Regular Sensing

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FSTTCS 2015+Submitted Work

Sensing - Motivation

- [redacted]
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- [redacted] si [redacted] ha [redacted] ts I [redacted] puts, resp [redacted]
- Con [redacted] ular I/O sp [redacted] $2^I \times 2^O$ [redacted] corresp [redacted] $\mathcal{T}$.
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Sensing - Motivation

- Consider an alphabet $2^I \times 2^O$ for some finite sets I and O of inputs and outputs, respectively.
- Consider a regular I/O specification over $2^I \times 2^O$, and a corresponding transducer $\mathcal{T}$.
- Reading the truth value of each signal in I requires activating some sensor.
- The complexity of $\mathcal{T}$ can be measured by the expected number of sensors $\mathcal{T}$ uses.

Related Work

- Mean-payoff Games with Incomplete Information
  [P. Hunter, G. A. Pérez, and J.F. Raskin, 2013].
- Minimum attention controller synthesis for ω-regular objectives
  [K. Chatterjee and R. Majumdar, 2011].
- Controller synthesis with budget constraints
  [K. Chatterjee, R. Majumdar, and T. A. Henzinger, 2008].
- Synthesis with Incomplete Information
  [O. Kupferman and M.Y. Vardi, 97].

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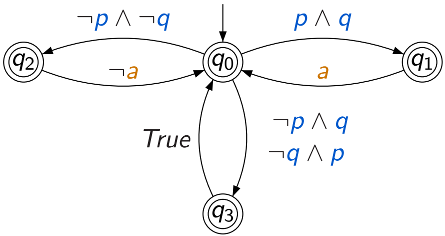

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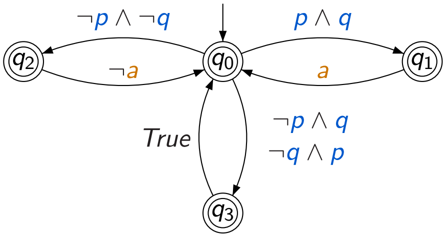

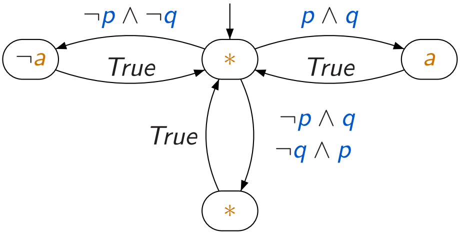

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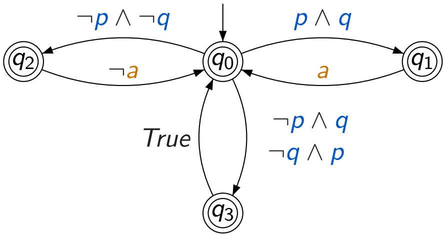

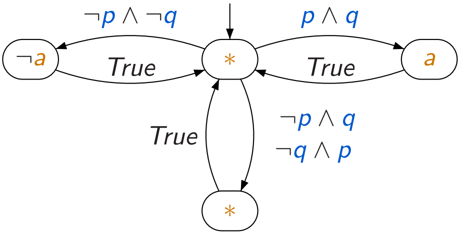

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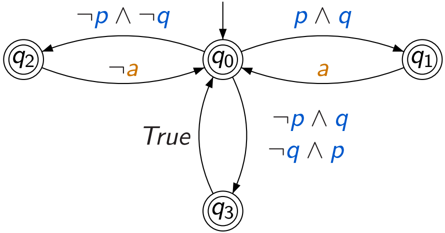

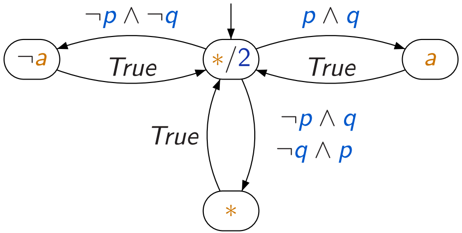

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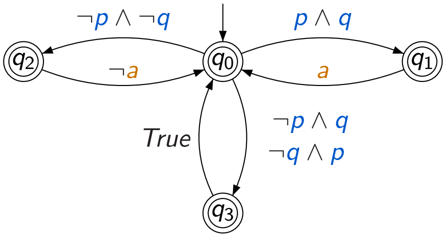

