Clearing Markets via Bundles

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Abstract. We study algorithms for combinatorial market design problems, where a set of heterogeneous and indivisible objects are priced and sold to potential buyers subject to equilibrium constraints. Extending the CWE notion introduced by Feldman et al. [STOC 2013], we introduce the concept of a *Market-Clearing Combinatorial Walrasian Equilibium* (MC-CWE) as a natural relaxation of the classical Walrasian equilibrium (WE) solution concept. The only difference between a MC-CWE and a WE is the ability for the seller to bundle the items prior to sale. This innocuous and natural bundling operation imposes a plethora of algorithmic and economic challenges and opportunities. Unlike WE, which is guaranteed to exist only for (gross) substitutes valuations, a MC-CWE always exists. The main algorithmic challenge, therefore, is to design computationally efficient mechanisms that generate MC-CWE outcomes that approximately maximize social welfare. For a variety of valuation classes encompassing substitutes and complements (including super-additive, single-minded and budget-additive valuations), we design polynomial-time MC-CWE mechanisms that provide tight welfare approximation results.

1 Introduction

The *resource allocation* problem lies at the heart of theoretical economics: how should scarce resources be allocated among individual agents with competing interests? Since the emergence of the Internet, which enables complex resource allocation on a grand scale, this has naturally become a central problem in computer science as well. Economists generally approach this problem by adopting the notion of *market equilibrium*. Broadly speaking, a market equilibrium is a set of resource prices that are stable in the sense that all agents are maximally happy with their allocations and no resources are left unallocated. A long line of work has been dedicated to addressing the existence of equilibrium prices, and it has been shown (see, e.g., [2]) that market equilibria exist very generally, as long as the market is *convex*.

While this result sounds appealing in its generality, the convexity assumption usually requires that resources be infinitely divisible. In many applications of interest, especially those with a computational aspect, resources are indivisible; in these cases the convexity assumption is inapplicable. Do the results from the convex environments carry over to non-convex environments? In general the answer is no: the existence of equilibrium prices is not guaranteed. As a result, the study of markets for indivisible goods tends to focus on specific cases for which such prices exists, such as when buyer values satisfy the *gross substitutes* condition.

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To be more precise, consider the following model. There are m indivisible, heterogeneous items to be allocated among n agents. Each agent $i \in [n]$ has preferences over bundles of items in the form of a valuation function v_i that maps every subset S of items into the value $v_i(S)$ that agent i derives from the bundle S (in monetary terms). Given a price vector $\mathbf{p} = (p_1, \ldots, p_m)$, a bundle S is said to be in agent i's *demand set* if S maximizes i's *utility* given $\mathbf{p}$, defined as the difference between $v_i(S)$ and $\sum_{j \in S} p_j$.

A *Walrasian equilibrium* (WE) is an assignment of *item prices* to the m items, and an assignment of the objects to the agents, such that: (1) every agent is allocated a bundle in his demand set, and (2) the market *clears*; i.e., all items are allocated. Such a solution is appealing; every agent is maximally happy despite competing preferences, the market clears, and the pricing structure is natural, simple, and transparent. Unfortunately, WE do not exist in general. A WE is guaranteed to exist only for the rather narrow class of gross substitutes (GS) valuations (a strict subset of submodular functions) [11]. This eliminates any hope for the applicability of WE to environments with valuations that exhibit complementarities, and many forms of substitutes as well.

Recently, [8] proposed a relaxation of WE, termed a *combinatorial Walrasian equilibrium* (CWE). In a CWE, the seller can choose to *bundle* objects prior to assigning prices. This is a natural power to afford the seller, since as the owner of the resources he has some inherent ability to define what is meant by an "item." The generated bundles induce a reduced market — a market in which the items for sale are the bundles generated by the seller. In addition to the bundling operation, the CWE further relaxes the WE notion in that it allows for items to remain unallocated (even when they are priced above zero). A CWE exists for any valuation profile, since the seller could bundle all objects into a single item. The important issue, then, is whether there exists a CWE that is (approximately) efficient with respect to social welfare. Indeed, [8] show there always exists a CWE with at least half of the optimal (unconstrained) welfare.

The CWE notion relaxes the WE notion in two ways: (i) it allows bundling, (ii) it does not require market clearance. While the bundling relaxation is central to the notion of CWE, the second relaxation warrants some discussion. The relaxation of market clearance is somewhat at odds with the notion of a two-sided market equilibrium: prices might not be stable from the seller's perspective. After all, if an object (i.e. bundle) does not sell, the seller may be tempted to decrease its price in order to to sell it and increase revenue. The concept of CWE therefore implicitly requires that the seller pre-commit to (sub-optimal) prices, in addition to committing to a bundling of the items. With this in mind, we consider whether the relaxation of market clearance is truly necessary. It is easy to see that the bundling relaxation alone is enough to guarantee existence of an equilibrium, so the question becomes one of welfare. Can we hope to achieve the welfare bound of [8] without relaxing market clearance?

To answer this question we define *Market-clearing CWE* (MC-CWE), which allows the bundling operation, but requires market clearance. A MC-CWE is precisely a WE over the reduced market; it differs only in the ability of the seller to pre-bundle the items, and in particular it is a stronger (more restrictive) concept than CWE.

For a number of valuation classes, encompassing both substitutes and complements, we provide two types of results. The first finds the fraction of the optimal social welfare that can be obtained in a MC-CWE outcome. The second addresses the same problem