

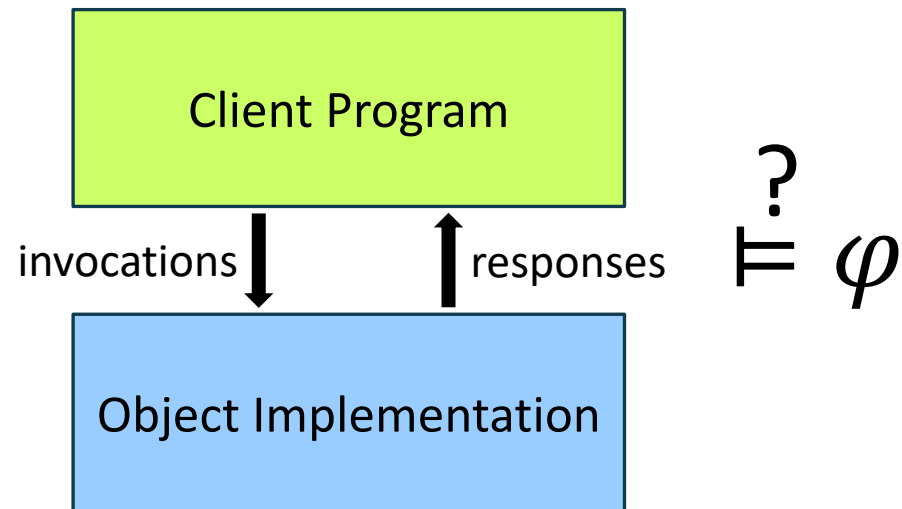
Hyperproperty-Preserving Register Specifications

