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Tutorial

# Weak memory models in programming language semantics

Ori Lahav



# About me

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- PhD from TAU in **logic in CS**
- Postdocs: **formal verification** (TAU) and **weak memory concurrency** (MPI-SWS)
- Now professor at TAU,  
main areas of research: programming languages theory, concurrency, verification
- ERC Starting Grant (hiring students/postdocs)

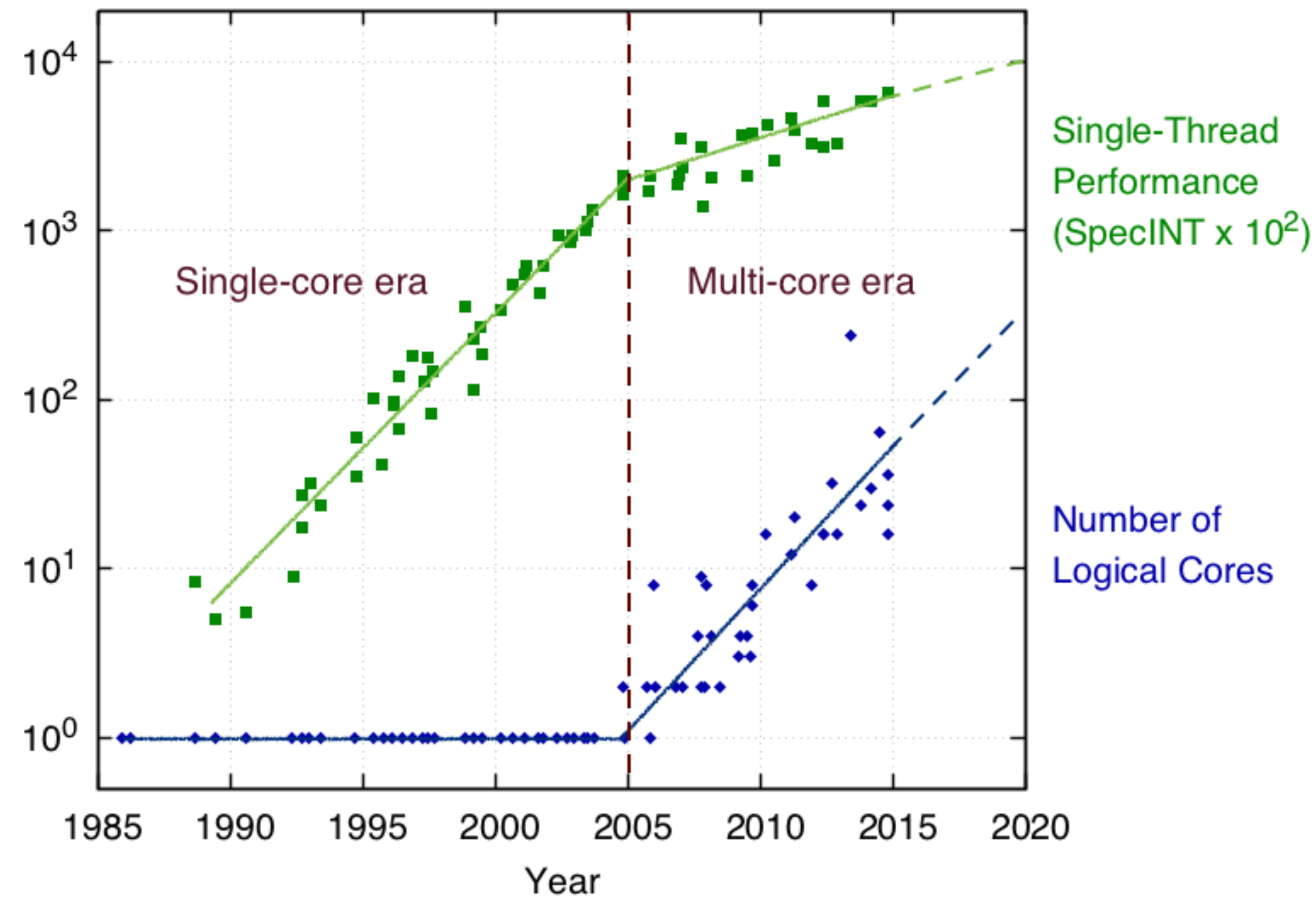


# Agenda

1. Introduction
2. The C/C++11 memory model
3. The out-of-thin-air problem & RC11
4. Implementability of (R)C11: compiler optimizations and mapping to hardware
5. Programmability guarantees: DRF theorems, library abstraction
6. Verification (short survey of problems and results)

