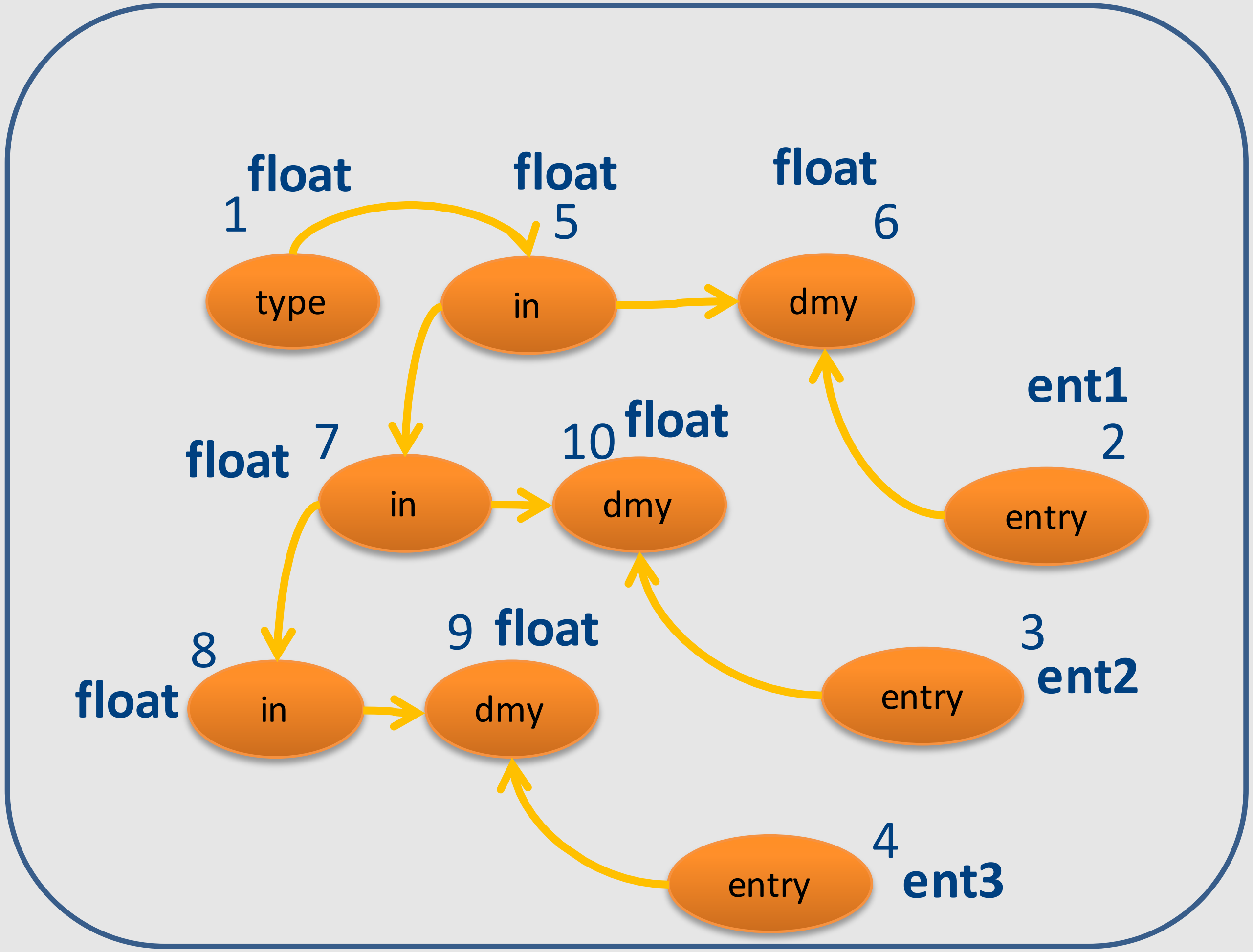
Dependencies

- A semantic equation a = b1,…,bm requires computation of b1,…,bm to determine the value of a

- The value of a depends on b1,…,bm
  - We write a → bi

Example

float x,y,z



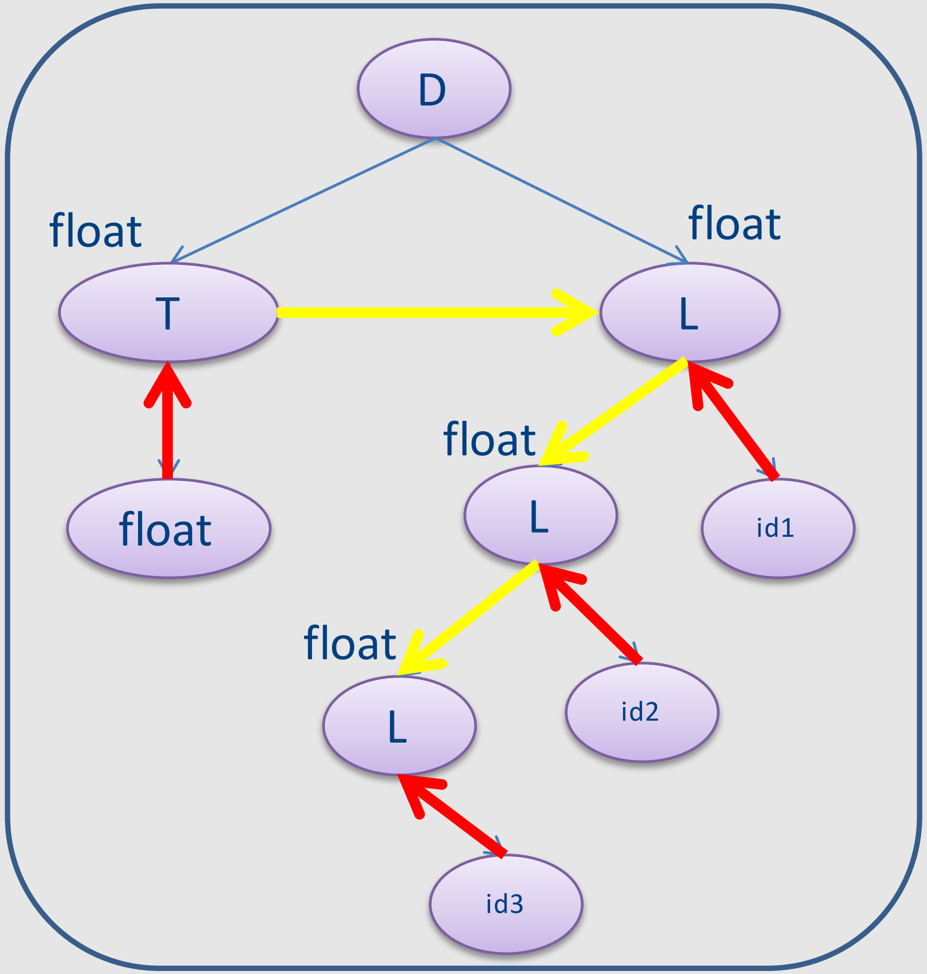
Inherited vs. Synthesized Attributes

- Synthesized attributes
  - Computed from children of a node
- Inherited attributes
  - Computed from parents and siblings of a node

- Attributes of tokens are technically considered as synthesized attributes

example

float x,y,z



Production	Semantic Rule
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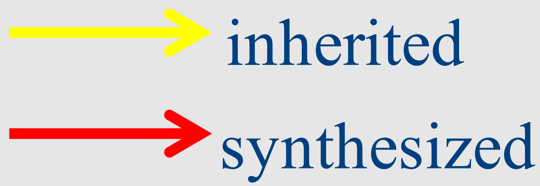
L → id	addType(id.entry,L.in)



S-attributed Grammars

- Special class of attribute grammars
- Only uses synthesized attributes (S-attributed)
- No use of inherited attributes

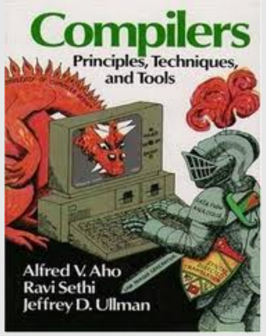
- Can be computed by any bottom-up parser during parsing
- Attributes can be stored on the parsing stack
- Reduce operation computes the (synthesized) attribute from attributes of children

L-attributed grammars

- L-attributed attribute grammar when every attribute in a production A → X1…Xn is
  - A synthesized attribute, or
  - An inherited attribute of Xj, 1 <= j <=n that only depends on
    - Attributes of X1…Xj-1 to the left of Xj, or
    - Inherited attributes of A

Intermediate Representation

Three-Address Code IR



Chapter 8

- A popular form of IR
- High-level assembly where instructions have at most three operands

Variable assignments

- var = constant ;
- var_1 = var_2 ;
- var_1 = var_2 **op** var_3 ;
- var_1 = constant **op** var_2 ;
- var_1 = var_2 **op** constant ;
- var = $constant_1$ **op** $constant_2$;
- Permitted operators are **+, -, *, /, %**

In the impl. var is replaced by a pointer to the symbol table

A compiler-generated temporary can be used instead of a var

Control flow instructions

- Label introduction

  ```
  _label_name:
  ```

  Indicates a point in the code that can be jumped to
- Unconditional jump: go to instruction following label L

  ```
  Goto L;
  ```
- Conditional jump: test condition variable t;
  if 0, jump to label L

  ```
  IfZ t Goto L;
  ```
- Similarly : test condition variable t;
  if not zero, jump to label L

  ```
  IfNZ t Goto L;
  ```

Procedures / Functions

- A procedure call instruction **pushes** arguments to stack and **jumps** to the function label
  A statement `x=f(a1,…,an);` looks like

```
Push a1; … Push an;
Call f;
Pop x; // pop returned value, and copy to it
```

- Returning a value is done by **pushing** it to the stack (`return x;`)

```
Push x;
```

- **Return control** to caller (and **roll up stack**)

```
Return;
```

TAC generation

- At this stage in compilation, we have
  - an AST
  - annotated with scope information
  - and annotated with type information
- To generate TAC for the program, we do recursive tree traversal
  - Generate TAC for any subexpressions or substatements
  - Using the result, generate TAC for the overall expression

cgen for binary operators

cgen(e_1 + e_2) = {

Choose a new temporary *t*

Let t_1 = **cgen**(e_1)

Let t_2 = **cgen**(e_2)

Emit($t = t_1 + t_2$)

Return *t*

}

cgen for `if-then-else`

cgen(if (e) s_1 else s_2)

Let _t = **cgen**(e)

Let L_{true} be a new label

Let L_{false} be a new label

Let L_{after} be a new label

Emit(IfZ _t Goto L_{false};)

cgen(s_1)

Emit(Goto L_{after};)

Emit(L_{false}:)

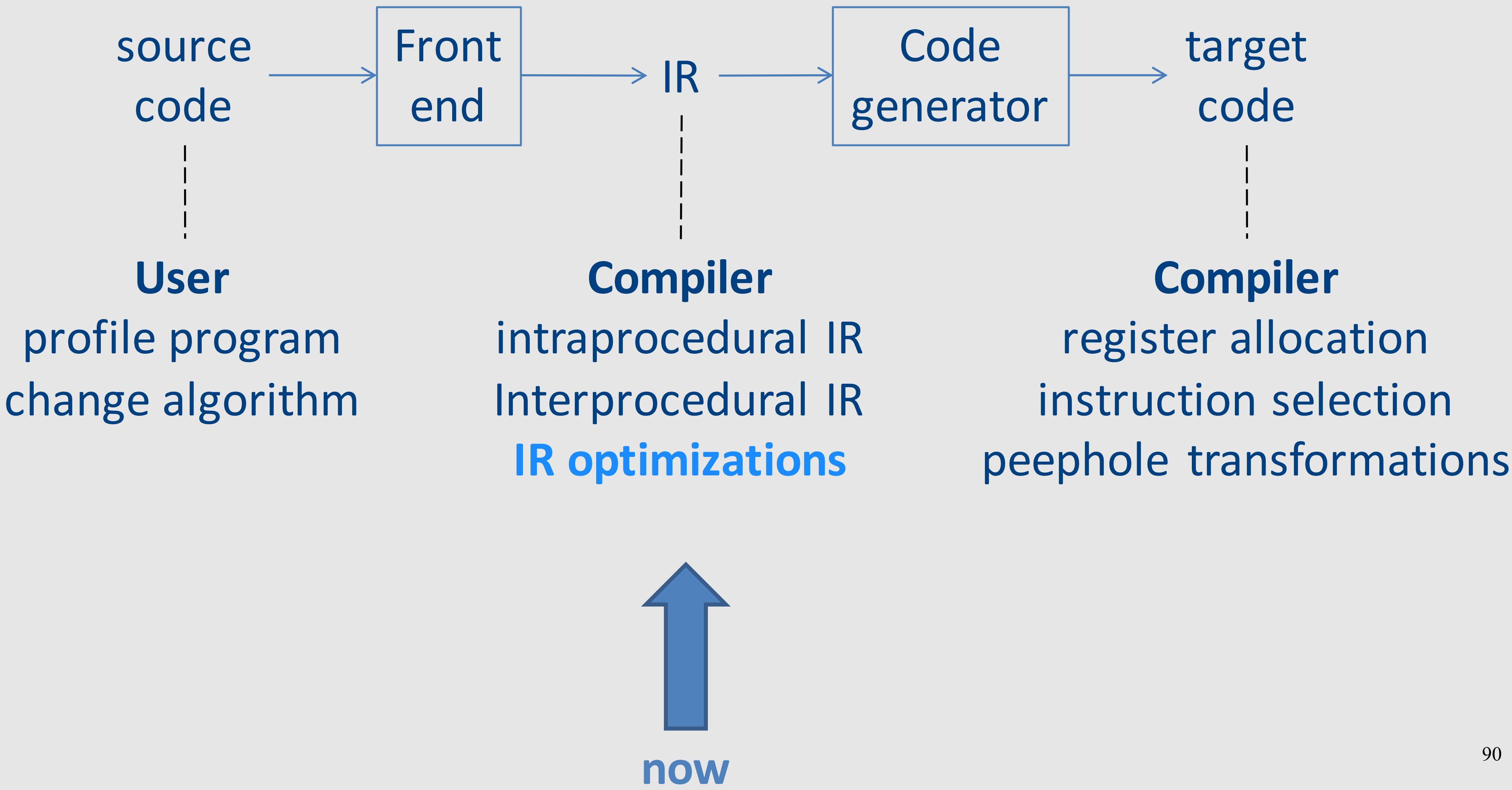
cgen(s_2)

