On the Accuracy of Passive Hyperbolic Localization in the Presence of Clock Drift

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***Abstract*—We discuss receiver clock correction and associated performance bounds for passive emitter localization using TDOA measurements from asynchronous sensor networks. In the considered system, passive receiving sensors are augmented with beacons at known locations that perform approximately periodical transmissions, used for calibration of the system synchronization. The precise transmission times as well as the transmission interval of the beacon messages are unknown. Similarly the transmissions of the target are irregular and do not, in general, occur simultaneous with the beacon transmissions. The clocks of each sensor are described by a linear error model. Based on that, we compare different snapshot based approaches for clock correction and derive theoretical limits for the localization of the target based on the modified Cramér-Rao lower bound (MCRLB). Simulation results are presented to illustrate the theoretical findings and show that our proposed estimators perform close to the MCRLB. Additional experimental results verify the analysis and the approach in a realistic large scale scenario.**

***Index Terms*—TDOA, Localization, Clock drift, Beacon, Cramér-Rao bound**

I. Introduction

Time measurement based localization systems either rely on very accurate synchronization between the anchor points, or, in asynchronous systems, require accurate estimation of the clock offsets to achieve optimum performance. Network infrastructure assisted passive localization has been in the center of various research activities for several decades. In comparison, results on the operation of such systems with asynchronous, drifting clocks and wireless synchronization are rather young.

The most common approach to localization is divided in two stages, where first a physical signal parameter is estimated, in our case the time of arrival (TOA), and second the target location is obtained using a second algorithm that takes the measurements, TOAs or the resulting time differences of arrival (TDOAs) as an input, e.g., [1]–[4]. Due to hardware imperfections and physical constraints, impairments of the first stage are not uncommon in practice. The requirement of nanosecond accuracy for decimeter level localization is challenging even for the most accurate clocks. For optimum localization performance, it is therefore necessary to apply correction methods to cope with the effects of asynchronous clocks. Accordingly, in [5] the idea of differential TDOA (DTDOA) has been introduced in order to correct for clock offsets. Further, in [6] the use of a calibration emitter, also known as beacon, has been proposed. However, the authors focus on inaccurate knowledge of sensor locations rather than asynchronous clocks. Later, [7] proposed a protocol for passive localization with asynchronous clocks where one of the anchors actively transmits a reference signal. The problem has also been investigated in a downlink fashion [8], where the transmitting anchors' clocks are assumed to be perfect, but the receiving target possesses a drifting clock. An extension of this scenario to cooperative localization is given in [9]. Most of the publications in this area apply the Cramér-Rao lower bound (CRLB) to derive theoretical performance limits. As described in [9], for problems with many interlinked parameters that all need to be estimated, the modified CRLB (MCRLB) often is more practical and provides more insight into the problem. It has been introduced in [10] and further thoroughly examined in [11] and [12]. In the present paper, the methodology of the modified bound is followed, as it constitutes a feasible way of deriving theoretical performance limits.

In contrast to the previous works, we consider a different system model, where the investigated system consists of distributed passive sensors, i.e., they can only receive, and additional beacons, that can only transmit. The beacons are not collocated with the sensors, sensor and beacon locations are known and the sensor clocks are running asynchronously and experience drift over time. Recently, in [13] a similar system has been considered. However, this paper does not present any theoretical bounds and assumes a very precisely timed beacon signal, where the exact time difference between two beacon transmissions is known, which we do not have in our case. Another implementation of such a system for wildlife tracking has been described in [14]. Previously, the sensors have been synchronized with GPS and therefore the clock drift has been neglected in practice, while for the present paper we consider the same system running with asynchronous, drifting clocks. For numerical evaluation we then obtain recorded TOAs from the fusion center of the system under this deteriorated conditions. Due to stringent constraints in battery size of the target and beacons, the rate of transmissions in the system is strictly limited. Further, target and beacon transmissions are not synchronized and therefore the time difference between the received messages can lead to large differences in the local clocks, during one set of measurements. Hence, we derive the necessary clock correction algorithms and evaluate their performance using simulations and a comparison with the MCRLB. For validation, we then apply the correction algorithms to the recorded TOAs.

The remainder of the paper is structured as follows. In Sec. II the model for the localization system and the erroneous clocks is defined. Subsequently, in Sec. III estimators for the correction are discussed. Then in Sec. IV the bounds for different clock correction approaches are derived. Finally, Sec. V provides simulation and experimental results to demonstrate and verify the performance of the discussed approaches. A short conclusion of the work is given in Sec. VI.

