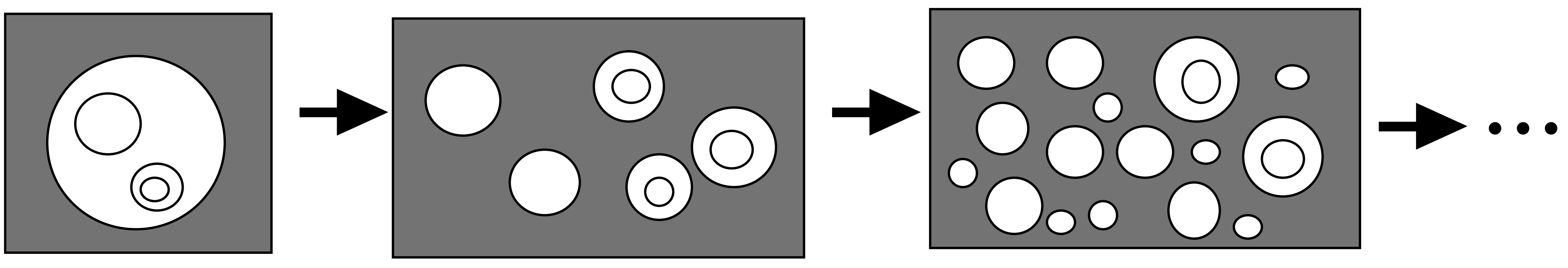
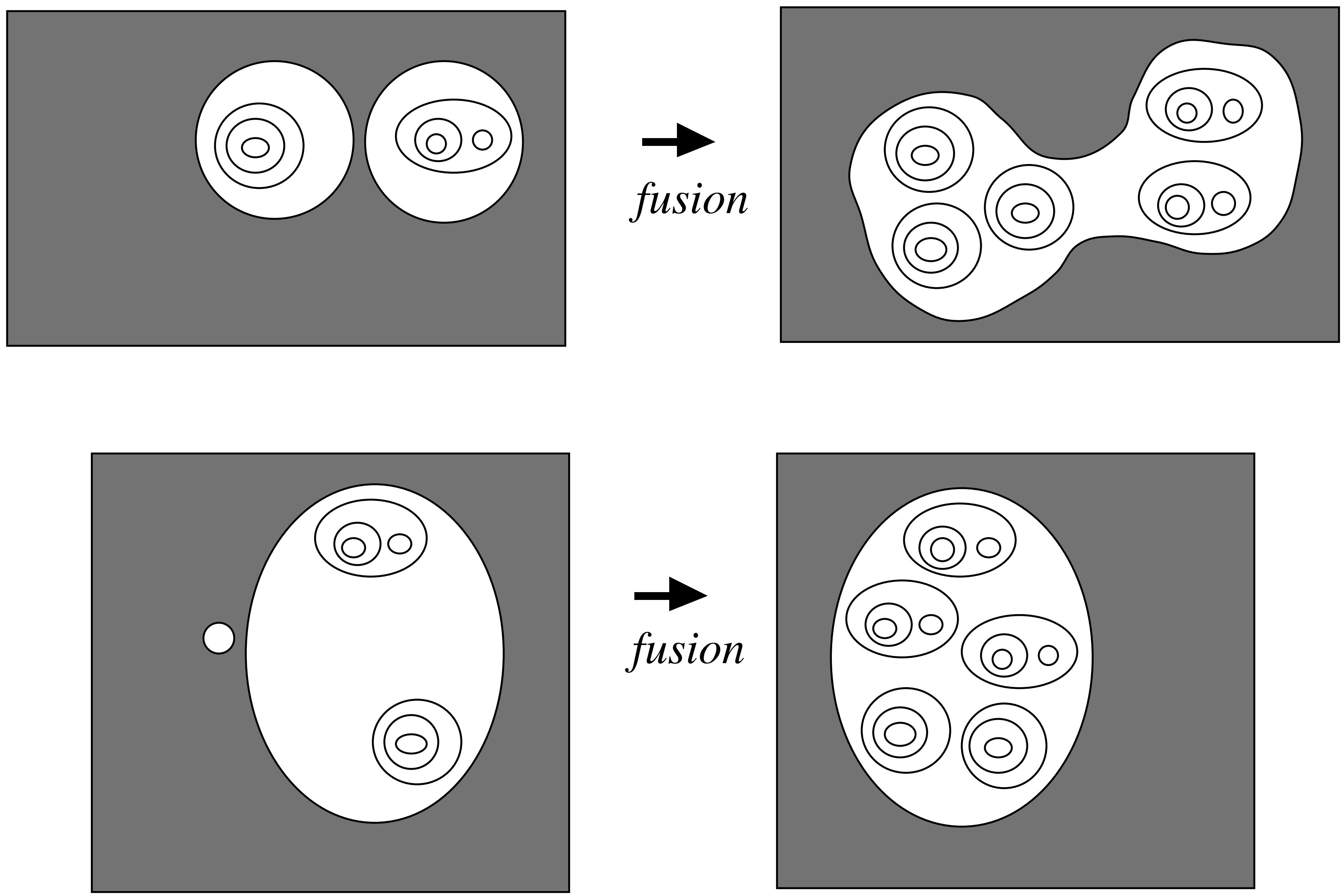
Termination