# Parallelism is here

“The Free Lunch Is Over: A Fundamental Turn Toward Concurrency in Software”/ Herb Sutter (2005)



# Concurrent programming is hard!

*“If you can get away with it, avoid using threads.  
Threads can be difficult to use, and they make programs  
harder to debug.”*

(Java documentation, ~25 years ago)



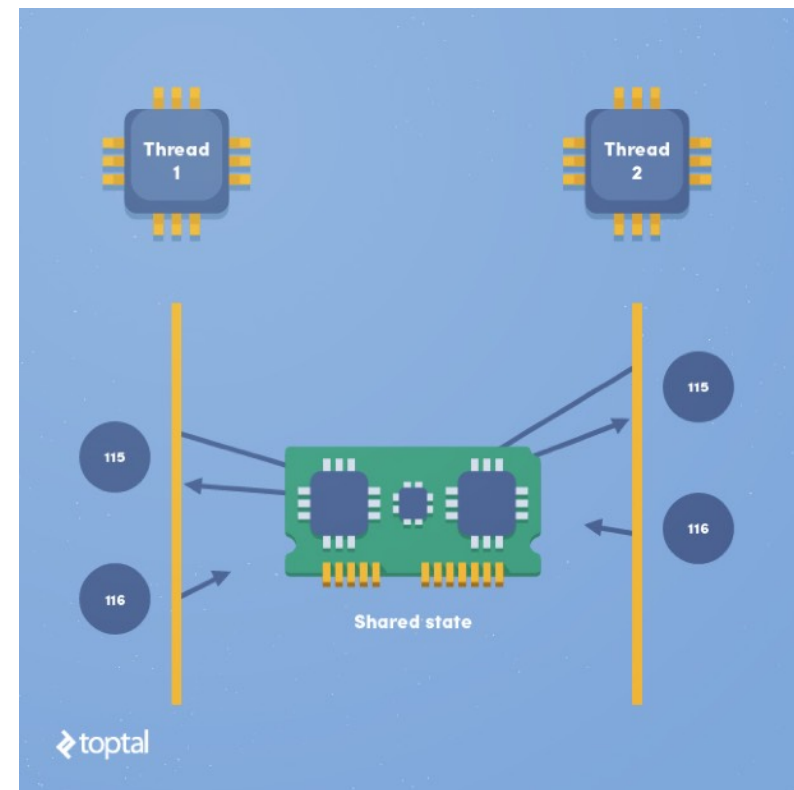
# Why?

- Requires a fundamentally different way of thinking
- Interference among threads
- Inevitable non-determinism
- Testing is ineffective
- Reproducing bugs and debugging is hard