Emit(Goto L_{after};)

Emit(L_{after}:)

IR Optimization

Optimization points

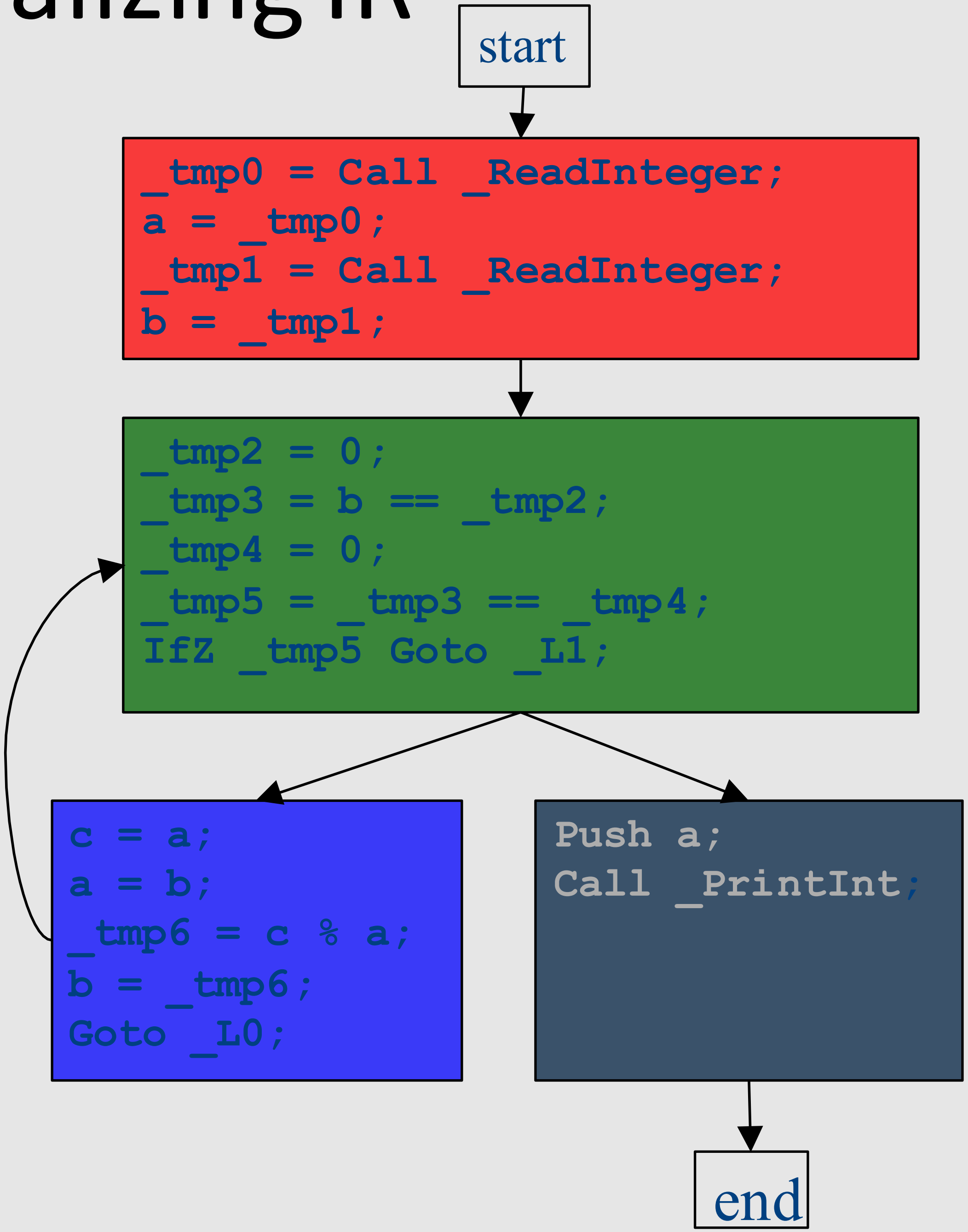


Overview of IR optimization

- Formalisms and Terminology
  - Control-flow graphs
  - Basic blocks
- Local optimizations
  - Speeding up small pieces of a procedure
- Global optimizations
  - Speeding up procedure as a whole
- The dataflow framework
  - Defining and implementing a wide class of optimizations

Visualizing IR

```
main:
    _tmp0 = Call _ReadInteger;
    a = _tmp0;
    _tmp1 = Call _ReadInteger;
    b = _tmp1;
_L0:
    _tmp2 = 0;
    _tmp3 = b == _tmp2;
    _tmp4 = 0;
    _tmp5 = _tmp3 == _tmp4;
    IfZ _tmp5 Goto _L1;
    c = a;
    a = b;
    _tmp6 = c % a;
    b = _tmp6;
    Goto _L0;
_L1:
    Push a;
    Call _PrintInt;
```



Control-Flow Graphs

- A control-flow graph (CFG) is a graph of the basic blocks in a function
- The term CFG is overloaded – from here on out, we'll mean “control-flow graph” and not “context free grammar”
- Each edge from one basic block to another indicates that control can flow from the end of the first block to the start of the second block
- There is a dedicated node for the start and end of a function

Common Subexpression Elimination

- If we have two variable assignments
  v1 = a op b
  …
  v2 = a op b
- and the values of v1, a, and b have not changed between the assignments, rewrite the code as
  v1 = a op b
  …
  v2 = v1
- Eliminates useless recalculation
- Paves the way for later optimizations

Common Subexpression Elimination

- If we have two variable assignments
  v1 = a op b   [or: v1 = a]
  …
  v2 = a op b   [or: v2 = a]
- and the values of v1, a, and b have not changed between the assignments, rewrite the code as
  v1 = a op b   [or: v1 = a]
  …
  v2 = v1
- Eliminates useless recalculation
- Paves the way for later optimizations

Copy Propagation

- If we have a variable assignment
  v1 = v2
  then as long as v1 and v2 are not reassigned, we can rewrite expressions of the form
  a = … v1 …
  as
  a = … v2 …
  provided that such a rewrite is legal

Dead Code Elimination

- An assignment to a variable v is called dead if the value of that assignment is never read anywhere
- Dead code elimination removes dead assignments from IR
- Determining whether an assignment is dead depends on what variable is being assigned to and when it's being assigned

Live variables

- The analysis corresponding to dead code elimination is called liveness analysis
- A variable is live at a point in a program if later in the program its value will be read before it is written to again
- Dead code elimination works by computing liveness for each variable, then eliminating assignments to dead variables

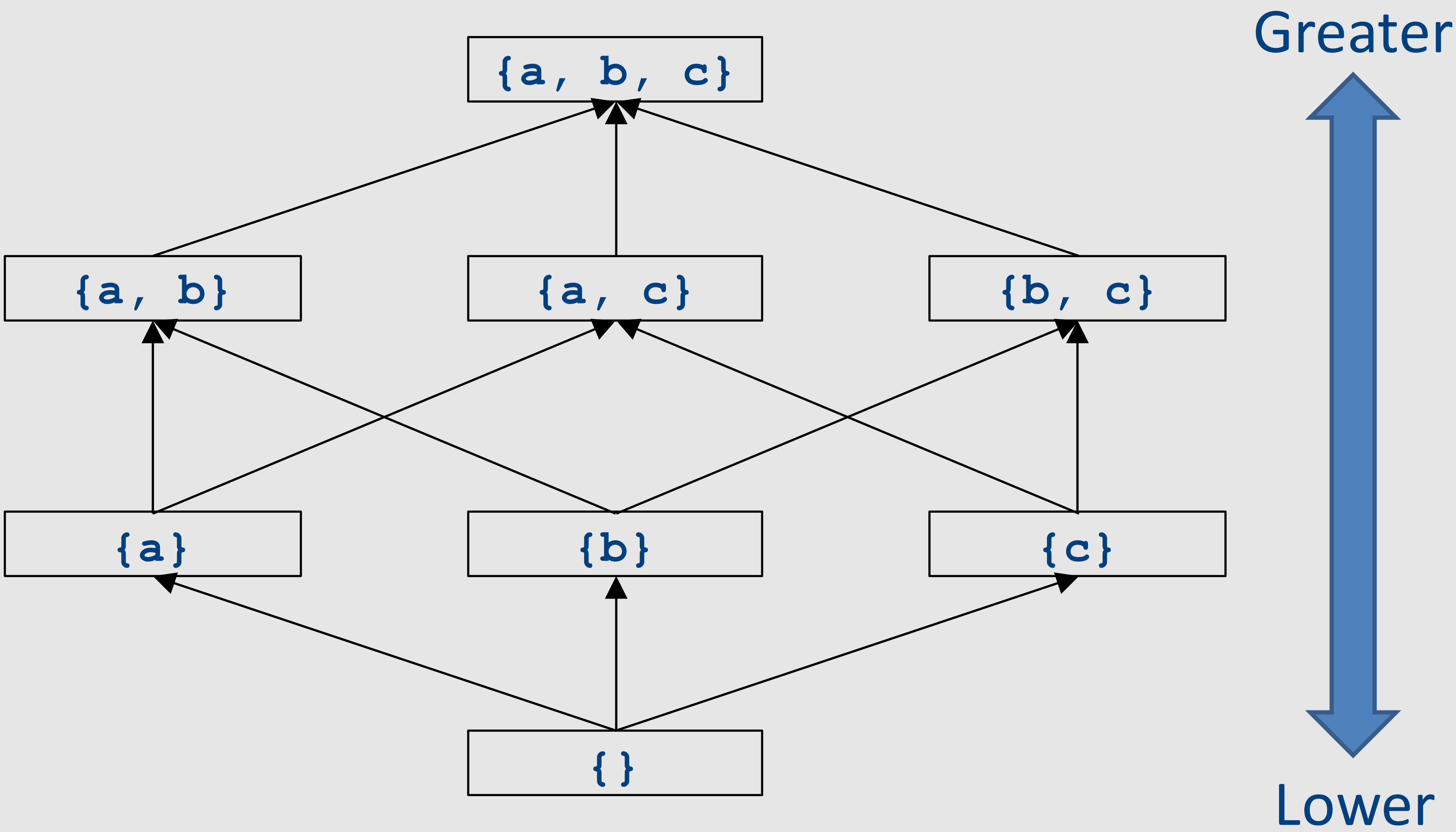
Local vs. global optimizations

- An optimization is local if it works on just a single basic block
- An optimization is global if it works on an entire control-flow graph of a procedure
- An optimization is interprocedural if it works across the control-flow graphs of multiple procedure
  - We won't talk about this in this course

Abstract Interpretation

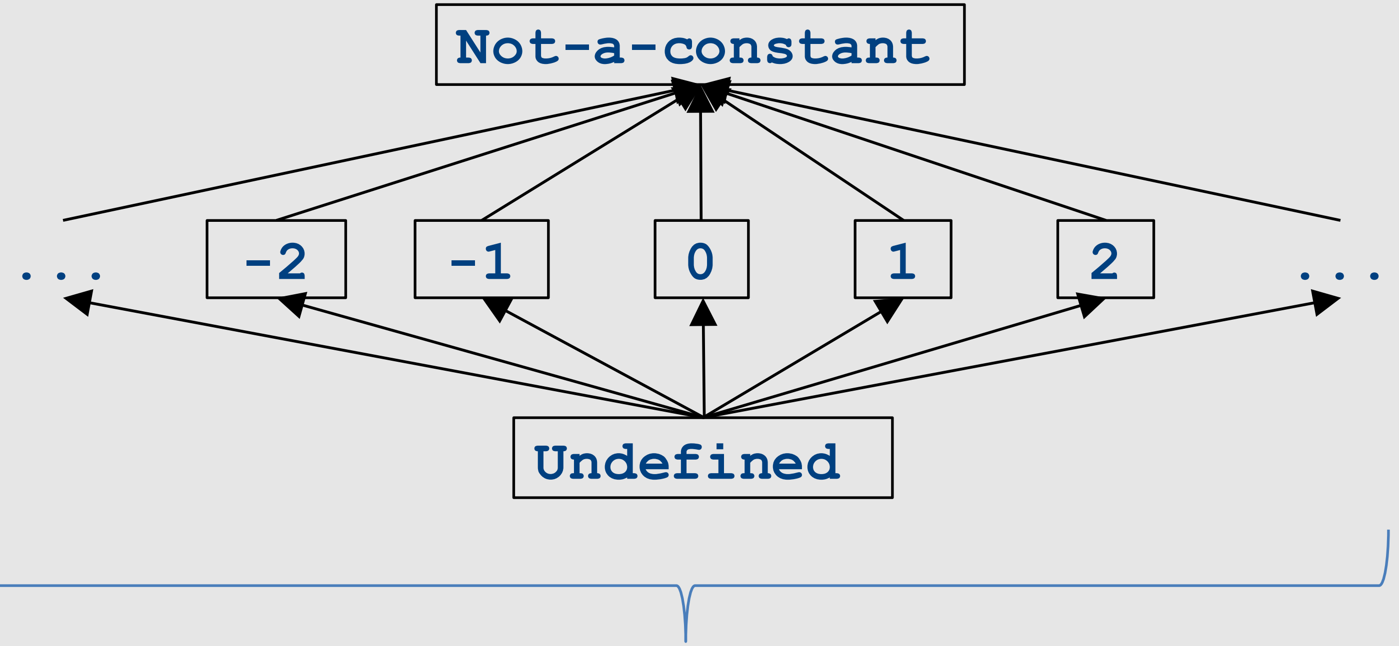
- Theoretical foundations of program analysis
- Cousot and Cousot 1977
- Abstract meaning of programs
  - Executed at compile time

Join semilattices and ordering



A semilattice for constant propagation

- One possible semilattice for this analysis is shown here (for each variable):