II. System and Clock Model

We consider a system of M distributed receiving sensors that are localizing a moving transmitter. The system is considered to be passive and has no communication between the sensors and the transmitting target. Additionally, two transmitting beacons at known locations are available for synchronization. We assume that each sensor i is independently able to estimate the TOA $t_{i,k}$ of the received signal from either the target $k = 0$ or the two beacons $k \in \{1, 2\}$, based on a pilot sequence. The location can then be determined using the TDOAs $\tau_{ij,k} = t_{i,k} - t_{j,k}$ between receiver i and the reference receiver j. In the following we apply [3] to find the solution and exclusively focus on the problem of correctly determining the TOAs and resulting TDOAs respectively. For that, the time offset caused by the drifting clocks at receiver i is modeled as [15]

$$t_i = \epsilon_i t + \phi_i \,, \tag{1}$$

where t is the real time, ϕ_i a constant initial offset at time instance $t^{(0)} = 0$ and ϵ_i the drift rate, which is assumed to be constant over the course of a measurement period and Gaussian distributed with respect to different clocks. The mean of ϵ_i is 1 and its standard deviation σ_{ϵ_i} depends on the specific clock type, it can reach as low as 10^{-12} for GPS disciplined oscillators (GPSDOs) and up to 10^{-5} for a temperature compensated crystal oscillator (TCXO). This especially means that in a system with several sensors, their clocks will often drift in different directions with respect to real time. Hence, assuming that target and beacon transmissions occur at a relatively low rate, e.g., 1 Hz, considerable relative errors between sensor clocks might be introduced. This can also lead to an association problem of the received transmissions at the different sensors. We assume that a mechanism exist to resolve this problem, such as unique message identifiers embedded in the signal. Another physical layer approach is to perform a plausibility check by comparing known and estimated beacon locations.

In order to perform clock synchronization between two sensors, two beacons at different locations are needed. However, a single beacon is sufficient to achieve a reasonable approximate solution as described later. Figure 1 shows the transmission sequence of the involved signal bursts at time instances $t^{(q)}$. A transmitted signal from beacon or target k at time index q

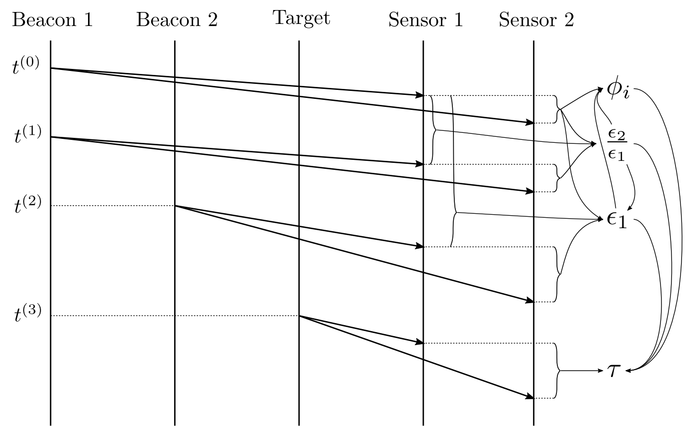


Fig. 1: Transmission sequence of a beacon augmented passive TDOA localization system using TOA measurements. Different combinations of measurements can be used to estimate the clock parameters and subsequently localize the target.

receives the sensor i at the TOA

$$t_{i,k}^{(q)} = \epsilon_i \left(t^{(q)} + \frac{d_{i,k}}{c} \right) + \phi_i + n_i^{(q)} \,, \tag{2}$$

where $n_i^{(q)}$ is a realization of zero-mean Gaussian noise with variance $\sigma_{n_i}^2$ and $d_{i,k}$ is the distance between the beacon or target and the sensor

$$d_{i,k} = \|\mathbf{p}_k - \mathbf{p}_i\|_2 \,, \tag{3}$$

with the two dimensional coordinate vector $\mathbf{p}_k = [x_k, \ y_k]^T$. The TDOA is then given as the difference

$$\begin{aligned} \tau_{ij,k} &= t_{i,k}^{(q)} - t_{j,k}^{(q)} \\ &= (\epsilon_i - \epsilon_j) t^{(q)} + \epsilon_i \frac{d_{i,k}}{c} - \epsilon_j \frac{d_{j,k}}{c} \\ &\quad + \phi_i - \phi_j + n_i^{(q)} - n_j^{(q)} \,. \end{aligned} \tag{4}$$

Localization algorithms eventually require the distance differences

$$\delta_{ij,k} = d_{i,k} - d_{j,k} \,, \tag{5}$$

in order to geometrically determine the target coordinates. Hence, correction of the parameters ϵ_i and ϕ_i is necessary. A possible way to remove parameters that are constant between two TDOA measurements is the differential time difference of arrival (DTDOA)

$$\Delta\tau_{ij,kl}^{(q,r)} = \tau_{ij,k}^{(q)} - \tau_{ij,l}^{(r)} \,. \tag{6}$$

Different types of DTDOA measurements are possible. A DTDOA between receptions of the same transmitter, e.g., the same beacon, at different time instance yields

$$\Delta\tau_{ij,kk}^{(q,r)} = (\epsilon_i - \epsilon_j)(t^{(q)} - t^{(r)}) + n_i^{(q)} - n_j^{(q)} - n_i^{(r)} + n_j^{(r)} \,. \tag{7}$$

Clearly, the constant term $\phi_i - \phi_j$ is cancelled out. Another type of DTDOA is between different transmitters k and l, i.e., two different beacons or one beacon and the target, at different time instances

$$\begin{aligned}\Delta\tau_{ij,kl}^{(q,r)} = \quad & (\epsilon_i - \epsilon_j)(t^{(q)} - t^{(r)}) \\ & + \epsilon_i\left(\frac{d_{i,k}}{c} - \frac{d_{i,l}}{c}\right) - \epsilon_j\left(\frac{d_{j,k}}{c} - \frac{d_{j,l}}{c}\right) \\ & + n_i^{(q)} - n_j^{(q)} - n_i^{(r)} + n_j^{(r)} ,\end{aligned} \tag{8}$$

again this eliminates the offset $\phi_i - \phi_j$. However the simple formation of DTDOAs is not able to remove ϵ_i and hence, for larger time spans between beacon and target transmissions, ignoring these erroneous clock rates leads to a drastic degradation of the system accuracy.