YOAV BEN SHIMON, ORI LAHAV, SHARON SHOHAM

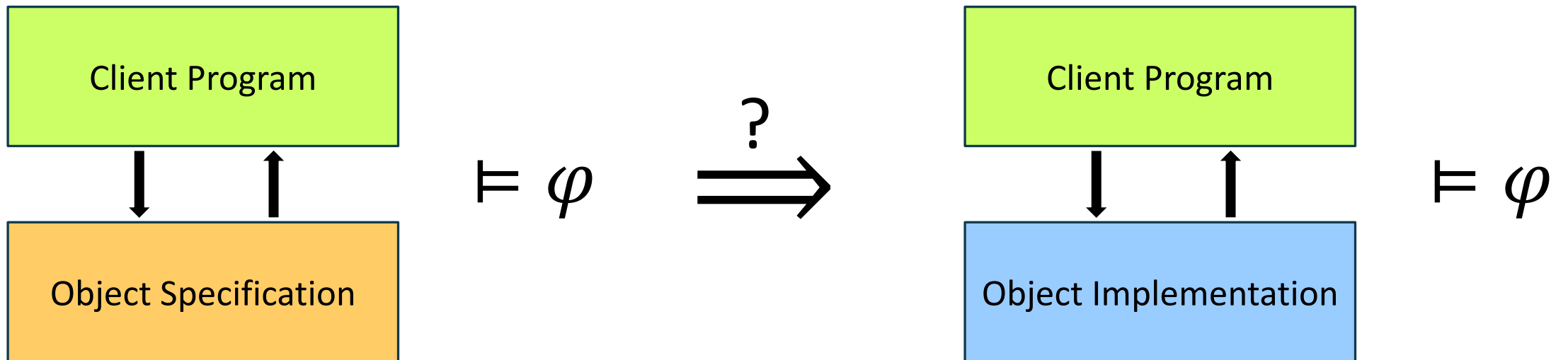
TEL AVIV UNIVERSITY



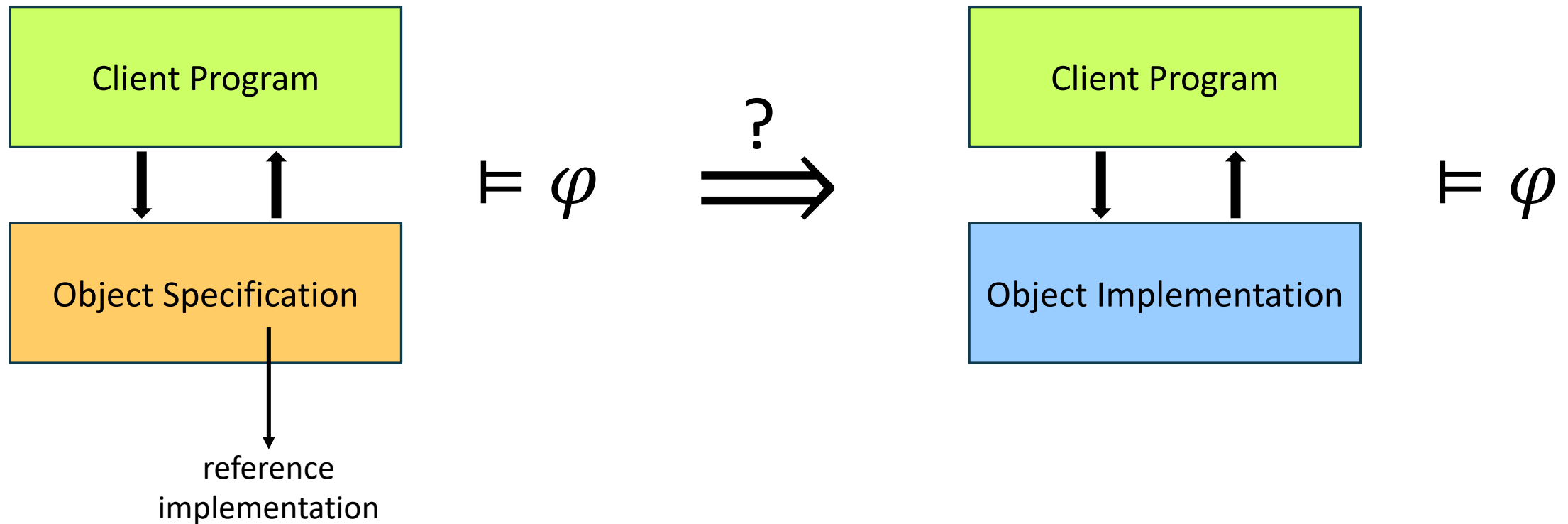
Verification via Abstraction



Verification via Abstraction

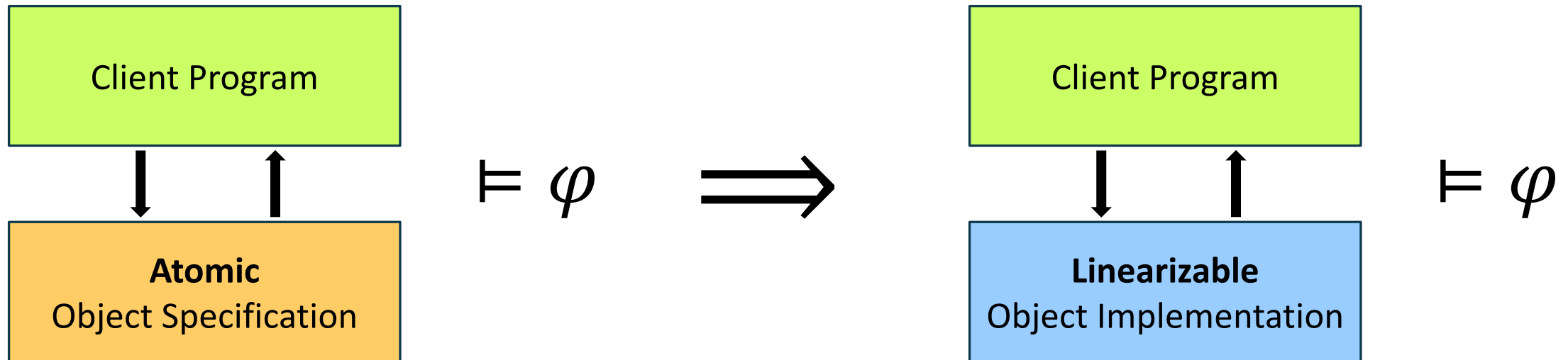


Verification via Abstraction



Abstraction via Linearizability

- If φ is a **trace property** (e.g., bad state not reachable):



- Does not work for **hyperproperties** [Golab, Higham, Woelfel '11] [Attiya, Enea '19]