The lattice is infinitely wide

Monotone transfer functions

- A transfer function f is monotone iff
  if $x \sqsubseteq y$, then $f(x) \sqsubseteq f(y)$
- Intuitively, if you know less information about a program point, you can't "gain back" more information about that program point
- Many transfer functions are monotone, including those for liveness and constant propagation
- Note: Monotonicity does **not** mean that $x \sqsubseteq f(x)$
  - (This is a different property called extensivity)

The grand result

- Theorem: A dataflow analysis with a **finite-height semilattice** and family of **monotone transfer functions** always terminates
- Proof sketch:
  - The join operator can only bring values up
  - Transfer functions can never lower values back down below where they were in the past (monotonicity)
  - Values cannot increase indefinitely (finite height)

Code Generation

From TAC IR to Assembly

- Shown in project & recitation

Instruction's AST: Pattern Tree

```
 R   result
 |
cst
    constant operand
```

- Load_Const cst, R // cost=1

```
R
|
a  memory location operand
```

- Load_Mem a, R // cost=3

```
  R
  |
  +
 / \
R   a
```

- Add_Mem a, R // cost=3

```
   R1
  /  \
R1    *
     / \
  cst   R2   register operand
```

- Add_Scaled_Reg cst, R1, R2 // cost=4

Instruction's AST: Pattern Tree

```
#1    R
      |
     cst
```

- Load_Const cst, R // cost=1

```
#2    R
      |
      a
```

- Load_Mem a, R // cost=3

```
#3    R
      |
      +
     / \
    R   a
```

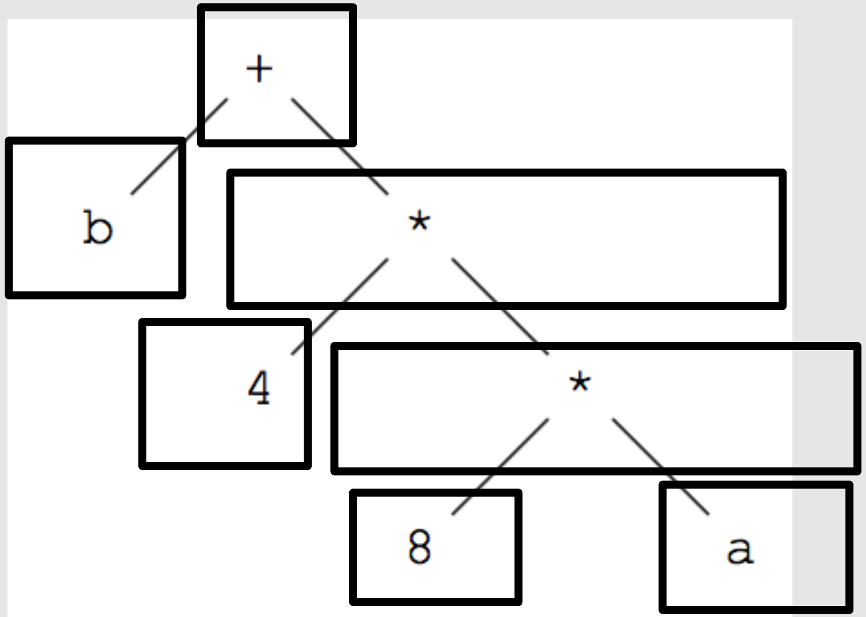
- Add_Mem a, R // cost=3

```
        R1
        |
#7      +
       / \
     R1   *   #7.1
         / \
       cst   R2
```

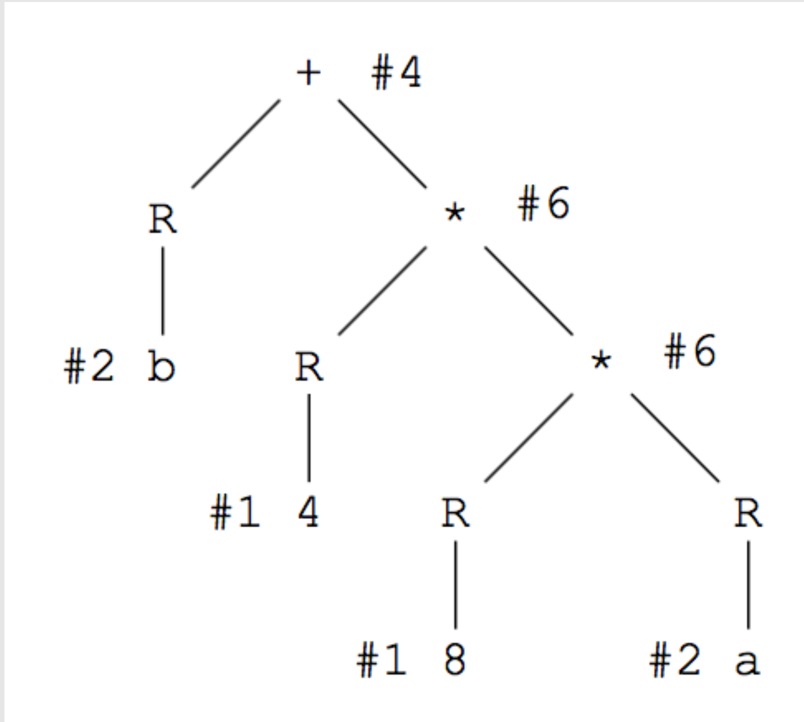
- Add_Scaled_Reg cst, R1, R2 // cost=4

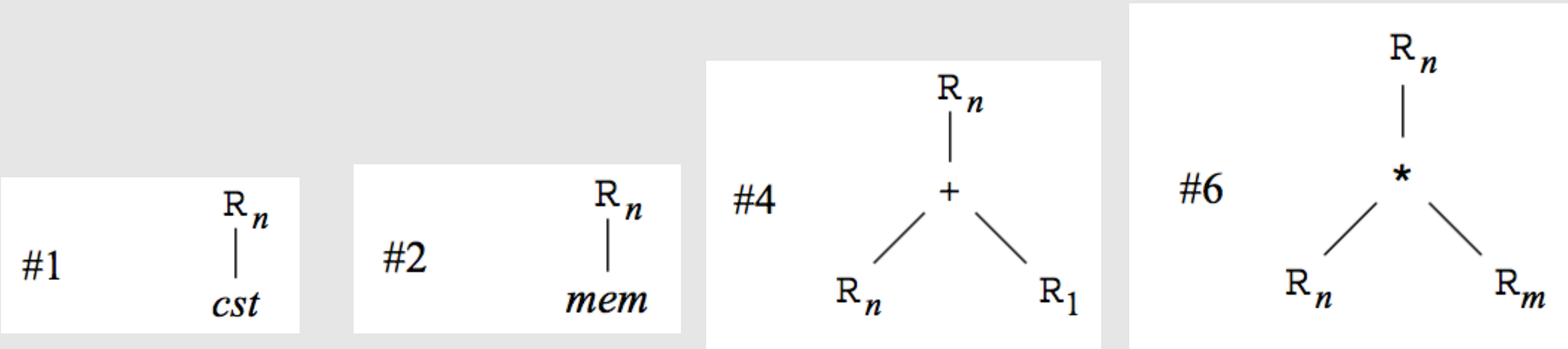
Example – Naïve rewrite

Input tree



Naïve Rewrite





Top-Down Rewrite Algorithm

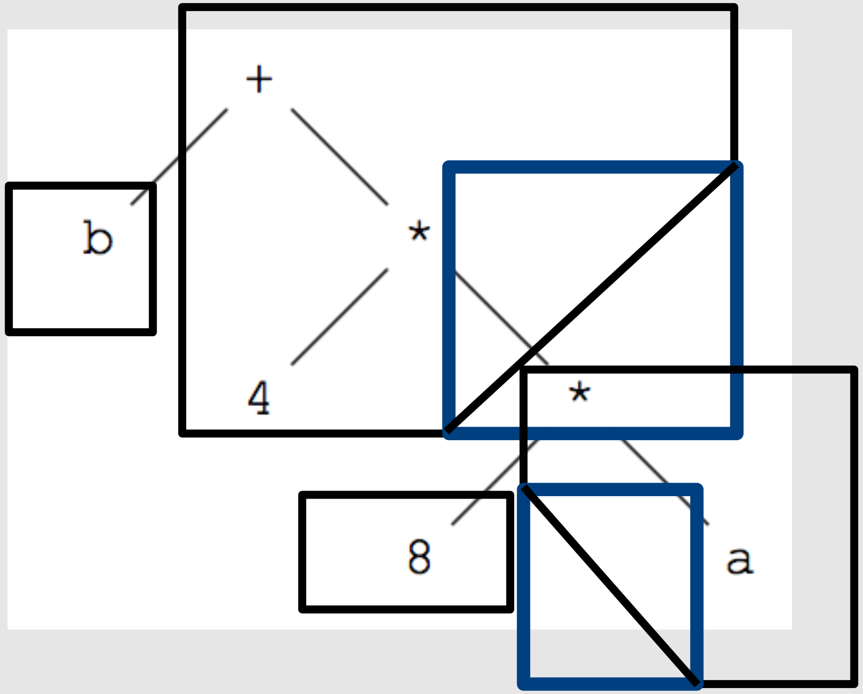
- aka **Maximal Munch**

- Based on **tiling**

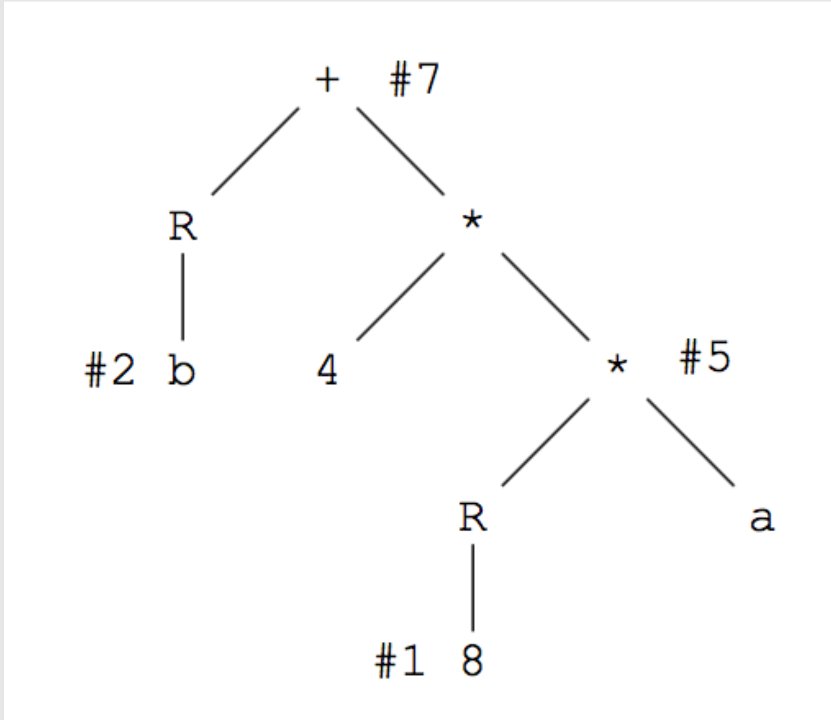
- Start from the root
- Choose largest tile
  - (covers largest number of nodes)
    - Break ties arbitrarily
- Continue recursively