III. Clock Error Correction

Next, four different estimators are defined that are useful in different situations, which depend on the amount of noise, the quality of the clocks and the time span between transmissions.

1) TDOA estimation without correction: Assuming that the clocks are of very high accuracy and synchronization is close to perfect, i.e, $\epsilon_i = 1$ and $\phi_i = 0$, simple TDOA estimation without further clock synchronization may be used based on (4).

2) TDOA estimation with offset correction: If very accurate clocks ($\epsilon_i = 1$) are available, that are not absolutely synchronized, i.e., $\phi_i \neq 0$, a DTDOA with a single beacon may be used based on (8). Equivalently, the difference $\phi_i - \phi_j$ can also be estimated explicitly using a single TDOA (4) from a beacon if the clock rates ϵ_i are known

$$\Delta\hat{\phi}_{1j} = \tau_{1j,k} - \frac{1}{c}\delta_{1j,k} , \tag{9}$$

where without loss of generality, one may set the time of transmission $t^{(0)} = 0$. The distance differences needed by the localization algorithm are then

$$\hat{\delta}_{1j,l} = c \cdot (\tau_{1j,l} - \tau_{1j,k}) + \delta_{1j,k} . \tag{10}$$

3) TDOA estimation with approximate offset and rate correction: When lower quality, unsynchronized clocks are used, i.e, $\epsilon_i \neq 1, \phi_i \neq 0$ it becomes necessary to correct for both parameters. This can be approximately achieved using a single beacon, however, two transmissions from that beacon are now required. The estimation of ϕ_i depends on ϵ_i, hence we begin with the estimation ϵ_i then subsequently estimate ϕ_i and correct the initially measured TOAs.

Let $i = 1$ be the reference sensor. The time $t^{(0)} - t^{(1)}$ between two beacon transmissions of a single beacon k can be estimated using a single sensor's clock. Compared to the true time difference between transmission this time difference is scaled

$$t^{(0)} - t^{(1)} = \frac{1}{\epsilon_1}(t_{1,k}^{(0)} - t_{1,k}^{(1)} - n_1^{(0)} + n_1^{(1)}) . \tag{11}$$

Considering this scaling, the DTDOA for the same beacon can be written as

$$\begin{aligned}\Delta\tau_{1j,kk}^{(0,1)} = & \left(1 - \frac{\epsilon_j}{\epsilon_1}\right)(t_{1,k}^{(0)} - t_{1,k}^{(1)}) \\ & + \frac{\epsilon_j}{\epsilon_1}(n_1^{(0)} - n_1^{(1)}) - n_j^{(0)} + n_j^{(1)} .\end{aligned} \tag{12}$$

From this, obviously not all clock rates can be estimated but it is possible to estimate them relative to the reference sensor, as $\beta_j = \frac{\epsilon_j}{\epsilon_1}$. The estimator of β_j can be written as

$$\hat{\beta}_j = \frac{-\Delta\tau_{1j,kk}^{(0,1)} + (t_{1,k}^{(0)} - t_{1,k}^{(1)})}{(t_{1,k}^{(0)} - t_{1,k}^{(1)})}, \quad j \in 2, \ldots, M . \tag{13}$$

Further, $\hat{\epsilon}_1 = 1$ is a reasonable choice based on the the mean value of the distribution. Therefore, the estimate of clock rates will become $\hat{\epsilon}_j = \hat{\beta}_j$. Using those estimated values for the rates, next the absolute clock offsets of all sensors is estimated

$$\hat{\phi}_j = t_{j,k}^{(0)} - \hat{\epsilon}_j \frac{d_{j,k}}{c}, \quad j \in 1, \ldots, M . \tag{14}$$

With all of this the TOAs of the target can be corrected as

$$\hat{\delta}_{1j,l} = c \cdot \left(\frac{t_{1,l}^{(3)} - \hat{\phi}_1}{\hat{\epsilon}_1} - \frac{t_{j,l}^{(3)} - \hat{\phi}_j}{\hat{\epsilon}_j}\right) , \tag{15}$$

and then used by the localization algorithm to determine the location $\mathbf{p}_0$. Neglecting estimation error, the clock correction essentially enforces equal clock rates in all sensors, which are identical to the true value of ϵ_1. A look at (4) reveals that this approach is reasonable as the term $(\epsilon_i - \epsilon_j)t^{(q)}$ becomes zero and the scaling in the term $\epsilon_i \frac{d_{i,k}}{c} - \epsilon_j \frac{d_{j,k}}{c}$ is less critical due to the errors in the rates being relatively small.