Example: Program using Shared Register

```
write(1);  
write(2);  
a ← coin();
```

```
b ← read();
```



Atomic:

Method `write(v)`

```
| X ← v;
```

```
| return;
```

Method `read()`

```
| out ← X;
```

```
| return out;
```

$\varphi = \text{Pr}[a = b] = \frac{1}{2}$ for
any **strong adversary**



sees coin toss results
controls scheduling & non-determinism

Example: Program using Shared Register

```
write(1);  
write(2);  
a ← coin();
```

|||

```
b ← read();
```



Atomic:

```
Method write(v)  
| X ← v;  
| return;
```

```
Method read()  
| out ← X;  
| return out;
```



$\varphi = \text{Pr}[a = b] = \frac{1}{2}$ for
any **strong adversary**



sees coin toss results
controls scheduling & non-determinism

Example: Program using Shared Register

```
write(1);  
write(2);  
a ← coin();
```

```
||  
b ← read();
```



```
Double-load:  
Method write(v)  
| X ← v;  
| return;  
Method read()  
| out1 ← X;  
| out2 ← X;  
| if * then return out1;  
| else return out2;
```

$\varphi = \text{Pr}[a = b] = \frac{1}{2}$ for
any **strong adversary**

↓
sees coin toss results
controls scheduling & non-determinism

→ Linearizable

Example: Program using Shared Register

```
write(1);  
write(2);  
a ← coin();
```

||

```
b ← read();
```



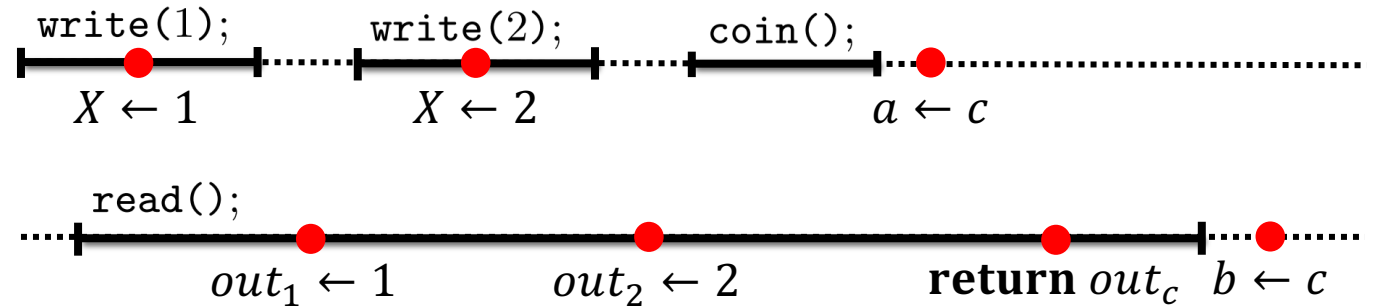
Double-load:

```
Method write(v)  
| X ← v;  
| return;  
Method read()  
| out1 ← X;  
| out2 ← X;  
| if * then return out1;  
| else return out2;
```

≠

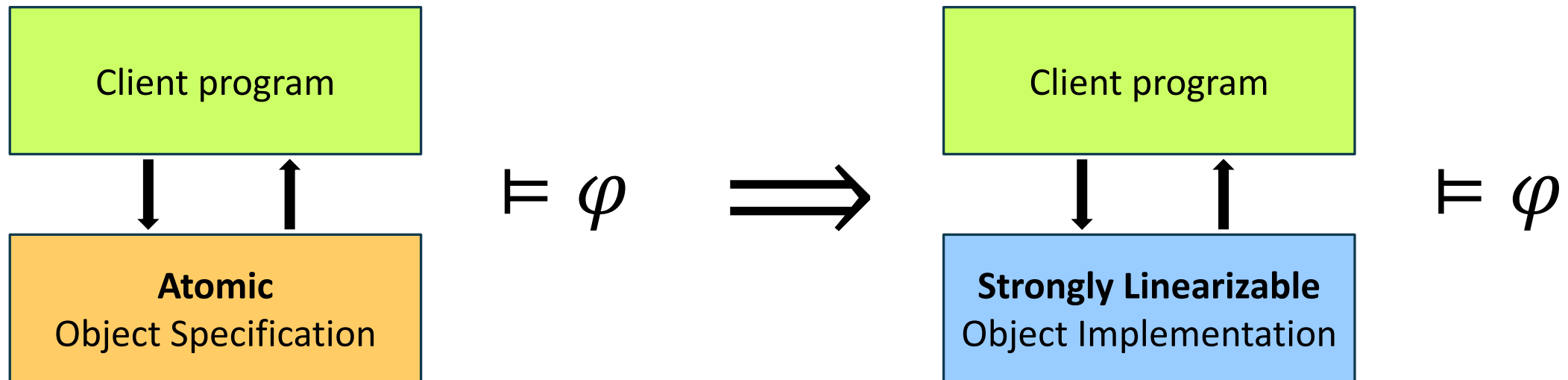
$\varphi = \text{Pr}[a = b] = \frac{1}{2}$ for
any **strong adversary**