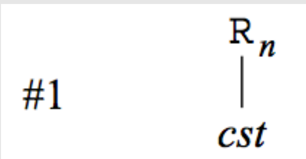
Top-down largest fit rewrite

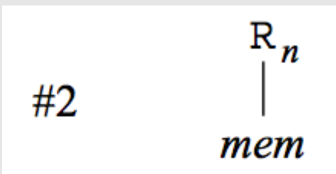
Input tree

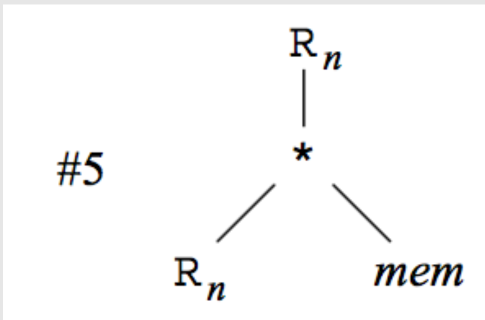


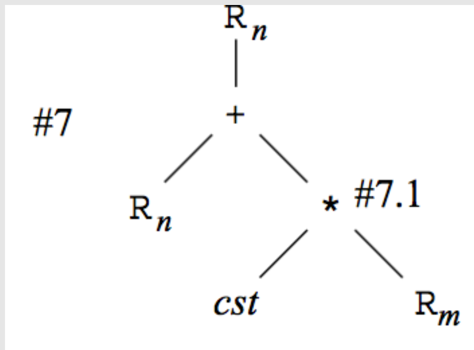
TDLF-Rewrite







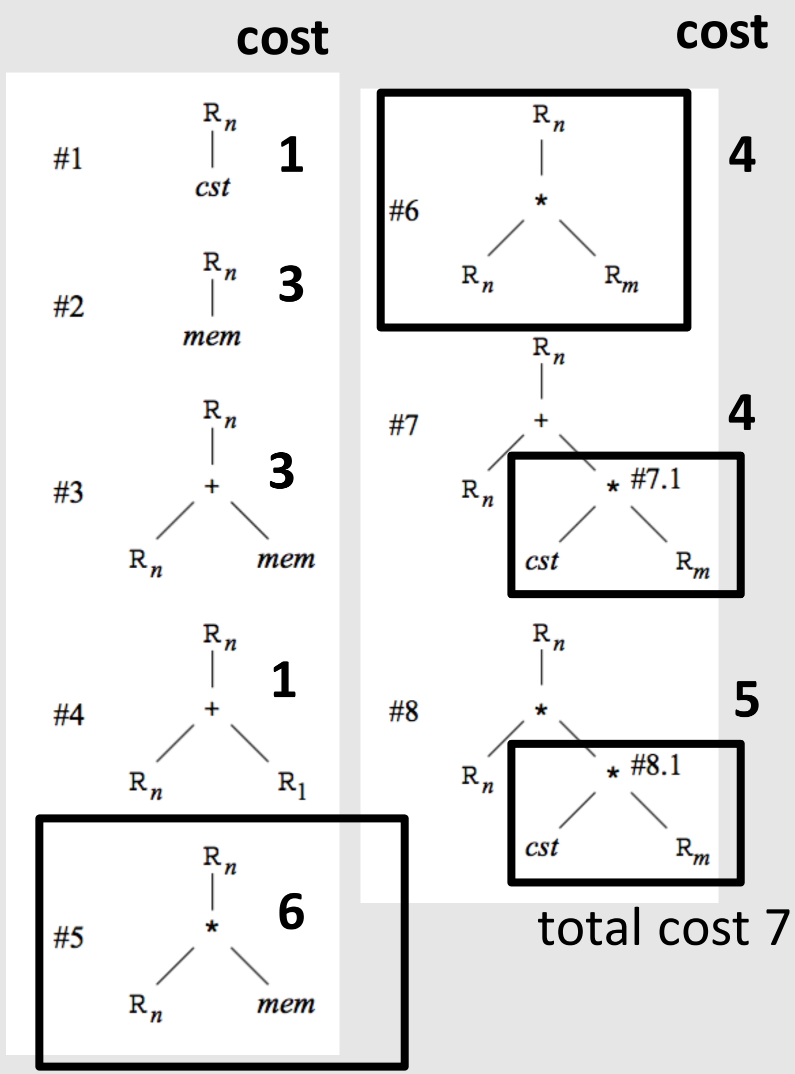




Instruction Selection with Dynamic Programming

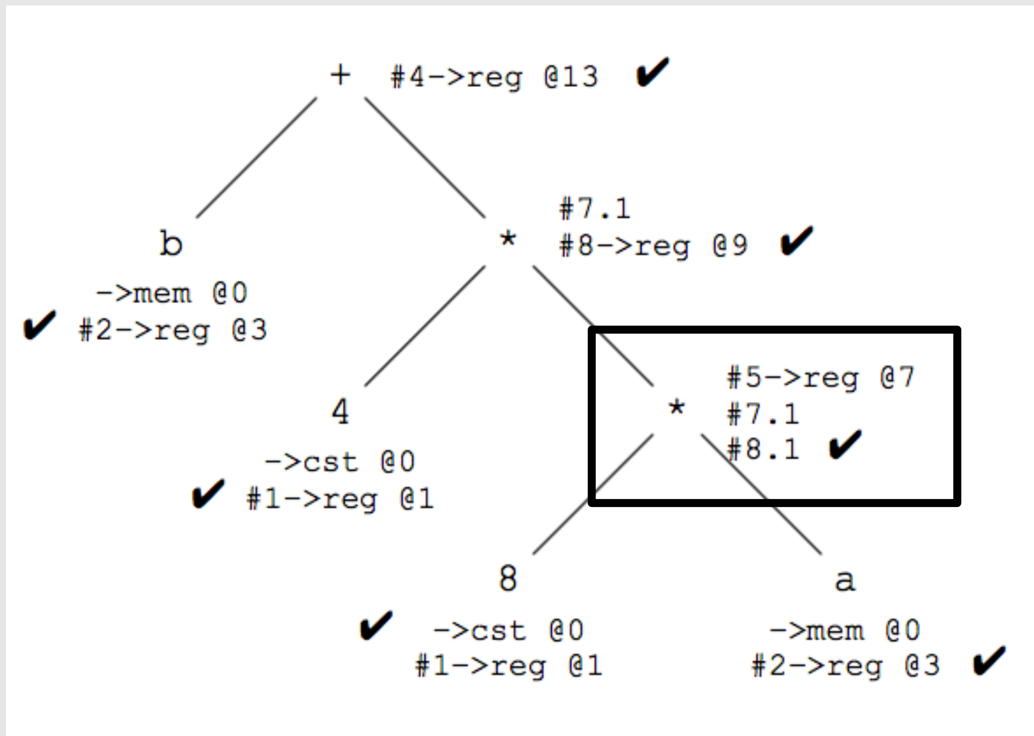
- Cost of sub-tree is sum of
  - The cost of the operator
  - The costs of the operands
- Idea: Compute the cost while detecting the patterns
- Label: Label→ Location @ cost
  - E.g., #5→reg @ 3

Example



total cost 7

total cost 8



Linearize code

- Standard AST→Code procedure
  - E.g., create the register-heavy code first

```
Load_Mem            a,R1        ; 3 units
Load_Const          4,R2        ; 1 unit
Mult_Scaled_Reg     8,R1,R2     ; 5 units
Load_Mem            b,R1        ; 3 units
Add_Reg             R2,R1       ; 1 unit
                    Total       = 13 units
```

Code generation for procedure calls

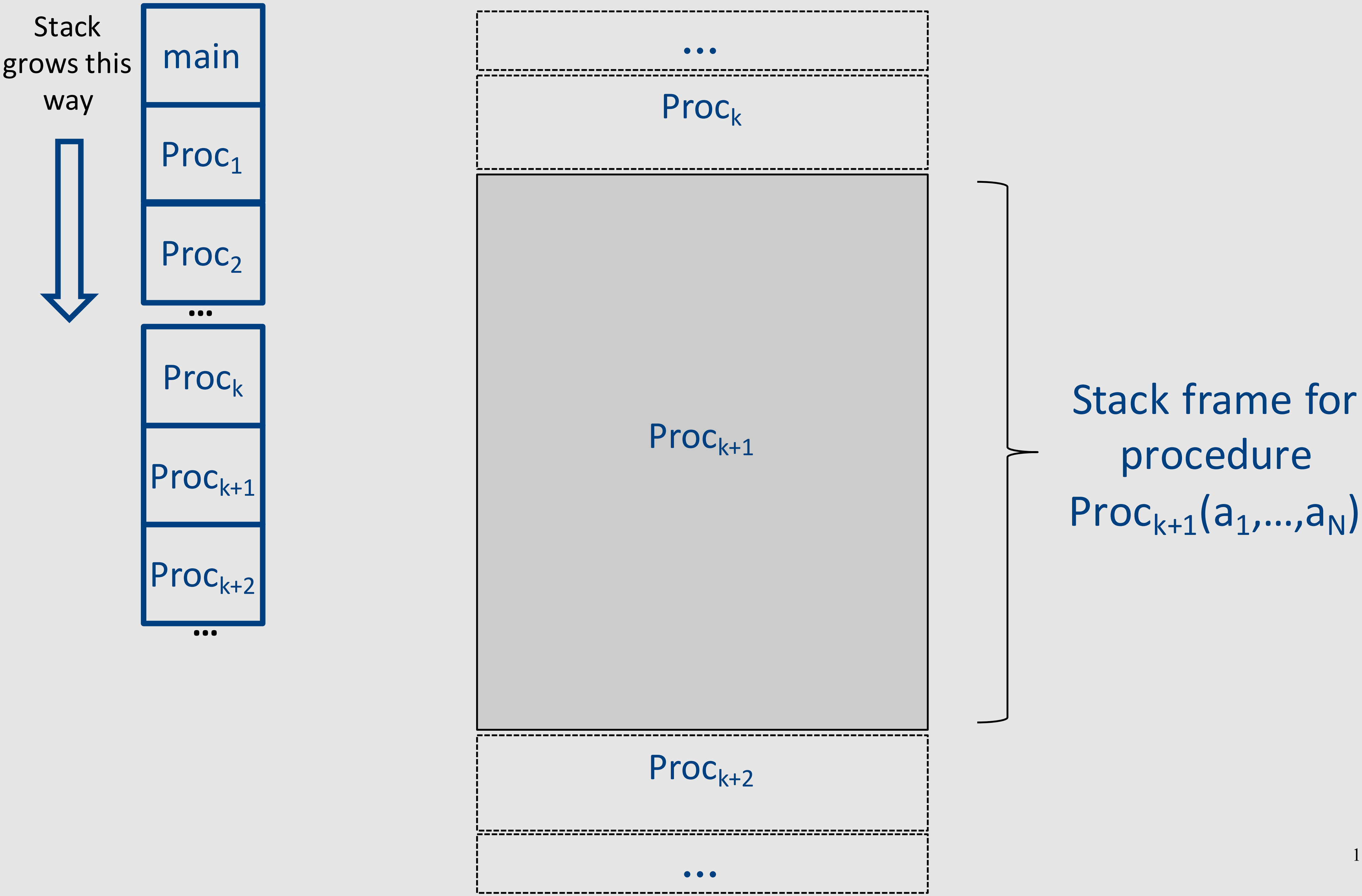
- Compile time generation of code for procedure invocations

- Activation Records (aka Stack Frames)

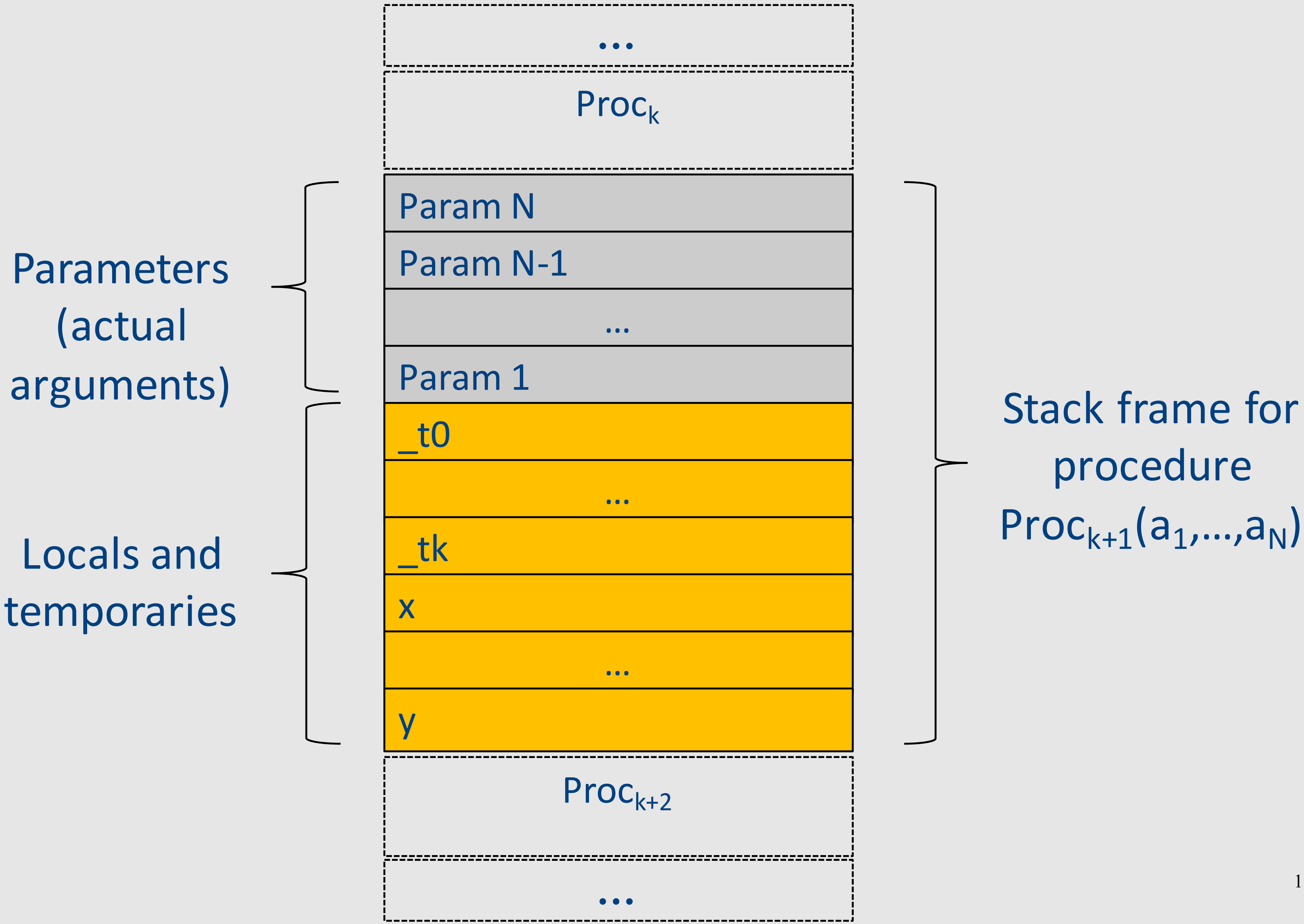
Supporting Procedures

- **Stack**: a new computing environment
  - e.g., temporary memory for **local variables**
- Passing information into the new environment
  - **Parameters**
- **Transfer** of **control** to/from procedure
- Handling return values