Rewriting

Fission




fusion
fusion

Better

- d(a) = depth(a)
- { {d(a) : a in A} : colony A }
- fission: depth decreases
- fusion: one deep item removed

DNFO

- $\neg\neg x \Rightarrow x$
- $\neg(x \vee y) \Rightarrow (\neg x) \wedge (\neg y)$
- $\neg(x \wedge y) \Rightarrow (\neg x) \vee (\neg y)$
- $x \wedge (y \vee z) \Rightarrow (x \wedge y) \vee (x \wedge z)$
- $(y \vee z) \wedge x \Rightarrow (y \wedge x) \vee (z \wedge x)$

DNF1

- $\neg\neg x \Rightarrow x$
- $\neg(x \vee y) \Rightarrow (\neg x) \wedge (\neg y)$
- $\neg(x \wedge y) \Rightarrow (\neg x) \vee (\neg y)$
- $x \wedge (y \wedge z) \Rightarrow (x \wedge y) \wedge z$
- $x \vee (y \vee z) \Rightarrow (x \vee y) \vee z$
- $x \wedge (y \vee z) \Rightarrow (x \wedge y) \vee (x \wedge z)$
- $(y \vee z) \wedge x \Rightarrow (y \wedge x) \vee (z \wedge x)$

DNF2

- $\neg\neg x \Rightarrow x$
- $\neg(x \vee y) \Rightarrow (\neg x) \wedge (\neg y)$
- $\neg(x \wedge y) \Rightarrow (\neg x) \vee (\neg y)$
- $(x \wedge y) \wedge z \Rightarrow x \wedge (y \wedge z)$
- $x \vee (y \vee z) \Rightarrow (x \vee y) \vee z$
- $x \wedge (y \vee z) \Rightarrow (x \wedge y) \vee (x \wedge z)$
- $(y \vee z) \wedge x \Rightarrow (y \wedge x) \vee (z \wedge x)$

DNF3

- $\neg\neg x \Rightarrow x$
- $\neg(x \vee y) \Rightarrow (\neg\neg\neg x) \wedge (\neg\neg\neg y)$
- $\neg(x \wedge y) \Rightarrow (\neg\neg\neg x) \vee (\neg\neg\neg y)$
- $x \wedge (y \vee z) \Rightarrow (x \wedge y) \vee (x \wedge z)$
- $(y \vee z) \wedge x \Rightarrow (y \wedge x) \vee (z \wedge x)$

DNF3

- $\neg\neg x \Rightarrow x$
- $\neg(x \vee y) \Rightarrow (\neg\neg\neg x) \wedge (\neg\neg\neg y)$
- $\neg(x \wedge y) \Rightarrow (\neg\neg\neg x) \vee (\neg\neg\neg y)$
- $x \wedge (y \vee z) \Rightarrow (x \wedge y) \vee (x \wedge z)$
- $(y \vee z) \wedge x \Rightarrow (y \wedge x) \vee (z \wedge x)$

$\neg\neg(a \wedge (b \vee c))$

DNF4

- $\neg\neg x \Rightarrow x$
- $\neg(x \vee y) \Rightarrow (\neg x) \wedge (\neg y)$
- $\neg(x \wedge y) \Rightarrow (\neg x) \vee (\neg y)$
- $x \wedge (y \vee z) \Rightarrow (x \wedge y) \vee (x \wedge z) \vee (x \wedge y) \vee (x \wedge z)$
- $(y \vee z) \wedge x \Rightarrow (x \wedge y) \vee (x \wedge z) \vee (x \wedge y) \vee (x \wedge z)$
- $x \vee x \Rightarrow x$

DNF5

- $\neg \neg x \Rightarrow x$
- $\neg(x \vee y) \Rightarrow (\neg x) \wedge (\neg y) \wedge (\neg x) \wedge (\neg y)$
- $\neg(x \wedge y) \Rightarrow (\neg x) \vee (\neg y) \vee (\neg x) \vee (\neg y)$
- $x \wedge (y \vee z) \Rightarrow (x \wedge y) \vee (x \wedge z)$
- $(y \vee z) \wedge x \Rightarrow (x \wedge y) \vee (x \wedge z)$
- $x \vee x \Rightarrow x$
- $x \wedge x \Rightarrow x$

DNF6

- $\neg\neg x \Rightarrow x$
- $\neg(x \vee y) \Rightarrow (\neg\neg\neg x) \wedge (\neg\neg\neg y) \wedge (\neg\neg\neg x) \wedge (\neg\neg\neg y)$
- $\neg(x \wedge y) \Rightarrow (\neg\neg\neg x) \vee (\neg\neg\neg y) \vee (\neg\neg\neg x) \vee (\neg\neg\neg y)$
- $x \vee x \Rightarrow x$
- $x \wedge x \Rightarrow x$

DNF7

- $\neg\neg x \Rightarrow x$
- $\neg(x \vee y) \Rightarrow (\neg x) \wedge (\neg y) \wedge (\neg x) \wedge (\neg y)$
- $\neg(x \wedge y) \Rightarrow (\neg x) \vee (\neg y) \vee (\neg x) \vee (\neg y)$
- $x \vee x \Rightarrow x$
- $x \wedge x \Rightarrow x$

Symbolic Computation

- $Dt = 1$
- $Dc = 0$
- $D(x+y) = Dx + Dy$
- $D(xy) = xDy + yDx$
- ...