↓
sees coin toss results
controls scheduling & non-determinism



Hyperproperty Preservation via Strong Linearizability

- If φ is a property of sets of traces generated by strong adversaries:



[Golab, Higham, Woelfel '11]

- Indeed, the *double-load register implementation* is not strongly linearizable

Strong Linearizability is Rarely Achievable

- Various impossibility results for strongly linearizable implementations
- Example: Crash-resilient lock-free message passing register implementation
 - No strongly linearizable implementation exists
 - In particular, ABD [Attiya, Bar-Noy, Dolev '95] is not strongly linearizable

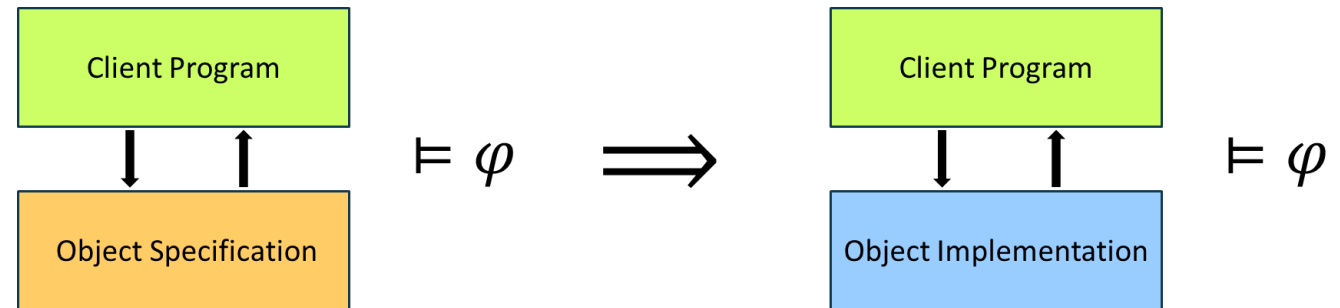
Strong Linearizability is Rarely Achievable

- Various impossibility results for strongly linearizable implementations
- Example: Crash-resilient lock-free message passing register implementation
 - No strongly linearizable implementation exists
 - In particular, ABD [Attiya, Bar-Noy, Dolev '95] is not strongly linearizable

■ Problem: how to reason about hyperproperties of clients that use non-strongly linearizable implementations, such as ABD?

Our Contributions

- Simple shared memory register specifications
 - In the form of (non-atomic) reference-implementations
 - Enable reasoning about hyperproperties of clients that use non-strongly linearizable implementations



Our Contributions

- Simple shared memory register specifications

- In the form of (non-atomic) reference-implementations

- Enable reasoning about hyperproperties of clients that use non-strongly linearizable implementations

- “Complete” for a range of linearizability classes, including:

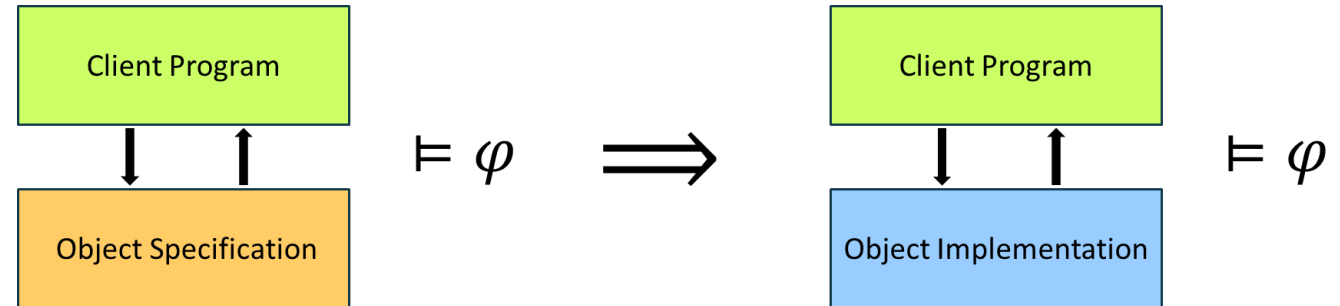
- Write strong-linearizability

- [Hadzilacos, Hu, Toueg '21]

- **Decisive** linearizability



Novel linearizability class



Linearizability

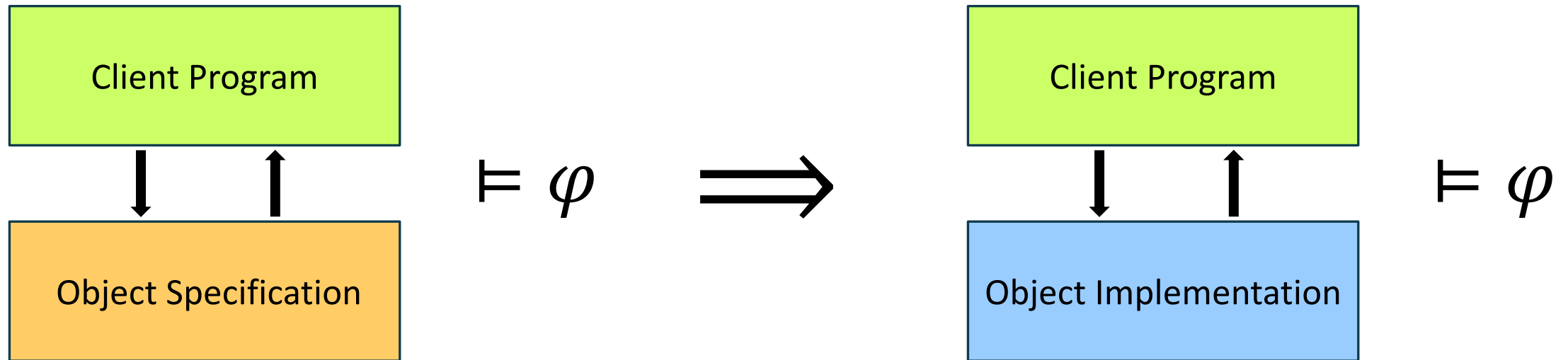
Decisive Linearizability

Write Strong-Linearizability

Strong Linearizability

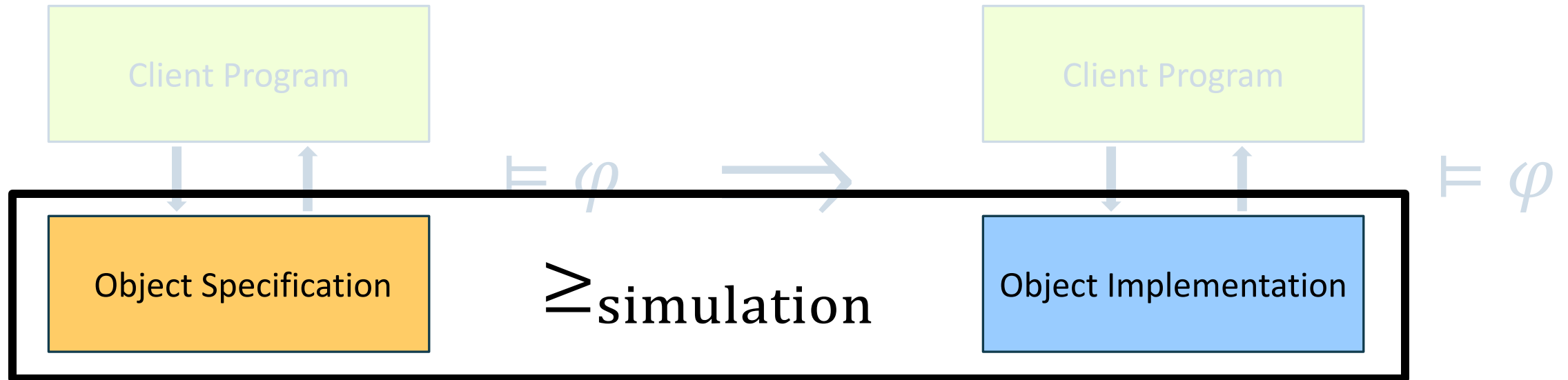
Hyperproperty Preservation via Simulation