Abstract Activation Record Stack



Abstract Stack Frame



Static (lexical) Scoping

```
main ( )
{
    int a = 0 ;
    int b = 0 ;
    {
        int b = 1 ;
        {
   B2       int a = 2 ;
            printf ("%d %d\n", a, b)
B0      }
  B1    {
   B3       int b = 3 ;
            printf ("%d %d\n", a, b) ;
        }
        printf ("%d %d\n", a, b) ;
    }
    printf ("%d %d\n", a, b) ;
}
```

a name refers to its (closest) enclosing scope

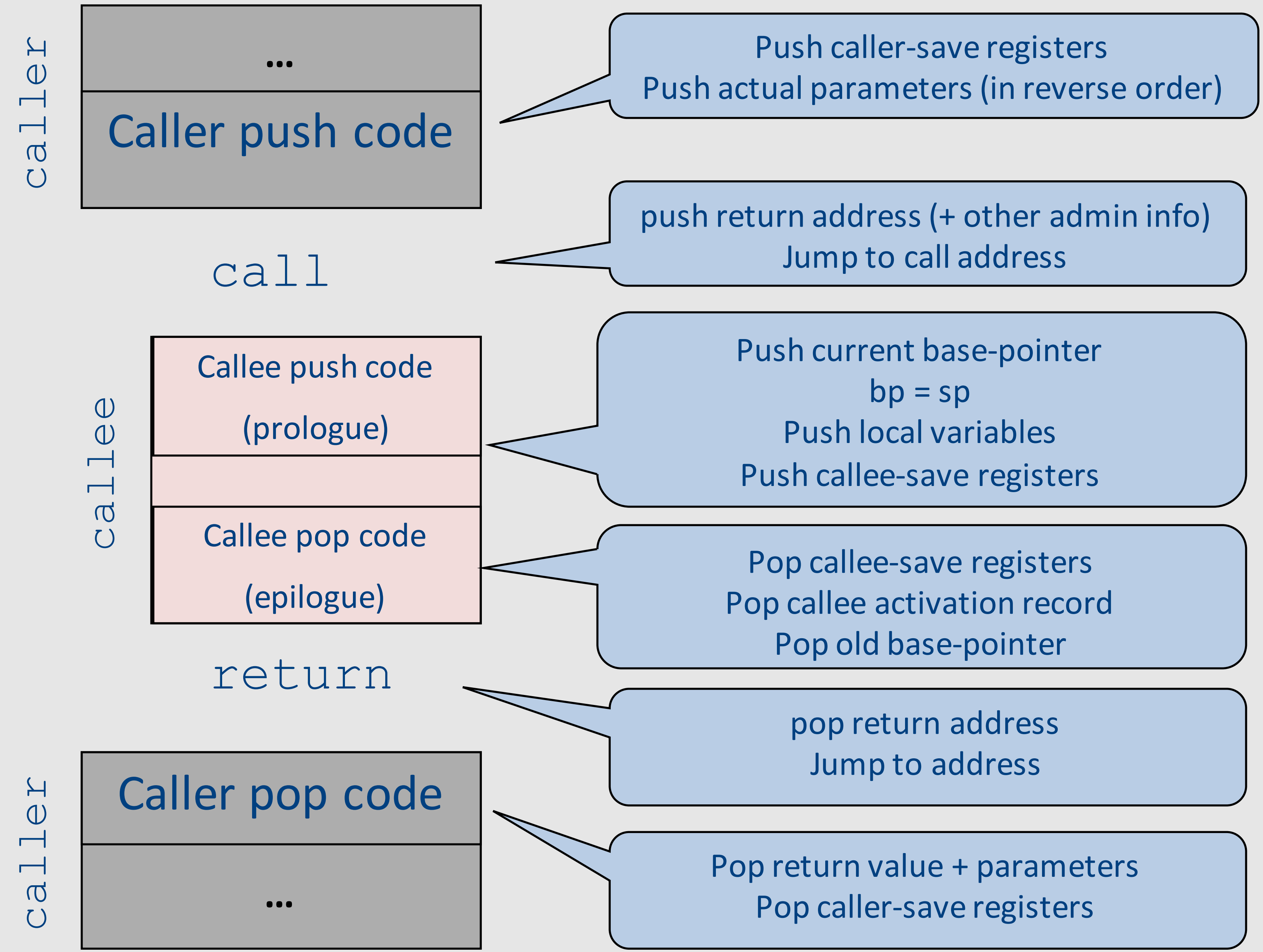
known at compile time

Declaration	Scopes
a=0	B0,B1,B3
b=0	B0
b=1	B1,B2
a=2	B2
b=3	B3

Dynamic Scoping

- Each identifier is associated with a global stack of bindings
- When entering scope where identifier is declared
  - push declaration on identifier stack
- When exiting scope where identifier is declared
  - pop identifier stack
- **Evaluating the identifier in any context binds to the current top of stack**
- Determined **at runtime**

Call Sequences



“To Callee-save or to Caller-save?”

- Callee-saved registers need only be saved when callee modifies their value
- Some heuristics and conventions are followed

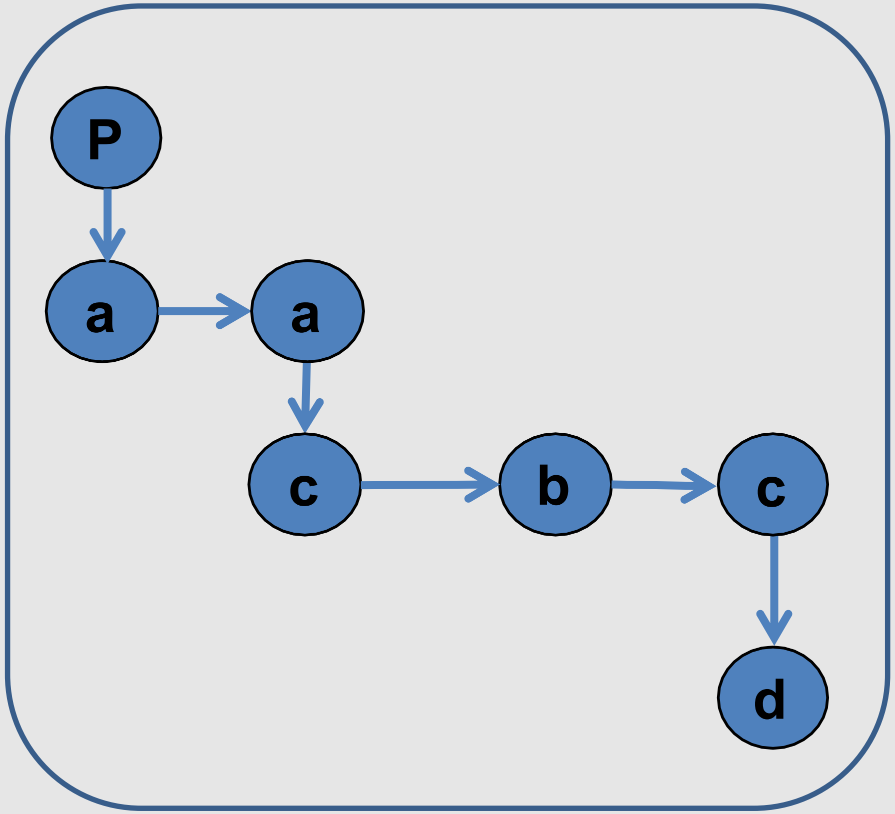
Nested Procedures

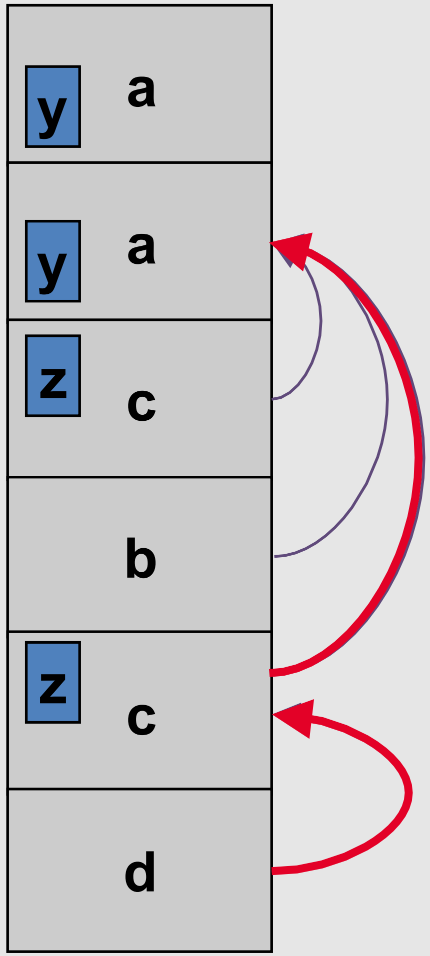
- problem: a routine may need to access variables of another routine that contains it statically
- solution: lexical pointer (a.k.a. access link) in the activation record
- lexical pointer points to the last activation record of the nesting level above it
  - in our example, lexical pointer of d points to activation records of c
- lexical pointers created at runtime
- number of links to be traversed is known at compile time

Lexical Pointers

```
program p(){
  int x;
  procedure a(){
    int y;
    procedure b(){ c() };
    procedure c(){
      int z;
      procedure d(){
        y := x + z
      };
      … b() … d() …
    }
    … a() … c() …
  }
  a()
}
```

Possible call sequence:
p → a → a → c → b → c → d





Register allocation

Register allocation

- Number of registers is **limited**
- Need to **allocate** them in a clever way
  - Using registers intelligently is a critical step in any compiler
    - A good register allocator can generate code orders of magnitude better than a bad register allocator

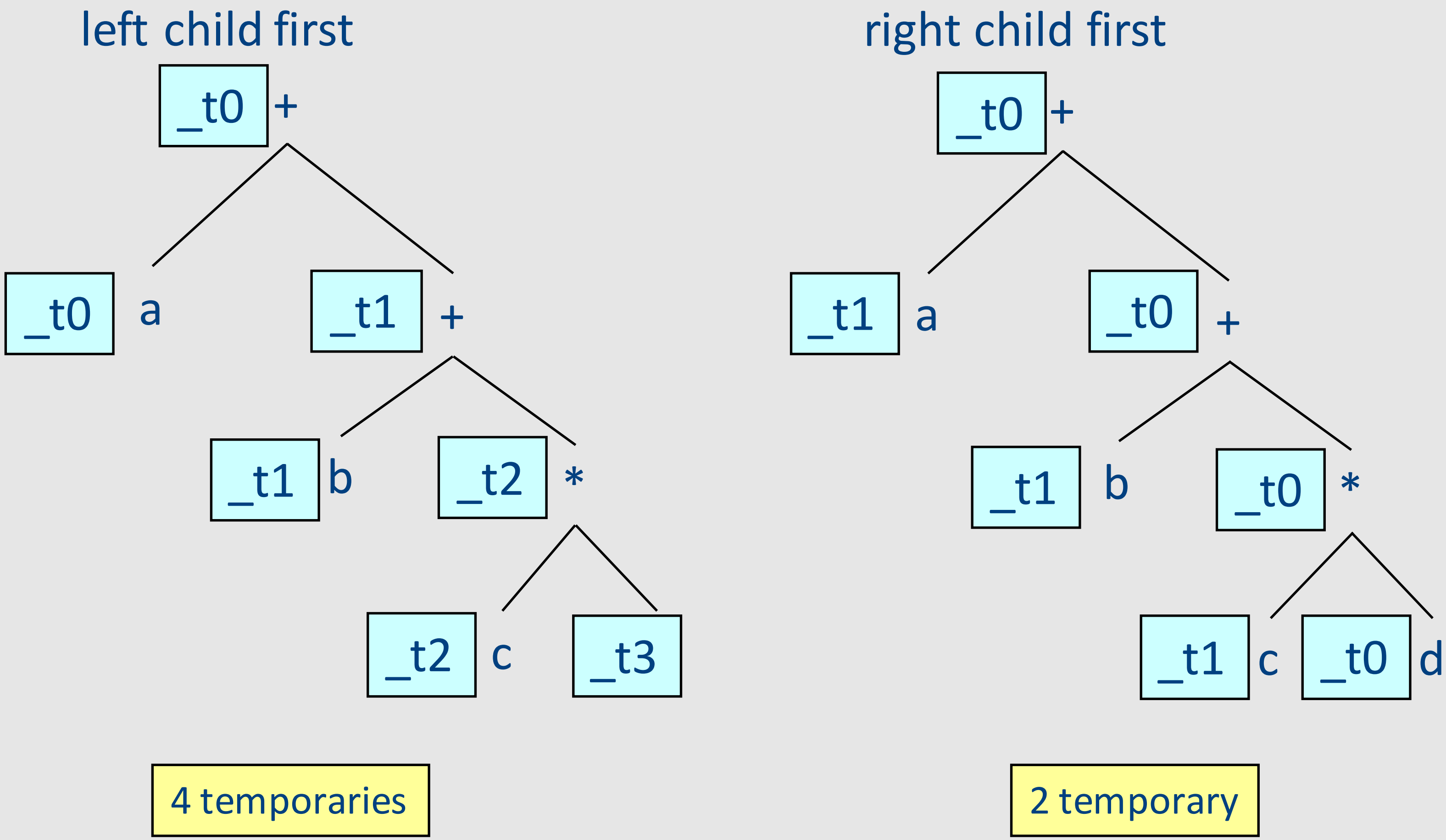
Sethi-Ullman translation

- Algorithm by Ravi Sethi and Jeffrey D. Ullman to emit optimal TAC
  - Minimizes number of temporaries
- Main data structure in algorithm is a stack of temporaries
  - Stack corresponds to recursive invocations of _t = **cgen**(e)
  - All the temporaries on the stack are live
    - Live = contain a value that is needed later on

