Question 1 Which of the following systems are confluent? Which are terminating?

$$\begin{array}{rcl} \neg\neg x & \to & x \\ \neg(x \wedge y) & \to & (\neg x) \vee (\neg y) \\ \neg(x \vee y) & \to & (\neg x) \wedge (\neg y) \end{array} \tag{1}$$

$$\begin{array}{rcl} \neg\neg x & \to & x \\ \neg(x \wedge y) & \to & (\neg\neg\neg x) \vee (\neg\neg\neg y) \\ \neg(x \vee y) & \to & (\neg\neg\neg x) \wedge (\neg\neg\neg y) \end{array} \tag{2}$$

$$\begin{array}{rcl} \neg\neg x & \to & x \\ \neg(x \wedge y) & \to & (\neg x) \vee (\neg y) \\ \neg(x \vee y) & \to & (\neg x) \wedge (\neg y) \\ x \wedge (y \vee z) & \to & (x \wedge y) \vee (x \wedge z) \\ (y \vee z) \wedge x & \to & (y \wedge x) \vee (z \wedge x) \end{array} \tag{3}$$

$$\begin{array}{rcl} \neg\neg x & \to & x \\ \neg(x \wedge y) & \to & (\neg\neg\neg x) \vee (\neg\neg\neg y) \\ \neg(x \vee y) & \to & (\neg\neg\neg x) \wedge (\neg\neg\neg y) \\ x \wedge (y \vee z) & \to & (x \wedge y) \vee (x \wedge z) \\ (y \vee z) \wedge x & \to & (y \wedge x) \vee (z \wedge x) \end{array} \tag{4}$$

$$\begin{array}{rcl} x \vee x & \to & x \\ x \wedge x & \to & x \\ \neg(x \wedge y) & \to & ((\neg x) \vee (\neg y)) \vee ((\neg x) \vee (\neg y)) \end{array} \tag{5}$$

$$\begin{array}{rcl} \neg\neg x & \to & x \\ x \vee x & \to & x \\ x \wedge x & \to & x \\ \neg(x \wedge y) & \to & ((\neg x) \vee (\neg y)) \vee ((\neg x) \vee (\neg y)) \\ \neg(x \vee y) & \to & ((\neg x) \wedge (\neg y)) \wedge ((\neg x) \wedge (\neg y)) \end{array} \tag{6}$$

$$\begin{array}{rcl} \neg\neg x & \to & x \\ x \vee x & \to & x \\ x \wedge x & \to & x \\ \neg(x \wedge y) & \to & ((\neg\neg\neg x) \vee (\neg\neg\neg y)) \vee ((\neg\neg\neg x) \vee (\neg\neg\neg y)) \\ \neg(x \vee y) & \to & ((\neg\neg\neg x) \wedge (\neg\neg\neg y)) \wedge ((\neg\neg\neg x) \wedge (\neg\neg\neg y)) \end{array} \tag{7}$$

Question 2 Let R and S be two rewrite systems with the following properties:

1. R is *confluent*.
2. S is *confluent*.
3. The reflexive-symmetric-transitive closures of their rewrite relations are identical ($\leftrightarrow_R^* = \leftrightarrow_S^*$).
4. The union of their rewrite relations ($\to_R \cup \to_S$) is terminating.
5. The right sides of every rule in R and S is in normal form.

Prove that their derivability relations are the same:

$$\to_R^* \quad = \quad \to_S^*$$