The Competition Complexity of Auctions:
A Bulow-Klemperer Result for Multi-Dimensional Bidders
[Abstract]

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A seminal result of Bulow and Klemperer [1989] demonstrates the power of competition for extracting revenue: when selling a single item to n bidders whose values are drawn i.i.d. from a regular distribution, the simple welfare-maximizing VCG mechanism (in this case, a second price-auction) with one additional bidder extracts at least as much revenue in expectation as the optimal mechanism. The beauty of this theorem stems from the fact that VCG is a *prior-independent* mechanism, where the seller possesses no information about the distribution, and yet, by recruiting one additional bidder it performs better than any prior-dependent mechanism tailored exactly to the distribution at hand (without the additional bidder).

In this work, we establish the first *full Bulow-Klemperer* results in *multi-dimensional* environments, proving that by recruiting additional bidders, the revenue of the VCG mechanism surpasses that of the optimal (possibly randomized, Bayesian incentive compatible) mechanism. For a given environment with i.i.d. bidders, we term the number of additional bidders needed to achieve this guarantee the environment's *competition complexity*.

Using the recent duality-based framework of Cai et al. [2016] for reasoning about optimal revenue, we show that the competition complexity of n bidders with additive valuations over m independent, regular items is at most $n + 2m - 2$ and at least $\log(m)$. We extend our results to bidders with additive valuations subject to downward-closed constraints, showing that these significantly more general valuations increase the competition complexity by at most an additive $m - 1$ factor. We further improve this bound for the special case of matroid constraints, and provide additional extensions as well.

CCS Concepts: •**Theory of computation → Algorithmic game theory and mechanism design;**

Additional Key Words and Phrases: Revenue maximization; prior-independence; simple auctions

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