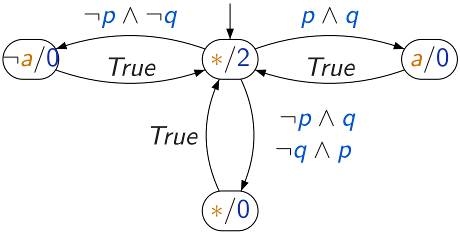

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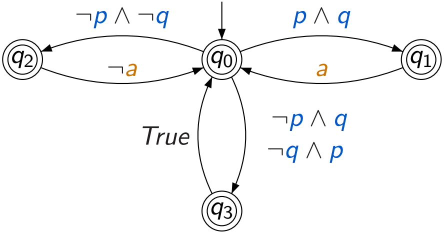

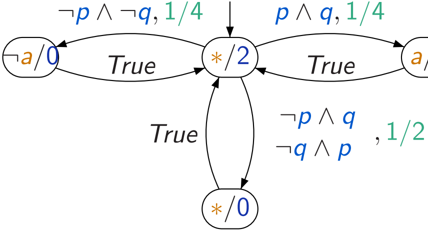

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- Other states sense $\emptyset$.
- Sensing cost of $\mathcal{T}$ is 1.

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- The sensing cost of $\mathcal{T}$ is the expected average cost on a uniformly-random word.

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- Output an I/O-*transducer* $\mathcal{T}$ that realizes $\mathcal{A}$, with minimal sensing.

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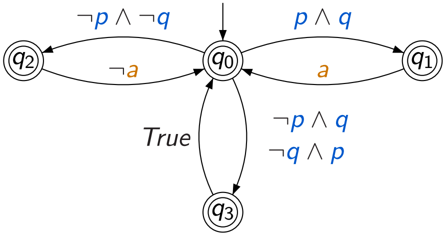

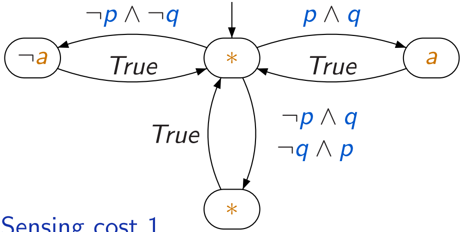

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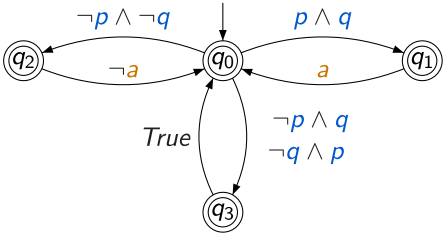

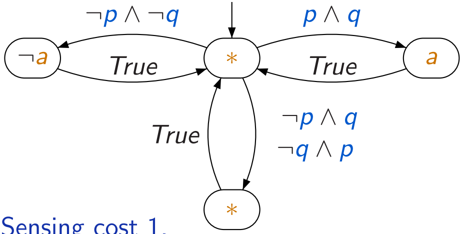

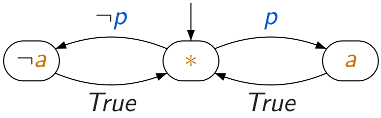

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Sensing cost $\frac{1}{2}$.

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- It is well known that a DFA specification is realizable iff there is an "embodied" strategy.
- However, a minimal-sensing transducer does not always correspond to an embodied strategy.
- For infinite words, the minimal sensing might not be attained. For example: $\mathsf{GF}(a \iff \mathsf{X}b)$

Lower Bound

Theorem

Given a DFA specification $\mathcal{A}$ and a threshold v, deciding whether $scost(\mathcal{A}) \leq v$ is EXPTIME-hard.

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We demonstrate the idea with DFAs.

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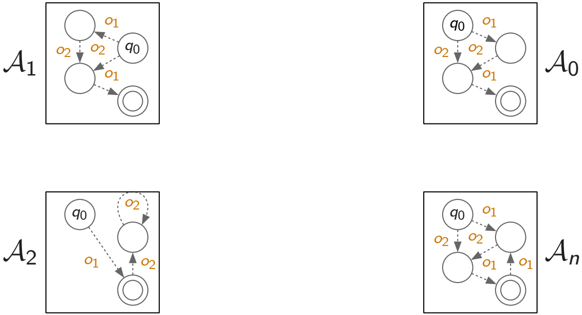