Rewriting

- $Dt \Rightarrow 1$
- $Dc \Rightarrow 0$
- $D(x+y) \Rightarrow Dx + Dy$
- $D(xy) \Rightarrow xDy + yDx$
- …

Factorial

- $x+0 \Rightarrow x$
- $x+s(y) \Rightarrow s(x+y)$
- $x*0 \Rightarrow 0$
- $x*s(y) \Rightarrow y + x*y$
- $f(0) \Rightarrow s(0)$
- $f(s(x)) \Rightarrow s(x)*f(x)$
-

Factorial

- $x+0 \Rightarrow x$
- $x+s(y) \Rightarrow s(x+y)$
- $x*0 \Rightarrow 0$
- $x*s(y) \Rightarrow y + x*y$
- $f(0) \Rightarrow s(0)$
- $f(s(x)) \Rightarrow s(x)*f(p(s(x)))$
- $p(s(x)) \Rightarrow x$

Termination

- If $s[x] \Rightarrow t[x]$ is a rule
- then $c[s[v]] \Rightarrow c[t[v]]$ is a rewrite
- Want $c[s[v]] > c[t[v]]$ in some wfo
- Want monotonicity
  - $s > t \Rightarrow f(\ldots,s,\ldots) > f(\ldots,t,\ldots)$

Exponential Interpretation

- $[Dx] = 3^{[x]}$
- $[t] = [c] = 3$
- $[x+y] = \ldots = [xy] = [x] + [y]$

Polynomial Interpretation

- $[Dx] = [x]^2$
- $[x+y] = \ldots = [xy] = [x] + [y]$
- Eventually positive
  - $x^2 + y^2 + 2xy - x^2 - y^2 - x - y = 2xy - x - y$
  - Derivatives: $2x-1$; $2y-1$

Multiset Path Order

- $s = f(s_1,\ldots,s_m)$ $t = g(t_1,\ldots,t_n)$
- $s > t$ if $s_i \gtrsim t$ for some i
- $s > t$ if
  - $(f,\{s_1,\ldots,s_m\}) >_{lex} (g,\{t_1,\ldots,t_n\})$
  - and $s > t_j$ for all j

Lexicographic Path Order

- $s = f(s_1,\ldots,s_m)$ $t = g(t_1,\ldots,t_n)$
- $s > t$ if $s_i \gtrsim t$ for some i
- $s > t$ if
  - $(f,s_1,\ldots,s_m) >_{lex} (g,t_1,\ldots,t_n)$
  - and $s > t_j$ for all j

Boyer & Moore

- if(if(x,y,z),u,v) ⇨ if(x,if(y,u,v),if(z,u,v))

Recursive Path Order

- $s = f(s_1,\ldots,s_m) \quad t = g(t_1,\ldots,t_n)$
- $s > t$ if $s_i \gtrsim t$ for some i
- $s > t$ if
  - $(f, s_1,\ldots,\{s_i,\ldots,s_m\}) >_{lex} (g, t_1,\ldots,\{t_i,\ldots,t_n\})$
  - and $s > t_j$ for all j

Simplification Order

- Suppose finite vocabulary
- Subterm: $f(\ldots,s,\ldots) > s$
- Monotonic: $s > t \Rightarrow f(\ldots,s,\ldots) > f(\ldots,t,\ldots)$
- Must be well-founded

Weak Simplification Order

- Weak subterm: $f(\ldots,s_i,\ldots) \gtrsim s_i$
- Weak monotonicity:
  $s_i \gtrsim t_i \Rightarrow f(\ldots,s_i,\ldots) \gtrsim f(\ldots,t_i,\ldots)$
- Well-quasi-order by Kruskal
- Enough for termination of rewriting
  - Why?

Total Order

- Suppose finite vocabulary
- Monotonic: $s > t \Rightarrow f(\ldots,s,\ldots) > f(\ldots,t,\ldots)$
- Well-founded iff subterm

Semantic Path Order

- $s = f(s_1,\ldots,s_m) \quad t = g(t_1,\ldots,t_n) \qquad >$
- $s > t$ if $s_i \gtrsim t$ for some i
- $s > t$ if
  - $(s, s_1,\ldots,s_m) >_{lex} (t, t_1,\ldots,t_n)$
  - and $s > t_j$ for all j
- require $s \Rightarrow t \Rightarrow f(\ldots s\ldots) \geq f(\ldots t\ldots)$

Proof

- Extend base order to a total w.f. order
- Consider minimal bad sequence
- Subterms are well-founded
- No use of $s_i \gtrsim t$ case
- So base order decreases and stabilizes