Example

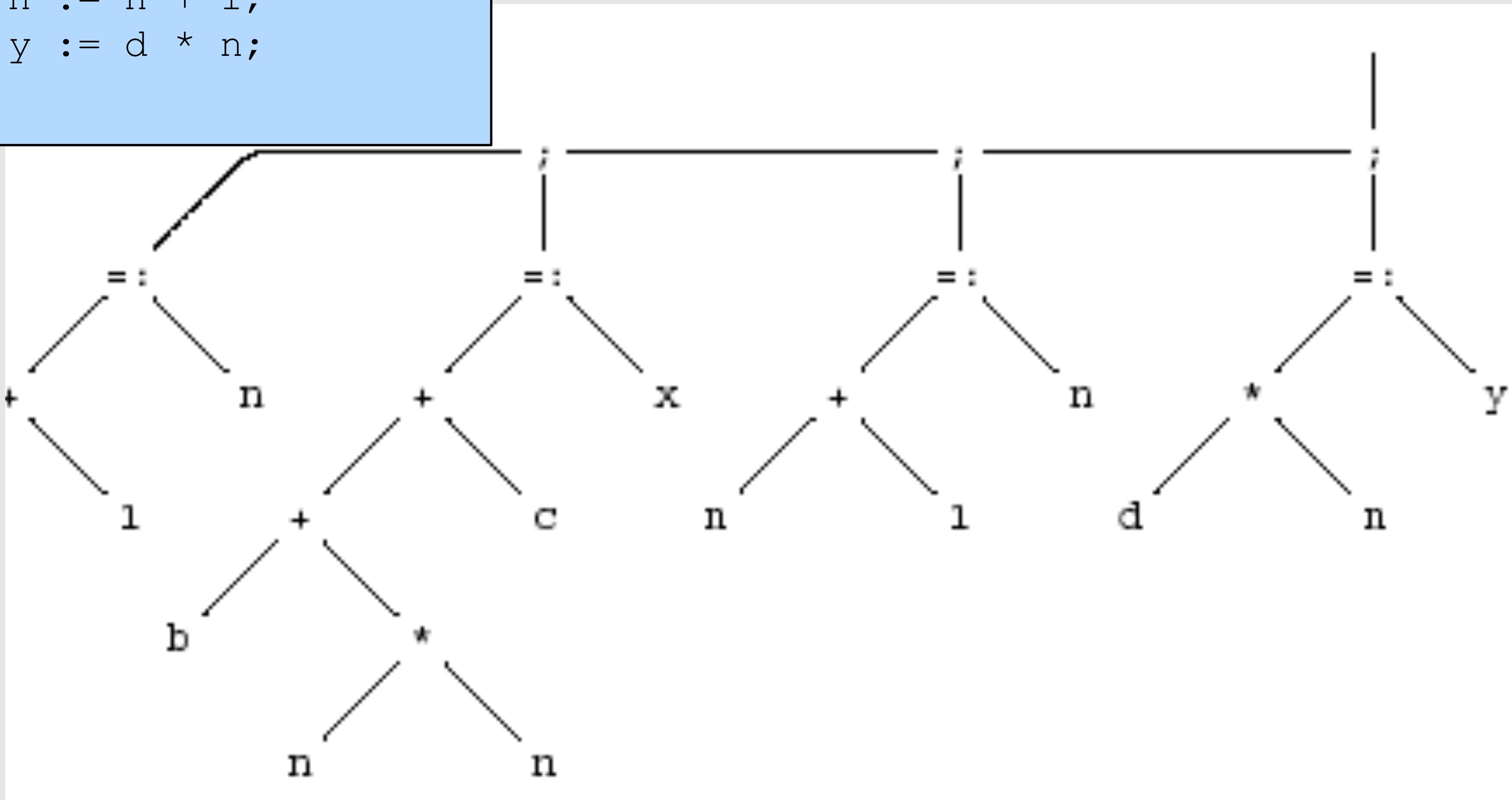
_t0 = **cgen**(a+(b+(c*d)))

*+ and * are commutative operators*



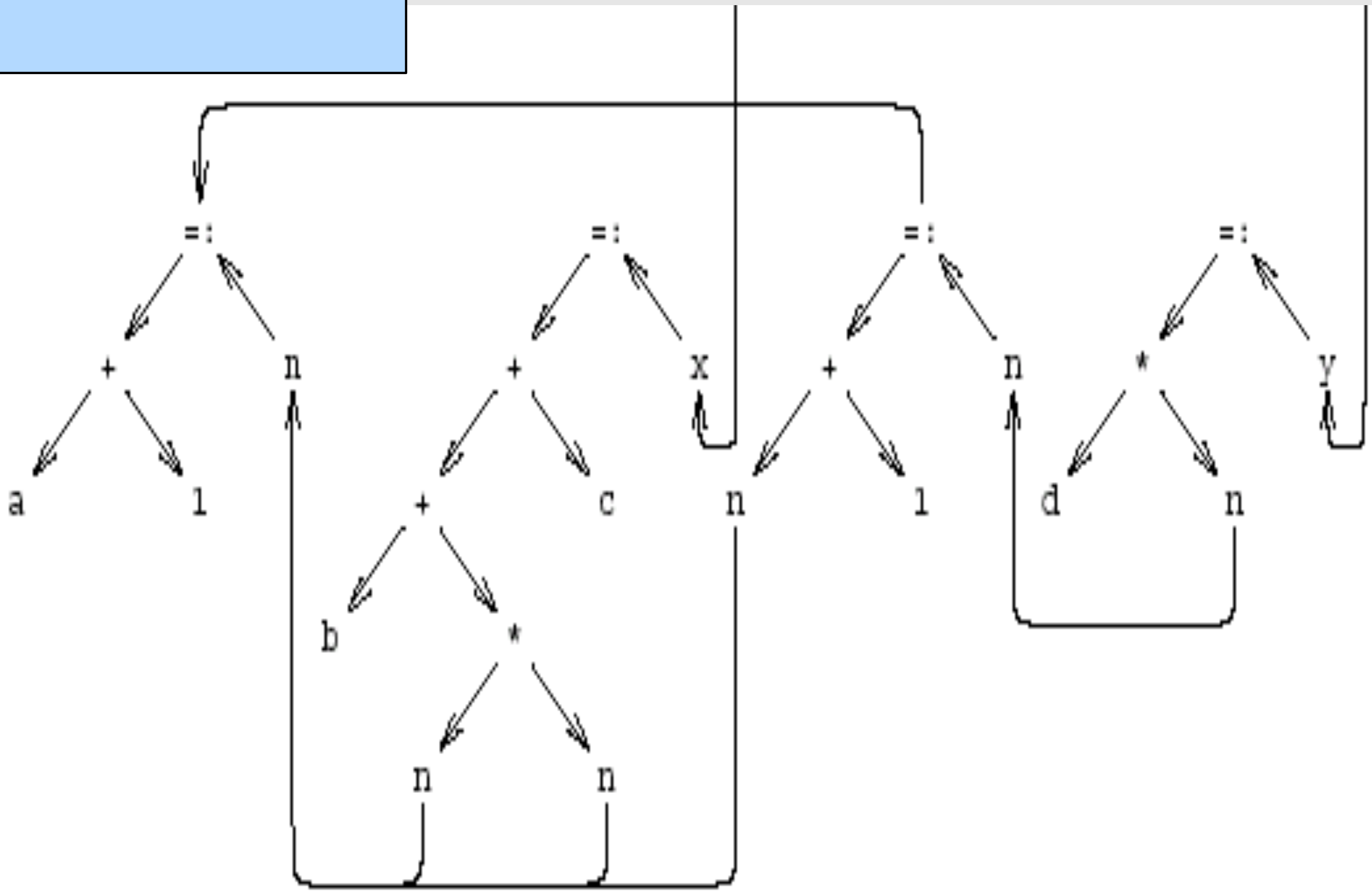
AST for a Basic Block

```
{
    int n;
    n := a + 1;
    x := b + n * n + c;
    n := n + 1;
    y := d * n;
}
```



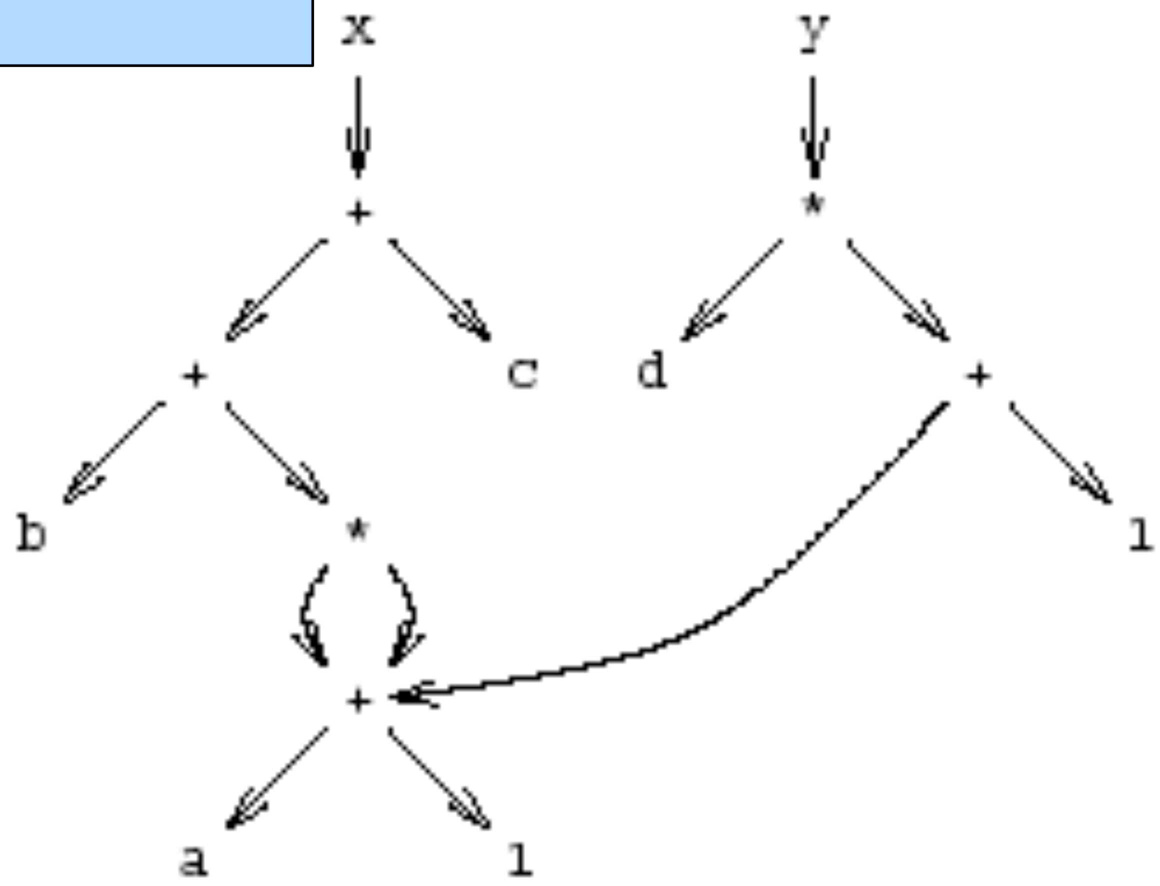
Dependency graph

```
{
    int n;
    n := a + 1;
    x := b + n * n + c;
    n := n + 1;
    y := d * n;
}
```

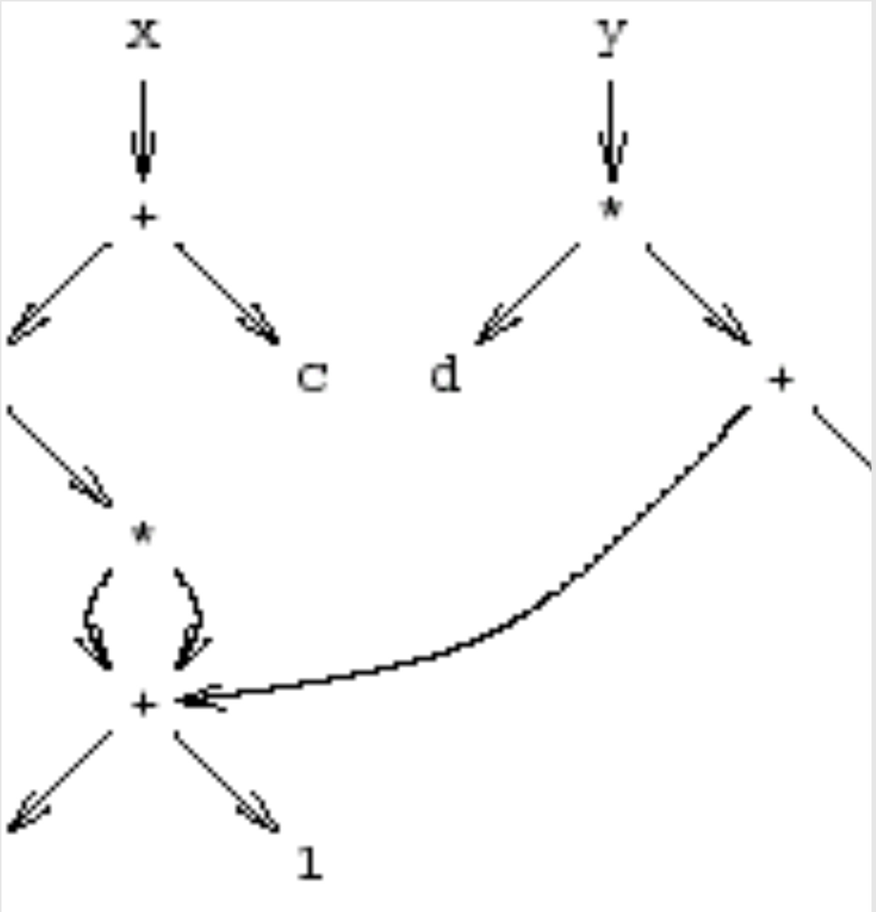


```
{
    int n;
    n := a + 1;
    x := b + n * n + c;
    n := n + 1;
    y := d * n;
}
```

Simplified Data Dependency Graph



Pseudo Register Target Code




```
Load_Mem     a,R1
Add_Const    1,R1
Load_Reg     R1,X1

Load_Reg     X1,R1
Mult_Reg     X1,R1
Add_Mem      b,R1
Add_Mem      c,R1
Store_Reg    R1,x

Load_Reg     X1,R1
Add_Const    1,R1
Mult_Mem     d,R1
Store_Reg    R1,y
```

“Global” Register Allocation

- Input:
  - Sequence of machine instructions (“assembly”)
    - Unbounded number of temporary variables
      - aka symbolic registers
  - “machine description”
    - # of registers, restrictions
- Output
  - Sequence of machine instructions using machine registers (assembly)
  - Some MOV instructions removed

Variable Liveness

- A statement x = y + z
  - **defines** x
  - **uses** y and z
- A variable x is live at a program point if its value (at this point) is used at a later point

y = 42	x undef, y live, z undef
z = 73	x undef, y live, z live
x = y + z	x is live, y dead, z dead
print(x);	x is dead, y dead, z dead

(showing state after the statement)

Main idea

- For every node n in CFG, we have out[n]
  - Set of temporaries live out of n
- Two variables *interfere* if they appear in the same out[n] of any node n
  - **Cannot be allocated to the same register**
- Conversely, if two variables do not interfere with each other, they can be assigned the same register
  - We say they have disjoint live ranges
- How to assign registers to variables?