- Preservation of hyperproperties \equiv forward simulation [Attiya, Enea '19]



Hyperproperty Preservation via Simulation

- Preservation of hyperproperties \equiv forward simulation [Attiya, Enea '19]



Complete Implementations

- \mathcal{C} - class of implementations
- An implementation I is
 - **\mathcal{C} -hard** if $I' \leq_{\text{simulation}} I$ for all $I' \in \mathcal{C}$
 - **\mathcal{C} -complete** if additionally, $I \in \mathcal{C}$
- Example: Atomic implementation is complete for the class of strongly linearizable implementations

Complete Implementations

- \mathcal{C} - class of implementations
- An implementation I is
 - **\mathcal{C} -hard** if $I' \leq_{\text{simulation}} I$ for all $I' \in \mathcal{C}$
 - **\mathcal{C} -complete** if additionally, $I \in \mathcal{C}$
- Example: Atomic implementation is complete for the class of strongly linearizable implementations
- Problem reformulation: devise simple complete implementations for (non-strong) linearizability classes

Complete Implementations

- \mathcal{C} - class of implementations
- An implementation I is
 - **\mathcal{C} -hard** if $I' \leq_{\text{simulation}} I$ for all $I' \in \mathcal{C}$
 - **\mathcal{C} -complete** if additionally, $I \in \mathcal{C}$
- Example: Atomic implementation is complete for the class of strongly linearizable implementations
- Problem reformulation: devise **simple** complete implementations for (non-strong) linearizability classes

Complete Implementations

- \mathcal{C} - class of implementations
- An implementation I is
 - **\mathcal{C} -hard** if $I' \leq_{\text{simulation}} I$ for all $I' \in \mathcal{C}$
 - **\mathcal{C} -complete** if additionally, $I \in \mathcal{C}$
- Example: Atomic implementation is complete for the class of strongly linearizable implementations
- Problem reformulation: devise **simple** complete implementations for (non-strong) linearizability classes
- Focus on registers

\mathcal{C} = Write Strong Linearizability [Hadzilacos, Hu, Toueg '21]

- Includes all single-writer register implementations
 - Specifically, single-writer ABD
- The “Write Strong Register” is complete:

| Method <code>write(<i>v</i>)</code> | Method <code>read()</code> |
|---|--|
| <code><i>X</i> ← <i>v</i>;</code> <code>return;</code> | <code>$\mathcal{V} \leftarrow \{X\};$</code> <code>do</code> <code> $\langle \mathcal{V}_{\text{prev}}, \mathcal{V} \rangle \leftarrow \langle \mathcal{V}, \mathcal{V} \cup \{X\} \rangle;$</code> <code>while $\mathcal{V} \neq \mathcal{V}_{\text{prev}};$</code> <code><i>out</i> ← pick $v \in \mathcal{V};$</code> <code>return <i>out</i>;</code> |

\mathcal{C} = Write Strong Linearizability [Hadzilacos, Hu, Toueg '21]

- Includes all single-writer register implementations
 - Specifically, single-writer ABD
- The “Write Strong Register” is complete:

| Method <code>write(v)</code> | Method <code>read()</code> |
|---|--|
| <code>X ← v;</code> <code>return;</code> | <code>V ← {X};</code> do <code>⟨V_{prev}, V⟩ ← ⟨V, V ∪ {X}⟩;</code> while <code>V ≠ V_{prev};</code> <code>out ← pick v ∈ V;</code> return <code>out;</code> |

- Captures hyperproperties of single-writer ABD
- What about multi-writer ABD?

