

# Interdependent Values without Single-Crossing

## [Abstract]

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We consider a setting where an auctioneer sells a single item to  $n$  potential agents with *interdependent values*. That is, each agent has her own private signal, and the valuation of each agent is a known function of all  $n$  private signals. This captures settings such as valuations for oil drilling rights, broadcast rights, pieces of art, and many more.

In the interdependent value setting, all previous work has assumed a so-called *single-crossing condition*. Single-crossing means that the impact of a private signal,  $s_i$ , on the valuation of agent  $i$ , is greater than the impact of  $s_i$  on the valuation of any other agent. It is known that without the single-crossing condition, an efficient outcome cannot be obtained. We ask what approximation to the optimal social welfare can be obtained when valuations do not exhibit single-crossing.

We show that, in general, without the single-crossing condition, one cannot hope to approximate the optimal social welfare any better than assigning the item to a random bidder. Consequently, we consider a relaxed version of single-crossing, *c-single-crossing*, with some parameter  $c \geq 1$ , which means that the impact of  $s_i$  on the valuation of agent  $i$  is at least  $1/c$  times the impact of  $s_i$  on the valuation of any other agent ( $c = 1$  is single-crossing). Using this relaxed notion, we obtain a host of positive results. These include a prior-free universally truthful  $2\sqrt{nc}^{3/2}$ -approximation to welfare, and a prior-free deterministic  $(n - 1)c$ -approximation to welfare. Under appropriate concavity conditions, we improve this to a prior-free universally truthful  $2c$  approximation to welfare as well as a universally truthful  $O(c^2)$ -approximation to the optimal revenue.

CCS Concepts: •**Theory of computation** → **Algorithmic game theory and mechanism design**;

Additional Key Words and Phrases: Interdependent values; single-crossing; welfare maximization; prior-free

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A full version of this paper is available at <https://www.cs.tau.ac.il/~alonarde/interdependent.pdf>.

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