Interference graph

- Nodes of the graph = variables
- Edges connect variables that interfere with one another
- Nodes will be assigned a color corresponding to the register assigned to the variable
- Two colors can't be next to one another in the graph

Graph coloring

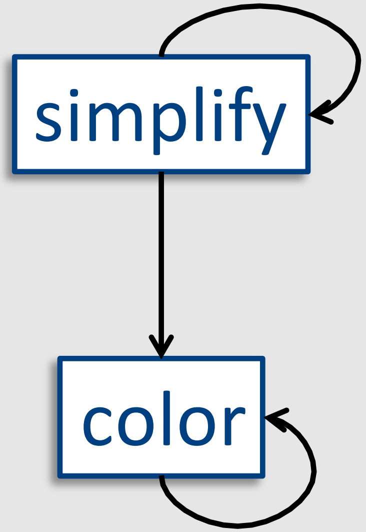
- This problem is equivalent to graph-coloring, which is NP-hard if there are at least three registers
- No good polynomial-time algorithms (or even good approximations!) are known for this problem
  - We have to be content with a heuristic that is good enough for RIGs that arise in practice

Coloring by simplification [Kempe 1879]

- How to find a **k**-coloring of a graph
- Intuition:
  - Suppose we are trying to ***k****-color a graph and find a node* with fewer than ***k*** *edges*
  - If we delete this node from the graph and color what remains, we can find a color for this node if we add it back in
  - Reason: fewer than *k neighbors* → *some color must be* left over

Coloring by simplification [Kempe 1879]

- How to find a k-coloring of a graph
- Phase 1: Simplification
  - Repeatedly simplify graph
  - When a variable (i.e., graph node) is removed, push it on a stack
- Phase 2: Coloring
  - Unwind stack and reconstruct the graph as follows:
  - Pop variable from the stack
  - Add it back to the graph
  - Color the node for that variable with a color that it doesn't interfere with



Handling precolored nodes

- Some variables are pre-assigned to registers
  - Eg: mul on x86/pentium
    - uses eax; defines eax, edx
  - Eg: call on x86/pentium
    - Defines (trashes) caller-save registers eax, ecx, edx
- To properly allocate registers, treat these register uses as special temporary variables and enter into interference graph as precolored nodes

Optimizing move instructions

- Code generation produces a lot of extra mov instructions

  mov t5, t9

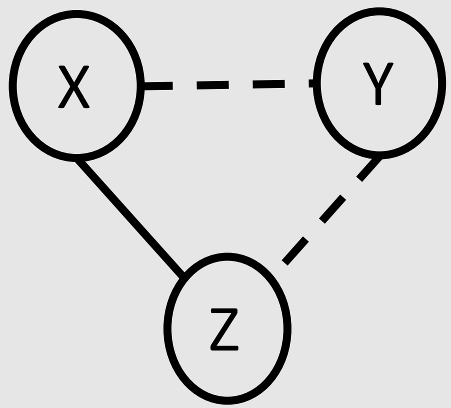
- If we can assign t5 and t9 to same register, we can get rid of the mov
  - effectively, copy elimination at the register allocation level
- Idea: if t5 and t9 are not connected in inference graph, coalesce them into a single variable; the move will be redundant
- Problem: coalescing nodes can make a graph un-colorable
  - Conservative coalescing heuristic

Constrained Moves

- A instruction T ← S is constrained
  - if S and T interfere
- May happen after coalescing

X ← Y

Y ← Z



- Constrained MOVs are not coalesced

Constrained Moves

- A instruction $T \leftarrow S$ is constrained
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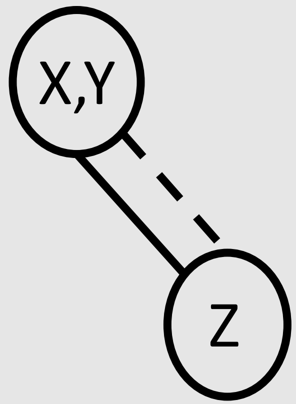
- May happen after coalescing

$X \leftarrow Y$

$Y \leftarrow Z$



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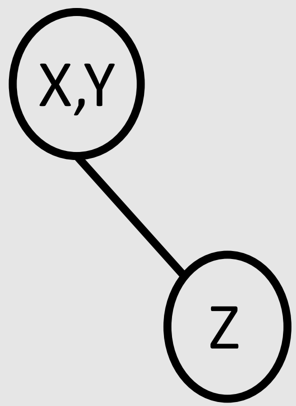
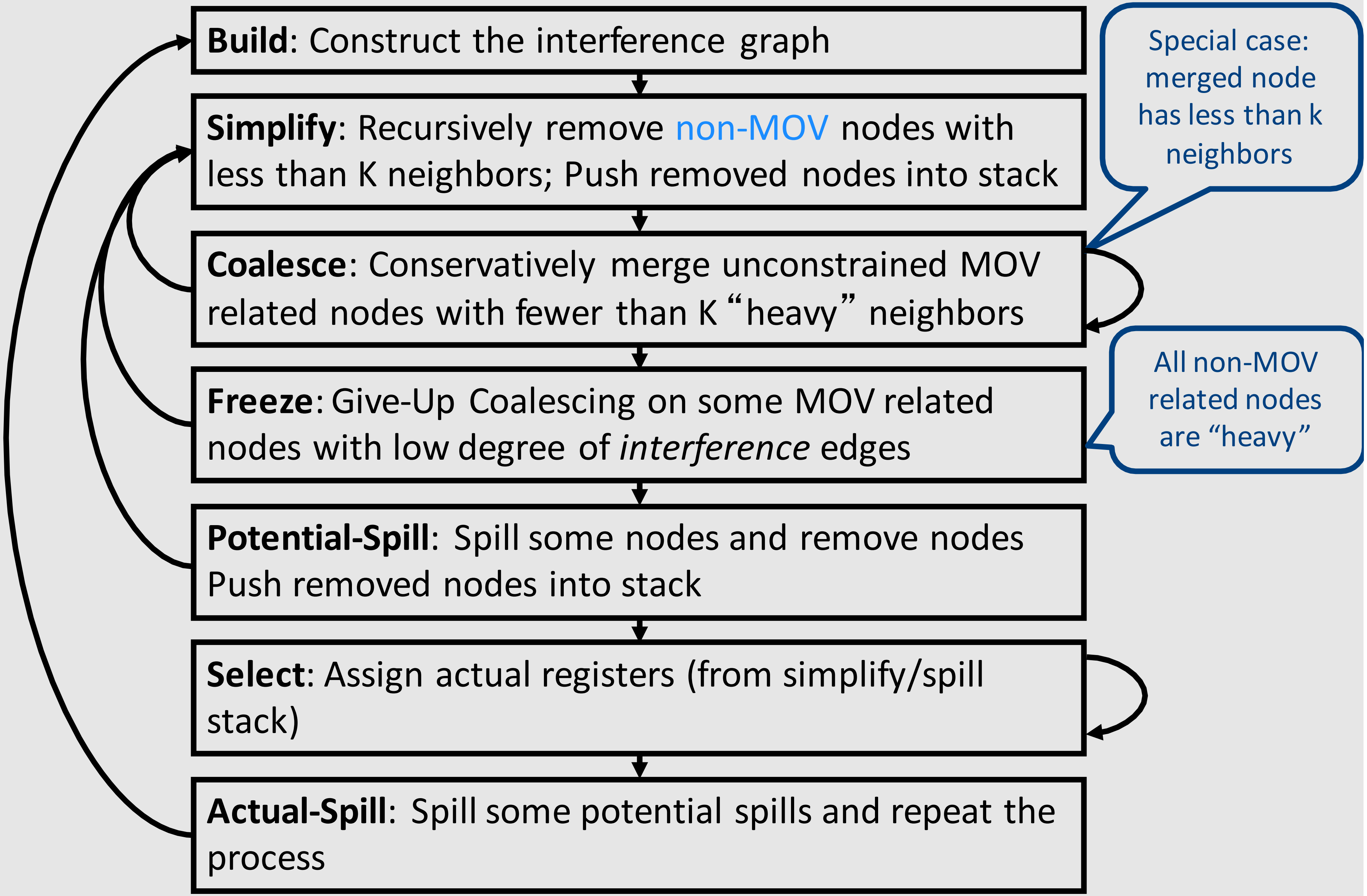


- Constrained MOVs are not coalesced

Graph Coloring with Coalescing

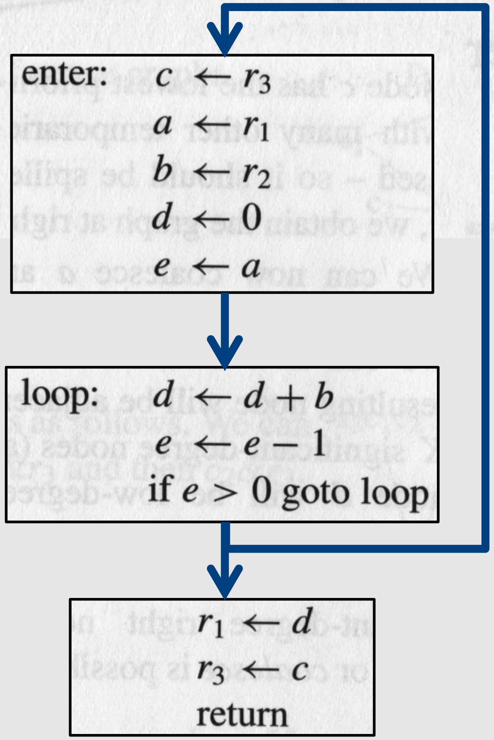


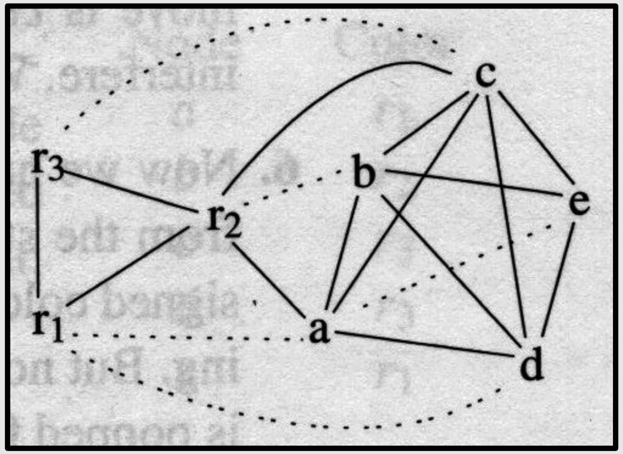
A Complete Example

```
int f(int a, int b) {
   int d=0;
   int e=a;
   do {d = d+b;
         e = e-1;
   } while (e>0);
   return d;
}
```

enter: $c \leftarrow r_3$
$a \leftarrow r_1$ — Callee-saved registers
$b \leftarrow r_2$ — Caller-saved registers
$d \leftarrow 0$
$e \leftarrow a$

loop: $d \leftarrow d + b$
$e \leftarrow e - 1$
if $e > 0$ goto loop
$r_1 \leftarrow d$
$r_3 \leftarrow c$
return (r_1, r_3 *live out*)





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if $e > 0$ goto loop
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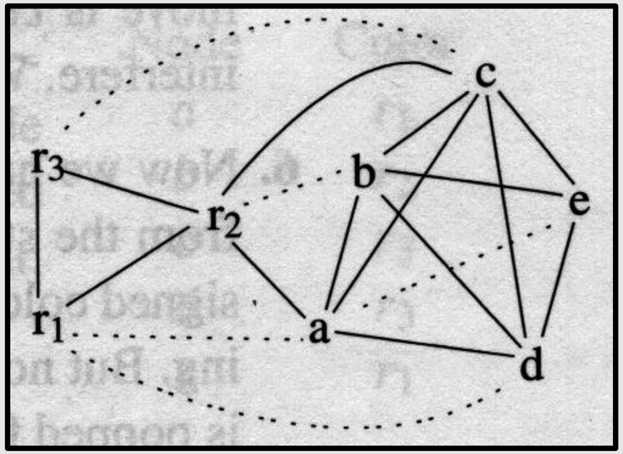
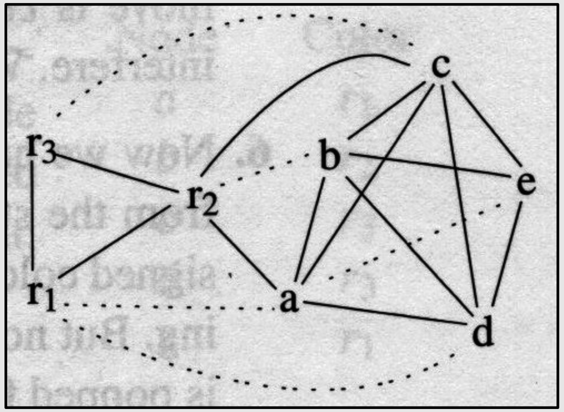
$r_3 \leftarrow c$
return (r_1, r_3 *live out*)

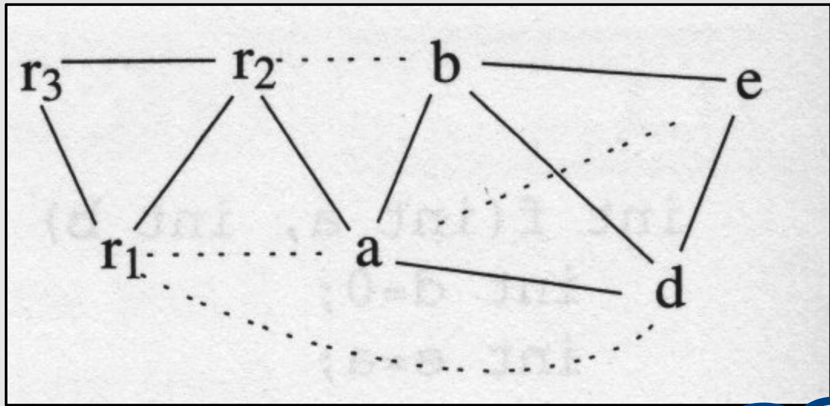
Node	Uses+Defs outside loop		Uses+Defs within loop		Degree		Spill priority
a	(2	+ 10 ×	0	) /	4	=	0.50
b	(1	+ 10 ×	1	) /	4	=	2.75
c	(2	+ 10 ×	0	) /	6	=	0.33
d	(2	+ 10 ×	2	) /	4	=	5.50
e	(1	+ 10 ×	3	) /	3	=	10.33