4) TDOA estimation with offset and rate correction: Finally, if the system dimensions become very large, e.g., satellite based systems, or the speed of the wave is low, e.g., ultrasound based systems, the error in the approximation of ϵ_1 can no longer be neglected any more. Fortunately, using a second beacon at a different location, an estimate for ϵ_1 can be obtained. Considering the time difference of two transmission from the two different beacons k and k'

$$\begin{aligned}t^{(0)} - t^{(2)} = & \frac{1}{\epsilon_1}(t_{1,k}^{(0)} - t_{1,k'}^{(2)}) - \left(\frac{d_{1,k}}{c} - \frac{d_{1,k'}}{c}\right) \\ & - \frac{1}{\epsilon_1}(n_1^{(0)} - n_1^{(2)}) ,\end{aligned} \tag{16}$$

this further results in

$$\begin{aligned}\Delta\tau_{1j,kk'}^{(0,2)} = & (1 - \beta_j)(t_{1,k}^{(0)} - t_{1,k'}^{(2)}) \\ & + \epsilon_1\beta_j\left(\frac{d_{1,k}}{c} - \frac{d_{1,k'}}{c}\right) - \epsilon_1\beta_j\left(\frac{d_{j,k}}{c} - \frac{d_{j,k'}}{c}\right) \\ & + \beta_j(n_1^{(0)} - n_1^{(2)}) - n_j^{(0)} + n_j^{(2)} ,\end{aligned} \tag{17}$$

and leads to an estimator for ϵ_1

$$\hat{\epsilon}_1 = c \cdot \frac{\Delta\tau_{1j,kk'}^{(0,2)} - (1 - \hat{\beta}_j)(t_{1,k}^{(0)} - t_{1,k'}^{(2)})}{\hat{\beta}_j(\delta_{1j,k} - \delta_{1j,k'})} . \tag{18}$$

where $\hat{\beta}_j$ is estimated using (13). Analogously to the last case, using (14) and (15), the necessary input values for the localization algorithm can be obtained.

IV. Modified CRLB

In this section, theoretical limits of the estimation of target location $\mathbf{p}_0$ using TDOAs from (4) are derived. For any unbiased estimator, a lower bound on the variance of the error is provided by the Cramér-Rao lower bound (CRLB) [16]. It is based on the inverse of the Fisher information matrix (FIM) which is in our case given as

$$[\mathbf{I}(\mathbf{u})]_{n,m} = -\operatorname{E}\left\{\frac{\partial^2 \ln p(\boldsymbol{\tau};\mathbf{u})}{\partial u_n \partial u_m}\right\}, \tag{19}$$

where $\boldsymbol{\tau} = [\tau_{12,k}, \ldots, \tau_{1M,k}]^T$ is the vector of TDOAs (4) of the target when sensor $i = 1$ is the reference sensor, and the vector $\mathbf{u}$ of parameters to be estimated.

For the problem at hand $\mathbf{u}$ consists of the target coordinates as well as the clock parameters. The relationship between these parameters makes an analytical derivation of the CRLB very challenging. An approach that provides a less tight bound but simplifies the computation is the MCRLB for vector parameter estimation [11]. To derive the MCRLB expression, the estimation parameters are split into $\mathbf{u}$, the vector of estimation parameters, and $\mathbf{v}$, the vector of unwanted parameters, which are defined differently for each case considered below. We assume $\mathbf{u}$ to be deterministic, and $\mathbf{v}$ to be random with known probability distribution function. Then, the conditional FIM $\mathbf{I}(\mathbf{v};\mathbf{u})$ is calculated using the expectation operator as

$$[\mathbf{I}(\mathbf{u};\mathbf{v})]_{n,m} = -\operatorname{E}_{\boldsymbol{\tau}|\mathbf{v}}\left\{\frac{\partial^2 \ln p(\boldsymbol{\tau}|\mathbf{v};\mathbf{u})}{\partial u_n \partial u_m}\right\}, \tag{20}$$

with the probability density function $p(\boldsymbol{\tau}|\mathbf{u};\mathbf{v})$

$$p(\boldsymbol{\tau}|\mathbf{v};\mathbf{u}) = \frac{1}{(2\pi)^{\frac{M-1}{2}} \det(\mathbf{C}^{-1})^{\frac{1}{2}}} \cdot \exp\left(-\frac{1}{2}(\boldsymbol{\tau}-\boldsymbol{\mu})^T \mathbf{C}^{-1}(\boldsymbol{\tau}-\boldsymbol{\mu})\right), \tag{21}$$

The conditional FIM can be calculated as [16]

$$[\mathbf{I}(\mathbf{u};\mathbf{v})]_{n,m} = \left[\frac{\partial \boldsymbol{\mu}}{\partial u_n}\right]^T \mathbf{C}^{-1}\left[\frac{\partial \boldsymbol{\mu}}{\partial u_m}\right] + \frac{1}{2}\operatorname{tr}\left(\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial u_n}\mathbf{C}^{-1}\frac{\partial \mathbf{C}}{\partial u_m}\right). \tag{22}$$