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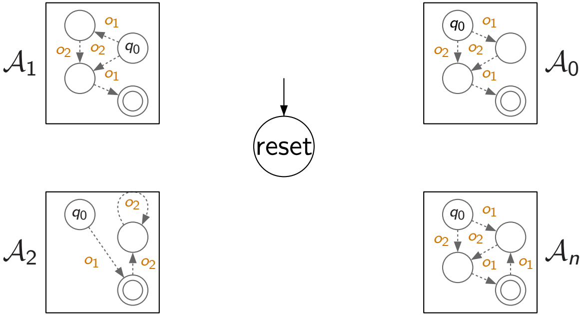

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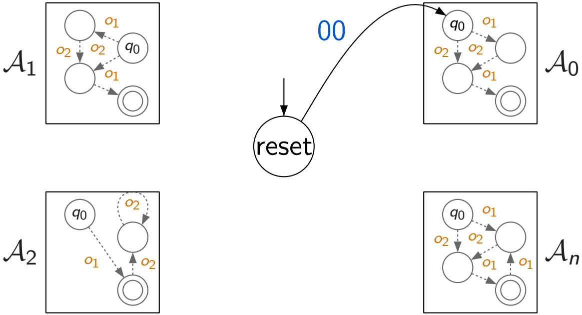

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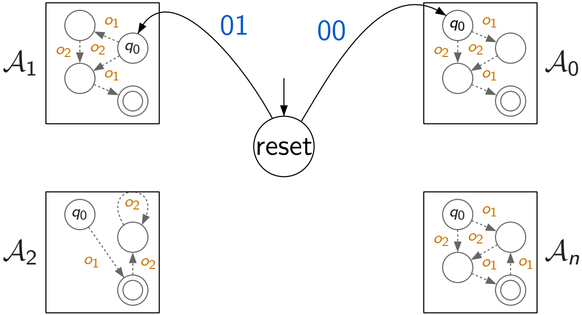

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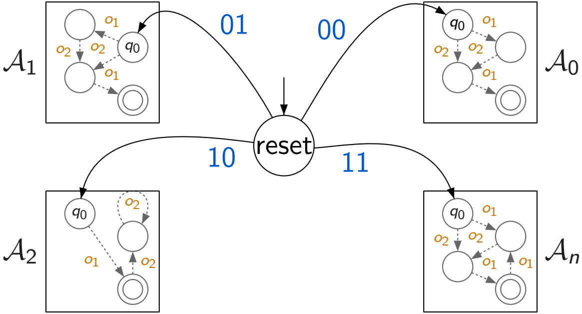

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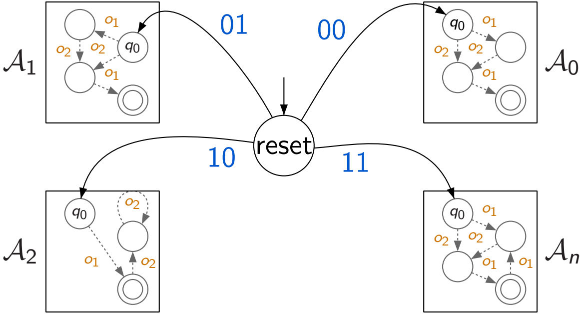

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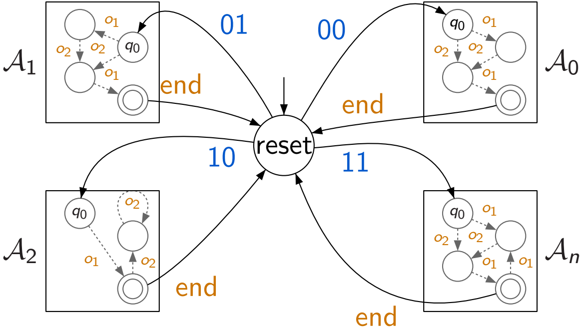

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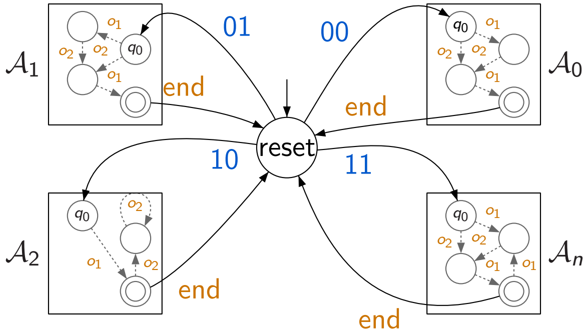
- The output *end* signifies that the word is finished.
- A transducer can ignore inputs in *reset* iff $\bigcap_i L(\mathcal{A}_i) \neq \emptyset$.