Example: Multiple Writers

```
write(1); | write(2);  
a ← coin(); |  
barrier(); | barrier();  
          | b ← read();
```

- When using multi-writer ABD, can force $a = b$
(Proof in the paper)

Example: Multiple Writers

```

write(1);   | write(2);
a ← coin(); |
barrier(); | barrier();
            | b ← read();
    
```

- When using multi-writer ABD, can force $a = b$ (Proof in the paper)
- Shared memory implementation capturing this behavior: **Try-not-to-store** register

Try-not-to-store:

Method `write(v)`

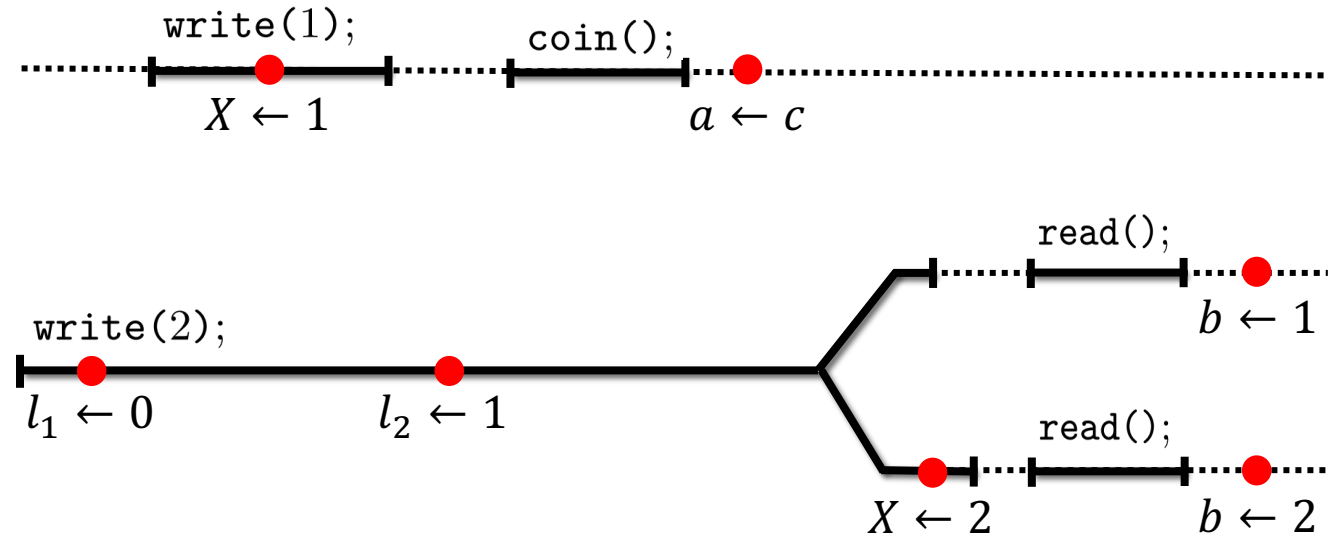
```

l1 ← X;
l2 ← X;
if * then
    if l1 ≠ l2 then
        return;
X ← v;
return;
    
```

Method `read()`

```

out ← X;
return out;
    
```



Reference Implementation for Multi-Writer ABD

- Combines ideas from the Write-strong and Try-not-to-store registers
 - Comparing version numbers instead of register values
 - Communicating “overwritten” writes to concurrent reads
 - Details in the paper
- Captures hyperproperties of multi-writer ABD

Reference Implementation for Multi-Writer ABD

- Combines ideas from the Write-strong and Try-not-to-store registers
 - Comparing version numbers instead of register values
 - Communicating “overwritten” writes to concurrent reads
 - Details in the paper
- Captures hyperproperties of multi-writer ABD
- Next: decisive linearizability, a new class of linearizable implementations for which this implementation is complete

Decisive Linearizability

- $e \sqsubseteq s$: execution e linearized by sequential history s



Decisive Linearizability

- $e \sqsubseteq s$: execution e linearized by sequential history s
- An implementation I is:
 - **Linearizable** if there exists a mapping $L: \text{executions}(I) \rightarrow \text{Seq}$ s.t. $\forall e. e \sqsubseteq L(e)$
 - **Strongly Linearizable** if $e_1 \leq_{\text{prefix}} e_2 \implies L(e_1) \leq_{\text{prefix}} L(e_2)$