According to [11], taking the expectation over $\mathbf{v}$, one can then obtain the modified Fisher information matrix (MFIM) $\mathbf{I}_{\text{mod}}(\mathbf{u})$ with

$$[\mathbf{I}_{\text{mod}}(\mathbf{u})]_{n,m} = \operatorname{E}_{\mathbf{v}}\{[\mathbf{I}(\mathbf{u};\mathbf{v})]_{n,m}\}. \tag{23}$$

This leads to the modified lower bound on the estimation variance

$$\operatorname{var}(\hat{u}_m) \geq \left[\mathbf{I}_{\text{mod}}(\mathbf{u})^{-1}\right]_{m,m}. \tag{24}$$

Next, this can be used to determine the bounds for the localization with the different clock correction approaches.

1) TDOA estimation without correction: In this case there is only a single transmission from the target $k = 0$ available. We consider $\mathbf{u} = [x_0, y_0]^T$ as estimation parameters and $\mathbf{v} = [\epsilon_1, \ldots, \epsilon_M, \phi_1, \ldots, \phi_M]^T$ as the unwanted parameters. To calculate the lower bound on the estimation error, we directly use the MCRLB from (24) with the mean $\boldsymbol{\mu} = [\mu_2, \ldots, \mu_M]^T$ and the covariance matrix $\mathbf{C}$ of $\boldsymbol{\tau}$

$$\mu_i = (\epsilon_1 - \epsilon_i)t^{(q)} + \epsilon_1\frac{d_{1,k}}{c} - \epsilon_i\frac{d_{i,k}}{c} + \phi_1 - \phi_i\,, \tag{25}$$

$$\mathbf{C} = \operatorname{diag}(\sigma_{n_2}^2, \ldots, \sigma_{n_M}^2) + \sigma_{n_1}^2 \mathbf{1}_{M-1}\mathbf{1}_{M-1}^T\,. \tag{26}$$

For the cases that beacon transmissions are available, this simple MCRLB is unacceptably loose and we resort to a two-step approach, where we first obtain bounds for the clock parameters and then, in the second step, derive the bound for the target location.

2) TDOA estimation with offset correction: In this case a single beacon $k = 1$ is available and the clock offsets ϕ_i can be estimated. Hence, the estimation parameter vector for the first step becomes $\mathbf{u} = [\phi_1, \ldots, \phi_M]^T$ and the vector of unwanted parameters $\mathbf{v} = [\epsilon_1, \ldots, \epsilon_M]^T$. Then, the MCRLB on the estimation of the clock offsets ϕ_i is calculated using the mean and covariance matrix from (25) and (26). In the second step, we have $\mathbf{u} = [x_0, y_0]^T$, and $\mathbf{v} = [\epsilon_1, \ldots, \epsilon_M]^T$. We model the clock offsets as $\hat{\phi}_i = \phi_i + \omega_i$, where $\hat{\phi}_i$ is an estimate with the error ω_i modeled as a Gaussian random variable, $\omega_i \sim \mathcal{N}(0, \sigma_{\phi_i}^2)$, where $\sigma_{\phi_i}^2$ is given by the bound from the first step. The second step mean and the covariance matrix then become

$$\mu_i = (\epsilon_1 - \epsilon_i)t^{(q)} + \epsilon_1\frac{d_{1,0}}{c} - \epsilon_i\frac{d_{i,0}}{c} + \hat{\phi}_i - \hat{\phi}_j\,, \tag{27}$$

$$\mathbf{C} = \operatorname{diag}(s_{1,\phi}^2, \ldots, s_{M,\phi}^2) + s_{1,\phi}^2 \mathbf{1}_{M-1}\mathbf{1}_{M-1}^T\,, \tag{28}$$

where $s_{i,\phi}^2 = \sigma_{n_i}^2 + \sigma_{\phi_i}^2$. This leads to the MCRLB of the target location.