A Complete Example

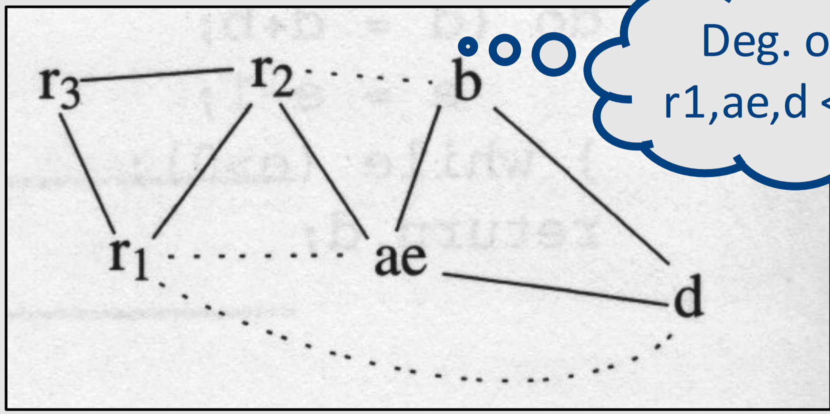


Spill c



c

a & e

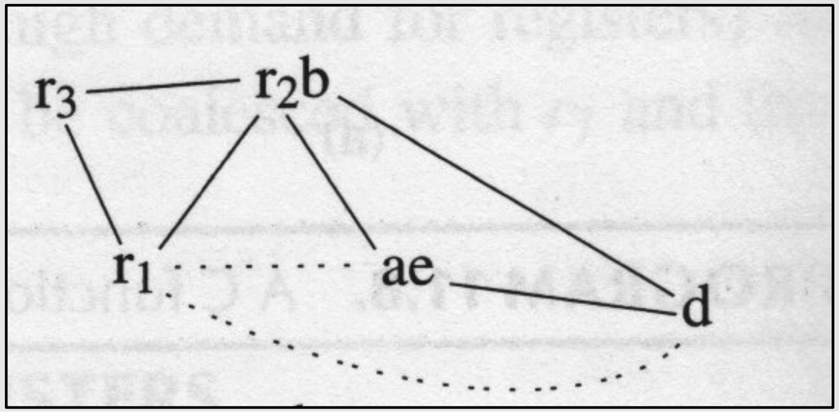


c

Deg. of r1,ae,d < K

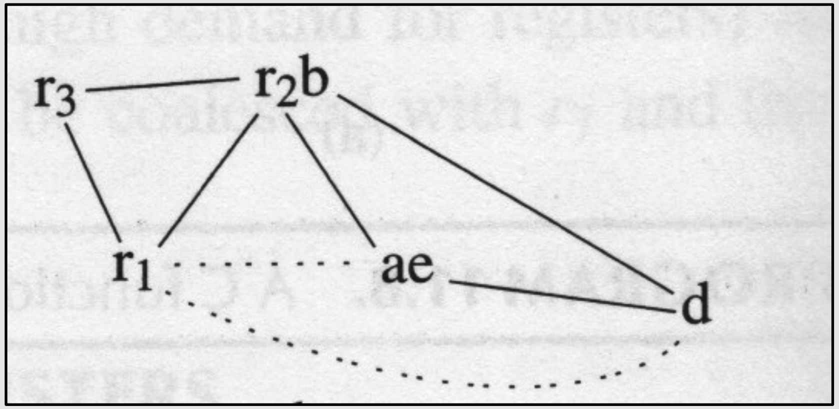
r2 & b

(Alt: ae+r1)



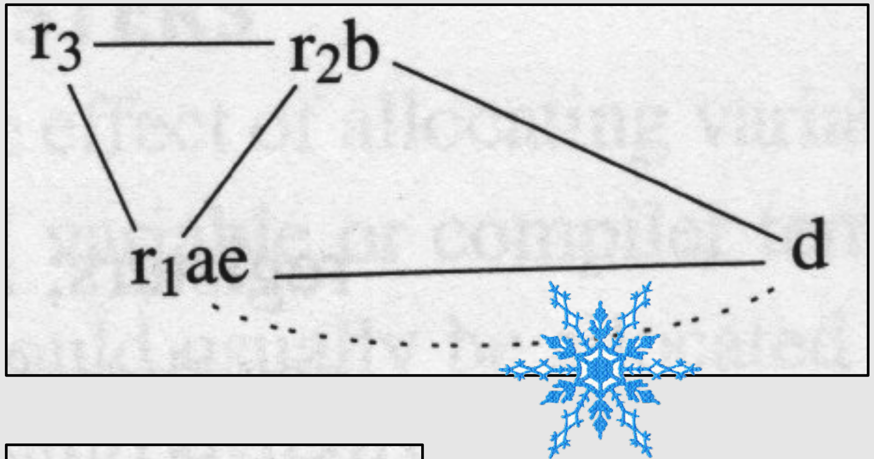
c

A Complete Example



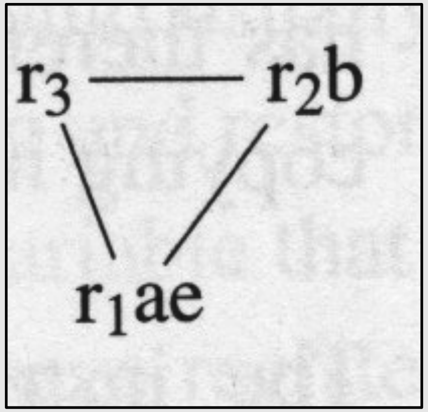
ae & r1

(Alt: …)c



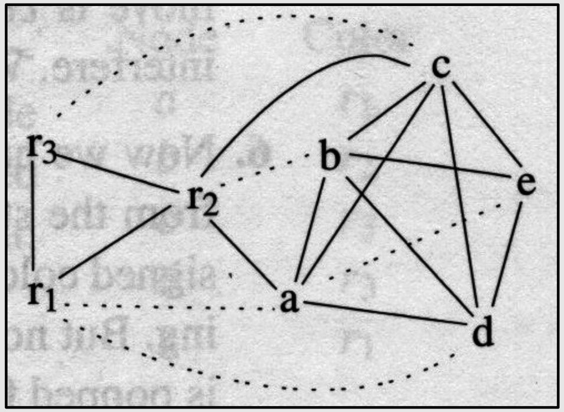
freeze $r_1$ae-d

Simplify d



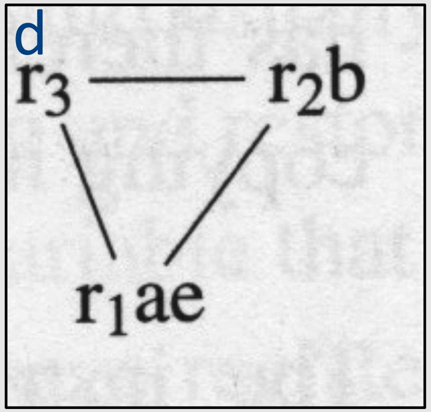
dc

pop c …



(Alt: ae+r1)

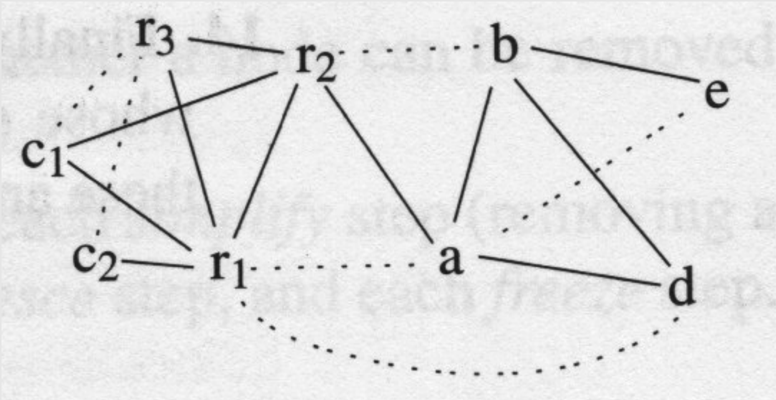
pop d



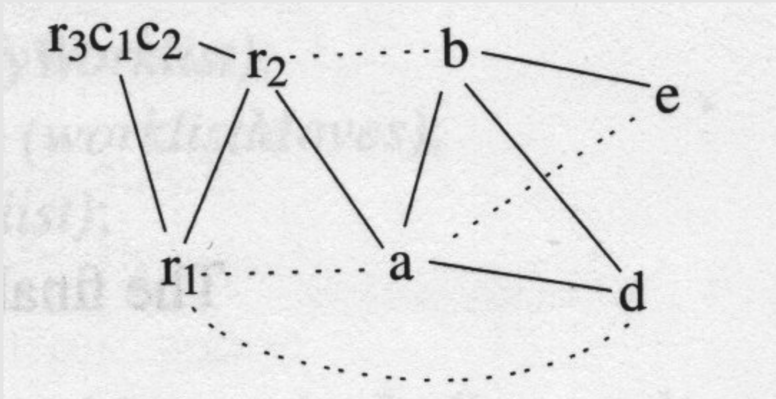
c

A Complete Example

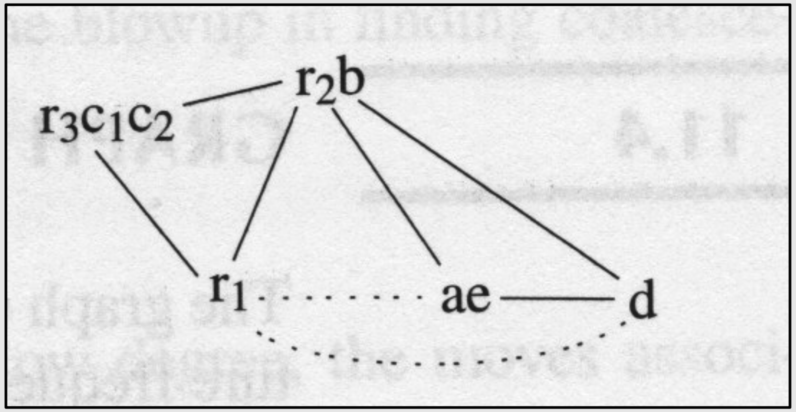
enter:	$c_1 \leftarrow r_3$
	$M[c_{loc}] \leftarrow c_1$
	$a \leftarrow r_1$
	$b \leftarrow r_2$
	$d \leftarrow 0$
	$e \leftarrow a$
loop:	$d \leftarrow d + b$
	$e \leftarrow e - 1$
	if $e > 0$ goto loop
	$r_1 \leftarrow d$
	$c_2 \leftarrow M[c_{loc}]$
	$r_3 \leftarrow c_2$
	return



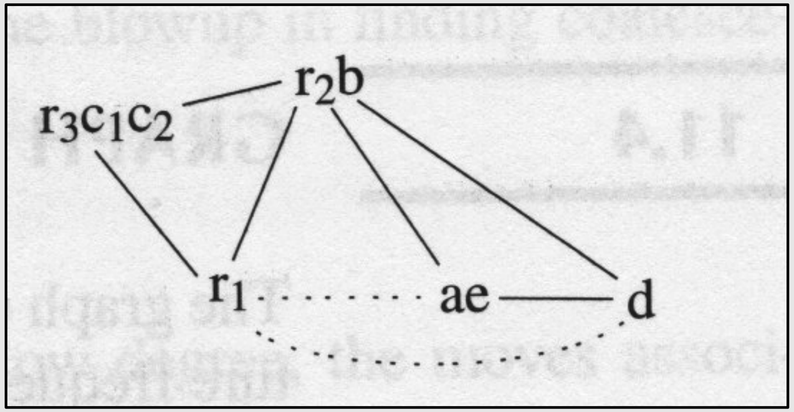
c1&r3, c2 &r3



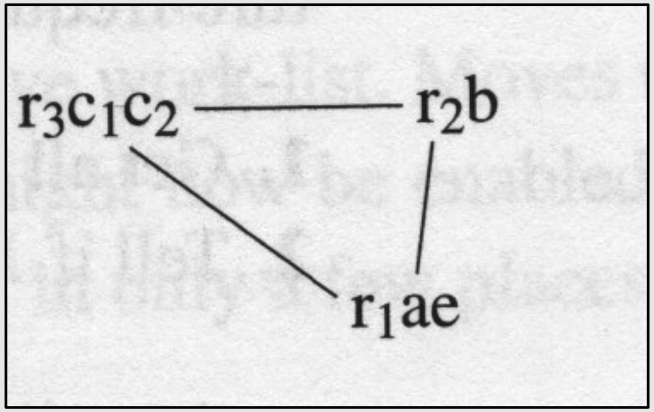
a&e, b&r2



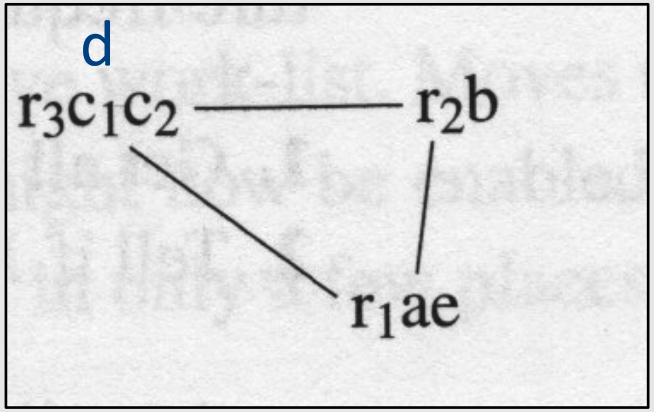
A Complete Example



ae & r1
Simplify d



Pop d



gen code

```
enter:  r3 ← r3
        M[c_loc] ← r3
        r1 ← r1
        r2 ← r2
        r3 ← 0
        r1 ← r1
loop:   r3 ← r3 + r2
        r1 ← r1 − 1
        if r1 > 0 goto loop
        r1 ← r3
        r3 ← M[c_loc]
        r3 ← r3
        return
```

"opt"

```
enter:  M[c_loc] ← r3
        r3 ← 0
loop:   r3 ← r3 + r2
        r1 ← r1 − 1
        if r1 > 0 goto loop
        r1 ← r3
        r3 ← M[c_loc]
        return
```