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- Can we also output a transducer that attains/approximates this value?
- Do *finite* transducers suffice for an approximation?

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- We reduce our problem to a variant of parity games combined with an MDP.

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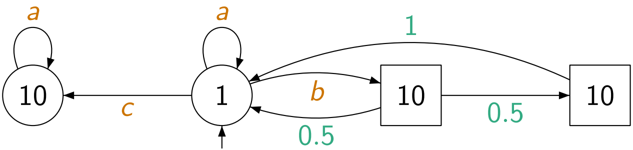
3. Construct from $\mathcal{D}$ a parity game $\mathcal{G}$, and assign a cost to each state according to the number of sensed inputs.
4. A winning strategy in $\mathcal{G}$ realizes the specification, and its expected mean-payoff against a stochastic environment is its sensing cost.

Parity MDPs

- A parity MDP is an MDP...



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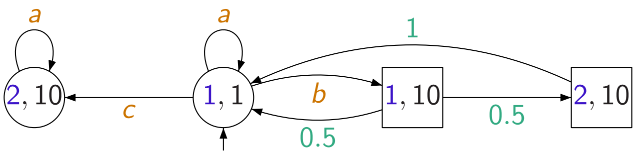

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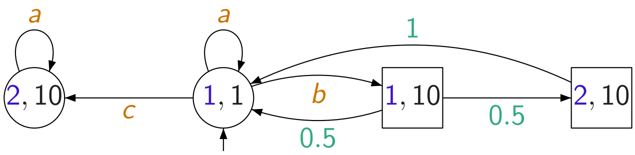

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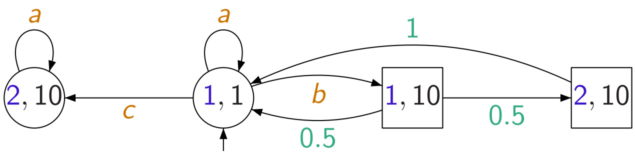

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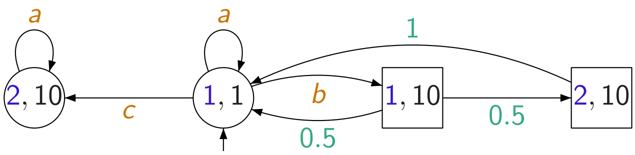
- Optimal strategy may not exist:



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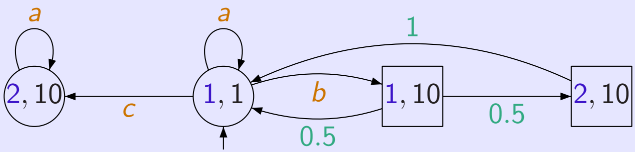

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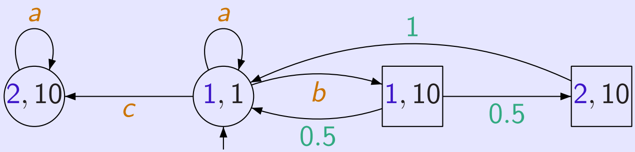

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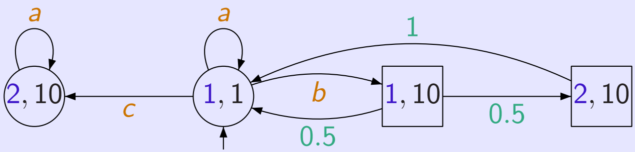

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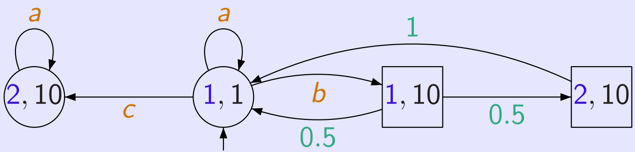

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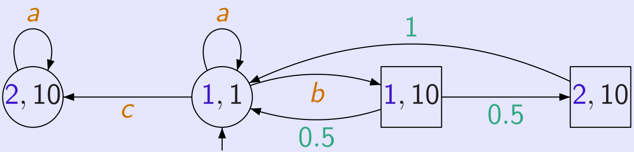
4. Otherwise, play *c* ("give up").



Summary of Results

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- Parity MDPs are a useful tool in modeling combinations of quantitative and Boolean properties.

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