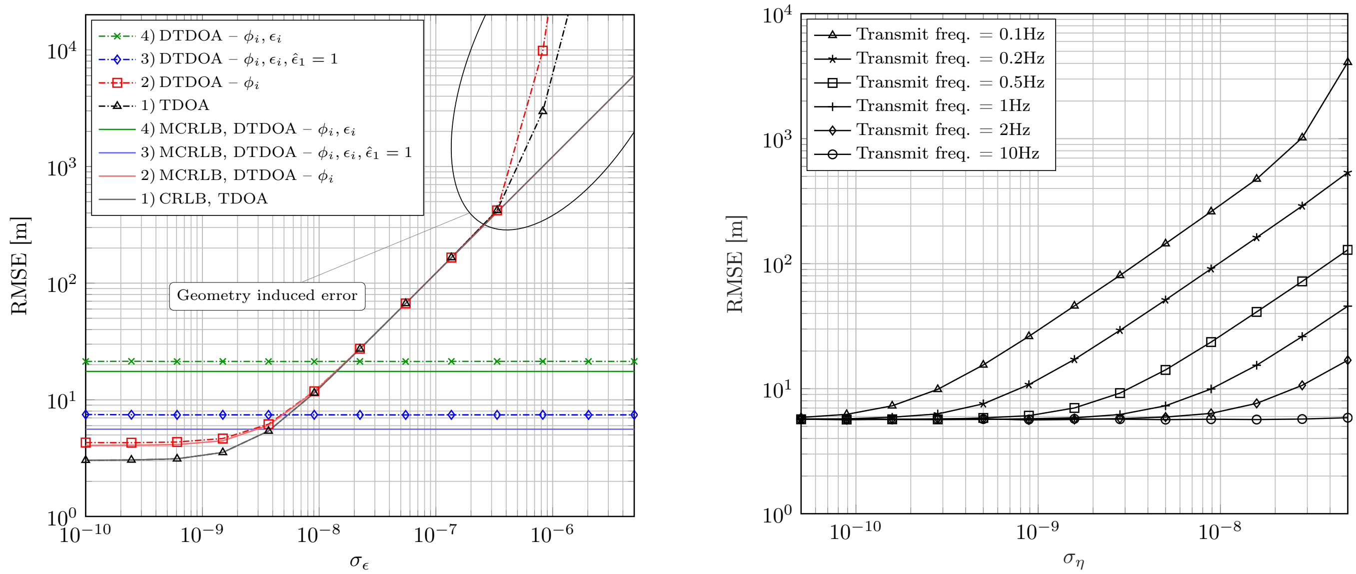
3) TDOA estimation with approximate offset and rate correction: When two transmissions of the beacon $k = 1$ are available, it is possible to estimate the clock offsets ϕ_i as well as the clock rates ϵ_i, except for one, e.g., ϵ_1. Thus, similar to the last case, in the first step, we define the vectors as $\mathbf{u} = [\epsilon_2, \ldots, \epsilon_M, \phi_1, \ldots, \phi_M]^T$ and $\mathbf{v} = [\epsilon_1]$.
In the second step, we model both the clock offset and rate as Gaussian random variables $\hat{\phi}_i = \phi_i + \omega_i$, $i = 1, \ldots, M$ and $\hat{\epsilon}_i = \epsilon_i + \xi_i$, $i = 2, \ldots, M$, where ω_i and ξ_i are the errors in the clock offset and rate estimation, modeled as Gaussian random variables $\omega_i \sim \mathcal{N}(0, \sigma_{\phi_i}^2)$ and $\xi_i \sim \mathcal{N}(0, \sigma_{\epsilon_i}^2)$. Variances $\sigma_{\phi_i}^2$ and $\sigma_{\epsilon_i}^2$ are derived using the the first step MCRLB. To calculate the MCRLB on the estimation of the target position we incorporate the clock offset and clock rate models into (4). The second step mean and covariance matrix for this case are

$$\mu_i = (\epsilon_1 - \hat{\epsilon}_i)t^{(q)} + \epsilon_1\frac{d_{1,0}}{c} - \hat{\epsilon}_i\frac{d_{i,0}}{c} + \hat{\phi}_i - \hat{\phi}_j\,, \tag{29}$$

$$\mathbf{C} = \operatorname{diag}(s_{1,\epsilon}^2, \ldots, s_{M,\epsilon}^2) + s_{1,\phi}^2 \mathbf{1}_{M-1}\mathbf{1}_{M-1}^T\,, \tag{30}$$

where $s_{i,\epsilon}^2 = \sigma_{\epsilon_i}^2(t^{(q)} + \frac{d_{i,0}}{c})^2 + \sigma_{n_i}^2 + \sigma_{\phi_i}^2$ and $s_{1,\phi}$ is the same as in the last case. Similarly, the second step parameter vectors are $\mathbf{u} = [x_0, y_0]^T$ and $\mathbf{v} = [\epsilon_1]$.

4) TDOA estimation with offset and rate correction: Finally, with the aid of two beacons $k \in \{1, 2\}$, it is possible to estimate the clock rates of all the sensors. Hence, in this case there are no unwanted parameters, neither in the first nor in the second step. The first step estimation parameter vector



(a) (b)

Fig. 2: (a) The MCRLB compared with simulations for different availability of beacon messages with $\phi_i = 0$. 1) Baseline, no beacons used. 2) Single beacon message, estimation of ϕ_i. 3) Two messages from the same beacon, estimation of ϕ_i and ϵ_i while $\hat{\epsilon}_1 = 1$. 4) Three messages from two beacons, full estimation of ϕ_i and ϵ_i. (b) Impact of time dependent clock rate modeled as a random walk, using method 3).

is $\mathbf{v} = [\epsilon_1, \ldots, \epsilon_M, \phi_1, \ldots, \phi_M]^T$. We calculate the variance of the clock parameters estimation error using the standard CRLB. Analogous to the last case in the second step, we model all the clock rates and offsets with Gaussian random variables. Thus, the bound for the target location is derived with the estimation parameter vector $\mathbf{u} = [x_0, y_0]^T$, mean

$$\mu_i = (\hat{\epsilon}_1 - \hat{\epsilon}_i)t^{(q)} + \hat{\epsilon}_1 \frac{d_{1,0}}{c} - \hat{\epsilon}_i \frac{d_{i,0}}{c} + \hat{\phi}_i - \hat{\phi}_j \,, \tag{31}$$

and covariance matrix

$$\mathbf{C} = \mathrm{diag}(s_{1,\epsilon}^2, \ldots, s_{M,\epsilon}^2) + s_{1,\epsilon}^2 \mathbf{1}_{M-1} \mathbf{1}_{M-1}^T \,. \tag{32}$$