Decisive Linearizability

- $e \sqsubseteq s$: execution e linearized by sequential history s
- An implementation I is:
 - **Linearizable** if there exists a mapping $L: \text{executions}(I) \rightarrow \text{Seq}$ s.t. $\forall e. e \sqsubseteq L(e)$
 - **Decisively Linearizable** if $e_1 \leq_{\text{prefix}} e_2 \implies L(e_1) \leq_{\text{subsequence}} L(e_2)$
 - **Strongly Linearizable** if $e_1 \leq_{\text{prefix}} e_2 \implies L(e_1) \leq_{\text{prefix}} L(e_2)$

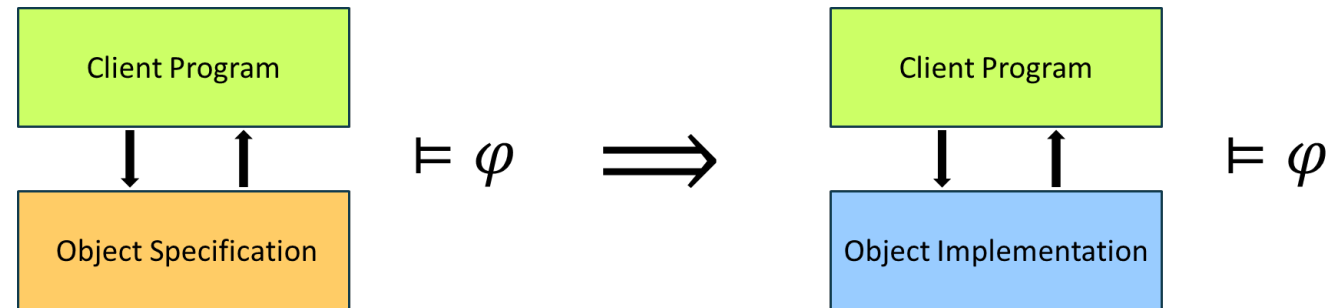


| | Lin. | Decisive | Strong |
|--|------|----------|--------|
| | ✓ | ✓ | ✓ |
| | ✓ | ✓ | ✗ |
| | ✓ | ✗ | ✗ |

Conclusion

- Simple shared memory register specifications:

- Write strong register
- Decisive register
- General construction (in the paper)



- Enable reasoning about hyperproperties of clients that use implementations satisfying certain linearizability criteria

- New linearizability class: decisive linearizability

- Applicable beyond registers

Linearizability

Decisive Linearizability

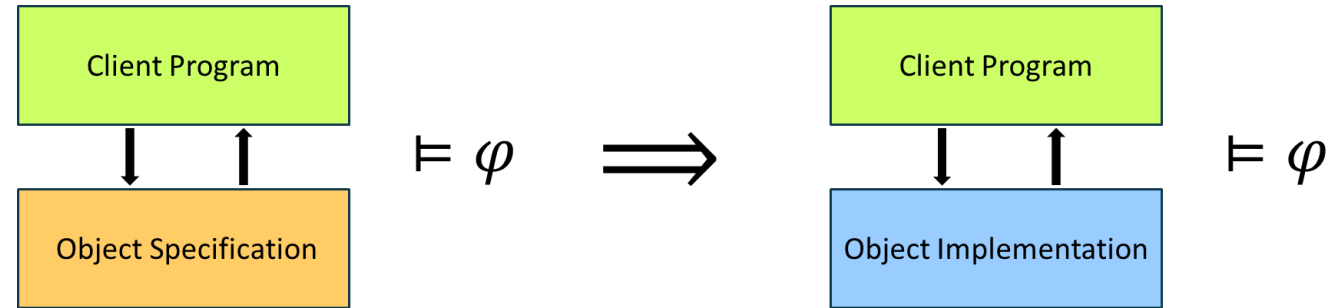
Write Strong-Linearizability

Strong Linearizability

Conclusion

- Simple shared memory register specifications:

- Write strong register
- Decisive register
- General construction (in the paper)



- Enable reasoning about hyperproperties of clients that use implementations satisfying certain linearizability criteria

- New linearizability class: decisive linearizability

- Applicable beyond registers

Linearizability

Decisive Linearizability

Write Strong-Linearizability

Strong Linearizability

Thank You!