

$$x - 0 \rightarrow x$$

$$s(x) - s(y) \rightarrow x - y$$

$$\text{quot}(0, s(y)) \rightarrow 0$$

$$\text{quot}(s(x), s(y)) \rightarrow s(\text{quot}(x - y, s(y)))$$

$$0 + y \rightarrow y$$

$$s(x) + y \rightarrow s(x + y)$$

$$(x - y) - z \rightarrow x - (y + z)$$

$$\text{le}(0, y) \rightarrow \text{true}$$

$$\text{le}(s(x), 0) \rightarrow \text{false}$$

$$\text{le}(s(x), s(y)) \rightarrow \text{le}(x, y)$$

$$\text{minus}(0, y) \rightarrow 0$$

$$\text{minus}(s(x), y) \rightarrow \text{if}_{\text{minus}}(\text{le}(s(x), y), s(x), y)$$

$$\text{if}_{\text{minus}}(\text{true}, s(x), y) \rightarrow 0$$

$$\text{if}_{\text{minus}}(\text{false}, s(x), y) \rightarrow s(\text{minus}(x, y))$$

$$\text{quot}(0, s(y)) \rightarrow 0$$

$$\text{quot}(s(x), s(y)) \rightarrow s(\text{quot}(\text{minus}(x, y), s(y)))$$

$$\text{le}(0, y) \rightarrow \text{true}$$

$$\text{le}(s(x), 0) \rightarrow \text{false}$$

$$\text{le}(s(x), s(y)) \rightarrow \text{le}(x, y)$$

$$\text{pred}(s(x)) \rightarrow x$$

$$\text{minus}(x, 0) \rightarrow x$$

$$\text{minus}(x, s(y)) \rightarrow \text{pred}(\text{minus}(x, y))$$

$$\text{gcd}(0, y) \rightarrow y$$

$$\text{gcd}(s(x), 0) \rightarrow s(x)$$

$$\text{gcd}(s(x), s(y)) \rightarrow \text{if}_{\text{gcd}}(\text{le}(y, x), s(x), s(y))$$

$$\text{if}_{\text{gcd}}(\text{true}, s(x), s(y)) \rightarrow \text{gcd}(\text{minus}(x, y), s(y))$$

$$\text{if}_{\text{gcd}}(\text{false}, s(x), s(y)) \rightarrow \text{gcd}(\text{minus}(y, x), s(x))$$

$le(0, y) \rightarrow \text{true}$

$le(s(x), 0) \rightarrow \text{false}$

$le(s(x), s(y)) \rightarrow le(x, y)$

$app(\text{nil}, y) \rightarrow y$

$app(\text{add}(n, x), y) \rightarrow \text{add}(n, app(x, y))$

$low(n, \text{nil}) \rightarrow \text{nil}$

$low(n, \text{add}(m, x)) \rightarrow \text{if}_{low}(le(m, n), n, \text{add}(m, x))$

$\text{if}_{low}(\text{true}, n, \text{add}(m, x)) \rightarrow \text{add}(m, low(n, x))$

$\text{if}_{low}(\text{false}, n, \text{add}(m, x)) \rightarrow low(n, x)$

$high(n, \text{nil}) \rightarrow \text{nil}$

$high(n, \text{add}(m, x)) \rightarrow \text{if}_{high}(le(m, n), n, \text{add}(m, x))$

$\text{if}_{high}(\text{true}, n, \text{add}(m, x)) \rightarrow high(n, x)$

$\text{if}_{high}(\text{false}, n, \text{add}(m, x)) \rightarrow \text{add}(m, high(n, x))$

$quicksort(\text{nil}) \rightarrow \text{nil}$

$quicksort(\text{add}(n, x)) \rightarrow app(quicksort(low(n, x)),$

$\text{add}(n, quicksort(high(n, x))))$