V. Results

Simulative as well as experimental results have been obtained for verification. In the simulation, 5 sensors have been placed using the same geometry as given in the experimental system, depicted in Fig. 3 (a). First, receiver clocks have been modeled according to (1) and the time between transmissions is 2 s. The standard deviation of the error in each TOA measurement is $\sigma_{n_i} = 10^{-8}$. It is then possible to compare the analytical expressions for the bounds with Monte Carlo simulation result. This is shown in Fig. 2 (a). From the plot it is found that if very good clocks are available it is recommendable to not apply any correction. That is due to the additional noise terms introduced into the solution due to additional measurements of the beacon signals. Note that this assumes that the offsets ϕ_i are close to zero. When the error in the TOA becomes large due to uncompensated differing clock rates, it leads to large outliers of the estimated location, caused by the nonlinear nature of the hyperbolic geometry. This can be observed in the upper right corner of Fig. 2 (a). Further, due to bad conditioning of the problem, as seen in (18), for the given system dimensions, full correction of all ϵ_i provides much worse performance than the approximation $\hat{\epsilon}_1 = 1$, which itself should be used only for clocks with $\sigma_\epsilon \geq 4 \times 10^{-9}$. However, this threshold obviously depends on the time between the beacon transmissions and also the measurement noise. Therefore, in a second simulation, the time dependence of the clock rates ϵ_i is modeled using a Brownian random walk approach where σ_η^2 is the variance of the additional Gaussian noise, that is cumulatively added. We then vary the time between the transmissions and observe the degradation of the localization in terms of root mean square error (RMSE). The resulting plot is presented in Fig. 2 (b). This can be used to select the rate of the beacon transmission based on the specification of the used sensor clocks.

Experimental measurements have been obtained from the ATLAS system [14] with a subset of 5 sensors as shown in Fig. 3 (a). Target tags, which are usually attached to birds for tracking, transmit a binary frequency shift keying signal every 1 s with a bandwidth of 2 MHz. Beacons transmit an identical signal every 2 s. Transmitted data is a 8192 long random code sequence, that yields a large correlation gain at the receiver and enables simultaneous channel access for a large number of tags. The sensors are based on software defined radio with Ettus USRP N200 frontends that contain a TCXO frequency reference specified with 2.5 ppm. However, according to the estimation for the recorded data set, all 5 sensor clock rates are within less than 1 ppm. Fig. 3 (b) shows the drift in the location of a static target when just the offset correction from Sec. III-2 or the offset and rate correction from Sec. III-3 is

