# 0368-4283 – Space-Bounded Computation

Tuesdays, 16:00-19:00 in Holtzblat 07.

Grading policy:

- Take-Home Exam – 50%. Students who want to do a reading project instead of the take-home exam should contact us.
- Homework – 50%.
- Bonuses for help in forum and active participation in class.

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Extended Syllabus

| Part I – The basics | | |
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| Basic Classes. Some representative problems. | The classes $\mathsf{DSpace}(s(n))$, $\mathsf{BPSpace}(s(n))$, $\mathsf{RSpace}(s(n))$ and $\mathsf{NSpace}(s(n))$. | [AB] |
| | Circuit classes: $\mathsf{NC}^k$, $\mathsf{AC}^k$ and $\mathsf{NC}$. | [AB] |
| | Some languages (and problems) we want to classify: addition, multiplication, parity, majority, sorting, undircted connectivity, directed connectivity (STCON), Det, Perm, Maximal Independent Set (MIS), Perfect Matching (PM), Polynomial identity testing (PIT). | |

| Part II – Random Walks over Undirected graphs | | |
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| From Combinatorics to Algebra | Undirected graphs as operators. | |
| | Spectral gap and rapid mixing. | |
| | Undirected connectivity is in $\mathsf{RL}$. | [AK+], [AS] |
| | Universal traversal sequences and universal exploration sequences. | |
| A random walk over an expander as a replacement to independent random samples | Expanders: Graphs with a large spectral gap. | |
| | The expander mixing lemma. | [V] |
| | The expander Chernoff bound. | [H] |
| | Bias samplers. | |
| | A comparison with other samplers (extractors, condensers, and more). | |

| Part III – The Zig-Zag product and its ramifications for Space Bounded computation | | |
|---|---|---|
| Growing a graph into an expander | Cayley graphs, Abelian Cayley graphs, Explicit exapnders with logarithmic degree. | |
| | The art of turning big problems to small: The Zig-Zag product. | [RVW] |
| | A “combinatorial” construction of fully explicit expanders. | [RVW] |
| | Undirected Connectivity is in Deterministic LogSpace. | [R] |
| | Explicit universal exploration sequences | [RVW] |
| Inverting the Laplacian in small space | The Laplacian of undirected graphs | [L] |
| | Approximating non-negative operators | [MR+] |
| | Sparsifiers. Sparsifying the clique: Derandomized Squaring | [MR+] |
| | Approximating the inverse of the Laplacian of an undirected graph | [MR+] |

| Part IV – Directed graphs. | | |
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| | Normal operators, Hermitian operators, diagonalizable operators. | |
| | The SVD decomposition vs. the Jordan normal form. | |
| | Viewing bipartite graphs as operators. The up and down Laplacian. Doing spectral analysis with directed graphs. | |
| | An eigenvalue-approximation problem hard for $\mathsf{NL}$. | |
| | The class $\mathsf{BQL}$. | |

| Part V – Pseudo-random Generators | | |
|---|---|---|
| Nisan's generator | Branching programs: The non-uniform analogue of $\mathsf{BPL}$. | |
| | Pseudo-Randomness. | |
| | Nisan's generator (with hash functions and pair-wise independence; with expanders and the expander mixing lemma; with extractors). | [N1,NZ] |
| | The INW generator. Curving the seed from the inside, or taking it for each level from the outside. | [INW] |
| | $\mathsf{BPL} \subseteq \mathsf{DTimeSpace}(\mathrm{poly}(n), \log^2 n)$ | [N2] |
| $\mathsf{RL}$ is in $\mathsf{DSpace}(\log^{1.5} n)$ | Pseudo-deterministic algorithms. Consistent sampling using shift and truncate. | [SZ] |
| | The Saks and Zhou derandomization algorithm. | [SZ] |
| PRGs against more restricted adversaries | A PRG against combinatorial rectangles. | |
| | A PRG against regular branching programs. | |
| | A PRG against half spaces. | |

References


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|---|---|---|
| [V] | Salil Vadhan | Pseudorandomness (link) |
| [LW] | Michael Luby and Avi Wigderson | Pairwise Independence and Derandomization (link) |
| [L] | László Lovász | Random Walks on Graphs: A Survey (link) |
| [AB] | Sanjeev Arora and Boaz Barak | Computational Complexity: A Modern Approach |
| [AK+] | R. Aleliunas, R. M. Karp, R. J. Lipton, L. Lovász and C. Rackof | Random walks, universal traversal sequences, and the complexity of maze problems |
| [AS] | Noga Alon and Benny Sudakov | Bipartite subgraphs and the smallest eigenvalue (link) |
| [H] | Alexander Healy | Randomness-Efficient Sampling within $\mathsf{NC}^1$ (link) |
| [RVW] | Omer Reingold, Salil Vadhan and Avi Wigderson | Entropy waves, the zig-zag graph product, and new constant-degree (link) |
| [R] | Omer Reingold | Undirected Connectivity in Log-Space (link) |
| [MR+] | J. Murtagh, O. Reingold, A. Sidford and S. Vadhan | Derandomization Beyond Connectivity: Undirected Laplacian Systems in Nearly Logarithmic Space (link) |
| [N1] | Noam Nisan | Pseudorandom generators for space-bounded computation (link) |
| [N2] | Noam Nisan | $\mathsf{RL} \subseteq \mathsf{SC}$ (link) |
| [INW] | Russel Impagliazzo, Noam Nisan and Avi Wigderson | Pseudorandomness for Network Algorithms (link) |
| [NZ] | Noam Nisan and David Zuckerman | Randomness is linear in space (link) |
| [SZ] | Michael Saks and Shiyu Zhou | $\mathsf{BP_H SPACE}(S) \subseteq \mathsf{DSPACE}(S^{3/2})$ (link) |