Some ordinals

Ordinals are typically defined as the set of all smaller ordinals; every set is bigger than its subsets.

$$
\begin{aligned}
0 &= \phi \\
n &= \{0, 1, \ldots, n-1\} \\
\omega &= \{0, 1, 2, \ldots\} \\
\omega + n &= \omega + (n-1) \cup \{\omega + (n-1)\} \text{ (natural } n\text{)} \\
\alpha + 1 &= \alpha \cup \{\alpha\} \text{ (ordinal } \alpha\text{)} \\
\omega 2 &= \omega \cup \{\omega + n | n < \omega\} \\
\omega n &= \cup_{i<\omega}(\omega(n-1) + i) \text{ (natural } n\text{)} \\
\omega^n &= \cup_{i<\omega}(\omega^{n-1} i) \text{ (natural } n\text{)} \\
\omega^\omega &= \cup_{n<\omega} \omega^n \\
\omega^\alpha &= \cup_{\beta<\alpha} \omega^\beta
\end{aligned}
$$

$$
\epsilon_0 = \omega^{\epsilon_0} = \bigcup_{n<\omega} \omega^{\left.\omega^{\cdot^{\cdot^{\omega}}}\right\} n \text{ times}}
$$

$$
\epsilon_0^{\epsilon_0} = \omega^{\epsilon_0^2} = \cup_{\alpha<\epsilon_0} \omega^{\epsilon_0 \alpha}
$$

$$
\epsilon_1 = \bigcup_{n<\omega} \epsilon_0^{\left.\epsilon_0^{\cdot^{\cdot^{\epsilon_0}}}\right\} n \text{ times}}
$$

$$
\epsilon_{\epsilon_0} = \cup_{\alpha<\epsilon_0} \epsilon_\alpha
$$