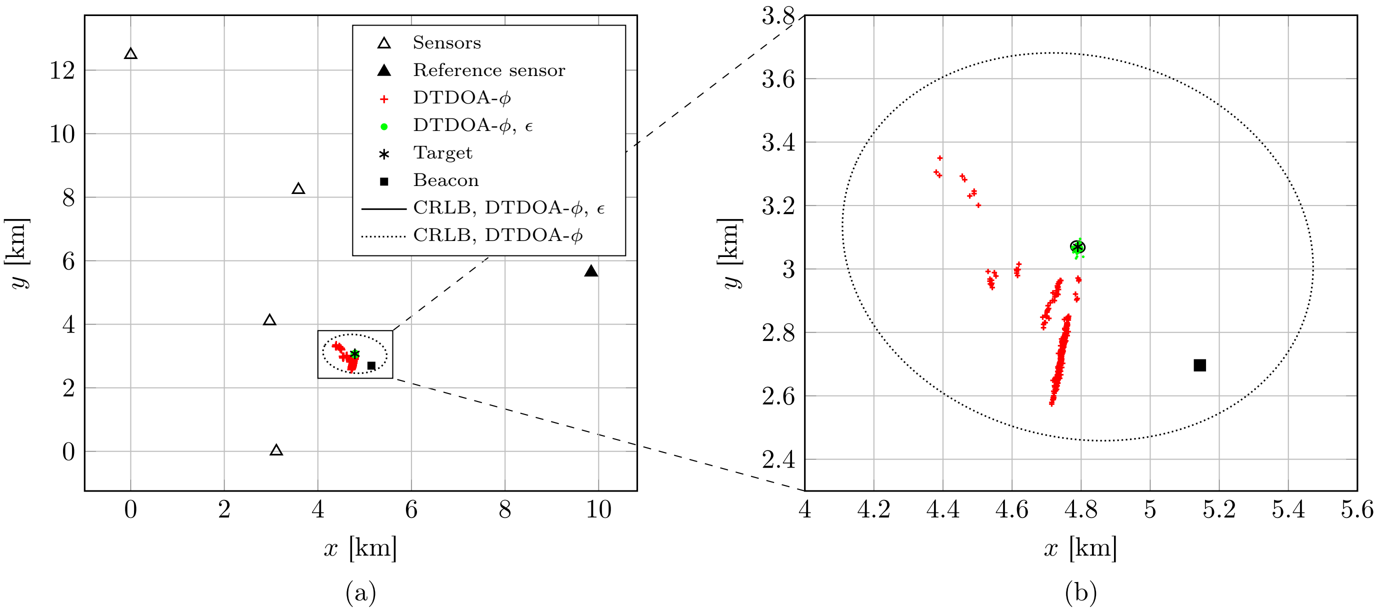


Fig. 3: (a) Geometry of the considered localization sensor network [14], (b) drift in the location of a stationary target with correction of ϕ_i compared to the case with correction of ϕ_i and ϵ_i while $\hat{\epsilon}_1 = 1$.

used. The RMSE for the two cases is 334.71 m and 8.40 m respectively.

VI. Conclusion

In this paper we have analyzed the sensitivity of a particular type of passive localization system with respect to different qualities of the sensor clocks. Four different cases have been identified and correction methods have been derived accordingly, together with the corresponding modified Cramér-Rao lower bounds. Results from simulation and a deployed experimental wildlife tracking system clearly illustrate that the considered system is able to perform well with drifting sensor clocks.

VII. Acknowledgments

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References

[1] W. H. Foy, "Position-location solutions by Taylor-series estimation," *IEEE Transactions on Aerospace and Electronic Systems*, no. 2, pp. 187–194, 1976.

[2] J. Smith and J. Abel, "Closed-form least-squares source location estimation from range-difference measurements," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 35, no. 12, pp. 1661–1669, 1987.

[3] Y. T. Chan and K. Ho, "A simple and efficient estimator for hyperbolic location," *IEEE Transactions on signal processing*, vol. 42, no. 8, pp. 1905–1915, 1994.

[4] Y. Huang, J. Benesty, G. W. Elko, and R. M. Mersereau, "Real-time passive source localization: A practical linear-correction least-squares approach," *IEEE transactions on Speech and Audio Processing*, vol. 9, no. 8, pp. 943–956, 2001.

[5] C. Yan and H. H. Fan, "Asynchronous self-localization of sensor networks with large clock drift," in *Mobile and Ubiquitous Systems: Networking & Services, 2007. MobiQuitous 2007. Fourth Annual International Conference on.* IEEE, 2007, pp. 1–8.

[6] K. Ho and L. Yang, "On the use of a calibration emitter for source localization in the presence of sensor position uncertainty," *IEEE Transactions on Signal Processing*, vol. 56, no. 12, pp. 5758–5772, 2008.

[7] Y. Wang, X. Ma, and G. Leus, "Robust time-based localization for asynchronous networks," *IEEE Transactions on Signal Processing*, vol. 59, no. 9, pp. 4397–4410, 2011.

[8] M. R. Gholami, S. Gezici, and E. G. Strom, "TDOA based positioning in the presence of unknown clock skew," *IEEE Transactions on Communications*, vol. 61, no. 6, pp. 2522–2534, 2013.

[9] M. Leng, W. P. Tay, C. M. S. See, S. G. Razul, and M. Z. Win, "Modified CRLB for cooperative geolocation of two devices using signals of opportunity," *IEEE Transactions on Wireless Communications*, vol. 13, no. 7, pp. 3636–3649, 2014.

[10] A. N. D'Andrea, U. Mengali, and R. Reggiannini, "The modified Cramer-Rao bound and its application to synchronization problems," *IEEE Transactions on Communications*, vol. 42, no. 234, pp. 1391–1399, 1994.

[11] F. Gini, R. Reggiannini, and U. Mengali, "The modified Cramer-Rao bound in vector parameter estimation," *IEEE Transactions on Communications*, vol. 46, no. 1, pp. 52–60, 1998.

[12] M. Moeneclaey, "On the true and the modified Cramer-Rao bounds for the estimation of a scalar parameter in the presence of nuisance parameters," *IEEE Transactions on Communications*, vol. 46, no. 11, pp. 1536–1544, 1998.

[13] J. Tiemann, F. Eckermann, and C. Wietfeld, "Multi-user interference and wireless clock synchronization in tdoa-based uwb localization," in *Indoor Positioning and Indoor Navigation (IPIN), 2016 International Conference on.* IEEE, 2016, pp. 1–6.

[14] A. Weller-Weiser, Y. Orchan, R. Nathan, M. Charter, A. J. Weiss, and S. Toledo, "Characterizing the accuracy of a self-synchronized reverse-GPS wildlife localization system," in *2016 15th ACM/IEEE International Conference on Information Processing in Sensor Networks (IPSN).* IEEE, 2016, pp. 1–12.

[15] Y.-C. Wu, Q. Chaudhari, and E. Serpedin, "Clock synchronization of wireless sensor networks," *IEEE Signal Processing Magazine*, vol. 28, no. 1, pp. 124–138, 2011.

[16] S. M. Kay, "Fundamentals of statistical signal processing: estimation theory," 1993.