Approximation Algorithms for Combinatorial Optimization Problems

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Final Exam

February 6, 2000

Please solve **four** out of the following **five** problems. The duration of the exam is **three** days. During the exam, do **not** discuss the content of the exam, and do **not** show it, to any person other than the instructor of the course.

Problem 1

(a) For every fixed k, describe a polynomial time algorithm that given a BIN PACKING instance that can be packed into k bins, produces a feasible packing that uses at most $k+1$ bins.

(b) Give the best approximation algorithm that you can find for the BIN PACKING problem restricted to instances in which all items are of size at least $\frac{1}{100}$.

Problem 2

(a) Give a $\frac{1}{2}$-approximation algorithm for the MAX DI-CUT problem *without* solving a semidefinite program. (An instance of the MAX DI-CUT problem is a *directed* graph $G=(V,E)$ with weights defined on its edges. The goal is to find a subset $S \subseteq V$ of the vertices such that the total weight of edges going from S to $V-S$ is maximized.) (Bonus: Can you get a performace ratio better than $\frac{1}{4}$ without solving a linear program?)

(b) An instance of the MIN 2-SAT problem is a 2-SAT formula. The goal is to find an assignment that *minimizes* the number of satisfied clauses. Give the best approximation algorithm that you can find for the problem, again without solving a semidefinite program. (Note that this is a *minimization* problem.)

Problem 3

(a) Given a graph $G=(V,E)$, we construct a graph G' by adding *two* intermediate vertices on each edge of G. Graphs obtained in this way are said to be *special*. Give the best approximation algorithm that you can find for the MAX CUT problem in special graphs.

(b) Describe a polynomial time algorithm that given a 3-colorable graph $G = (V, E)$ produces a coloring of G with a small number of colors such that no triangle in G is monochromatic. (The coloring produced by the algorithm is not necessarily valid, adjacent vertices may be given the same color. However, if $u, v, w \in V$ are three *distinct* vertices and $(u, v), (u, w), (v, w) \in E$, then u, v and w cannot all get the same color.) Try to make the number of colors used by your algorithm as small as possible.

Problem 4

(a) Give a 2-approximation algorithm for the multicut problem, as defined in class, restricted to instances that are trees.

(b) Give the best approximation algorithm that you can find for the following problem. The input is a weighted undeirected graph $G = (V, E)$ on n vertices, where n is divisible by 3. The goal is to find a forest of G of minimum weight such that number of vertices in each tree of this forest is divisible by 3.

Problem 5

An instance of the MINIMUM UNSATISFIED EQUALITIES AND INEQUALITIES in the Boolean variables $x_1, x_2, \ldots, x_n$ is a collection of equalities of the form $x_i = x_j$ and inequalities of the form $x_i \neq x_j$. The goal is to find a smallest sub-collection of equalities/inequalities whose removal leaves a set of relations that can all be simultaneously satisfied. Give the best approximation algorithm that you can find for the problem. (Note again that this is a minimization problem.)