# Concurrent programming

shared memory



interaction by reading and writing shared objects in memory

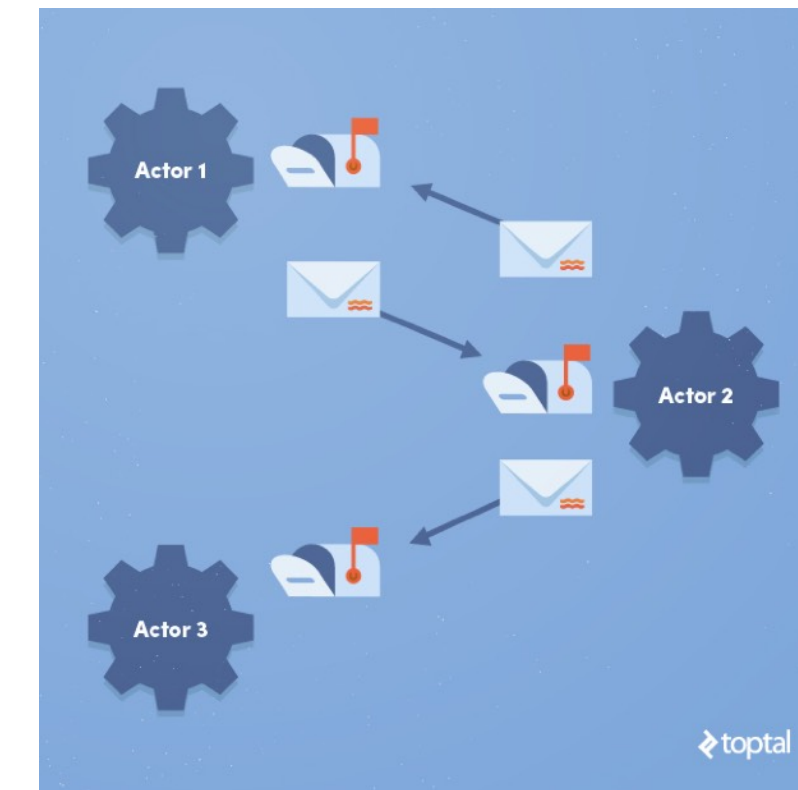
`store/write`

`load/read`

`read-modify-write (e.g. CAS, FADD)`

`lock & unlock`

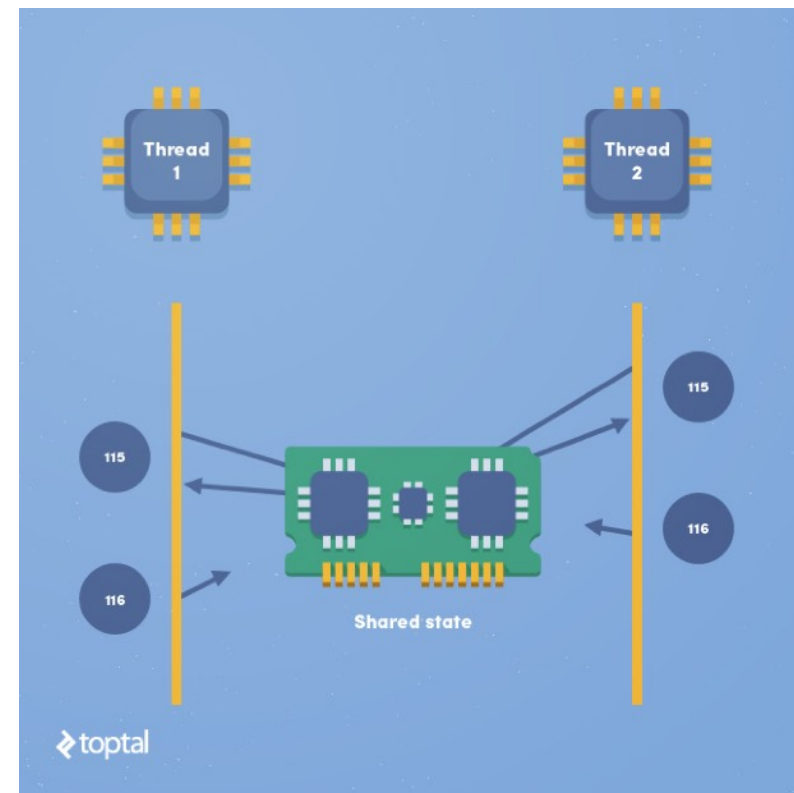
message passing



interaction by sending messages to each other through a communication channel

# Concurrent programming

shared memory



interaction by reading and writing shared objects in memory

`store/write`

`load/read`

`read-modify-write (e.g. CAS, FADD)`

`lock & unlock`





# Dekker's mutual exclusion



`wants_to_enter[0] ← false, wants_to_enter[1] ← false, turn ← 0 // or 1`

```
wants_to_enter[0] ← true
while (wants_to_enter[1]) {
  if (turn ≠ 0) {
    wants_to_enter[0] ← false
    while (turn ≠ 0) { // busy wait }
    wants_to_enter[0] ← true
  }
  // critical section
  ...
  turn ← 1
  wants_to_enter[0] ← false
  // remainder section
```

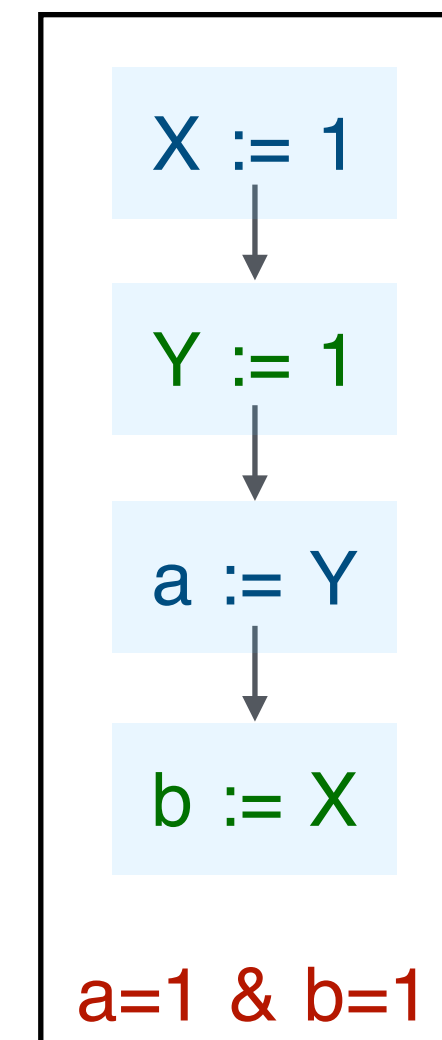
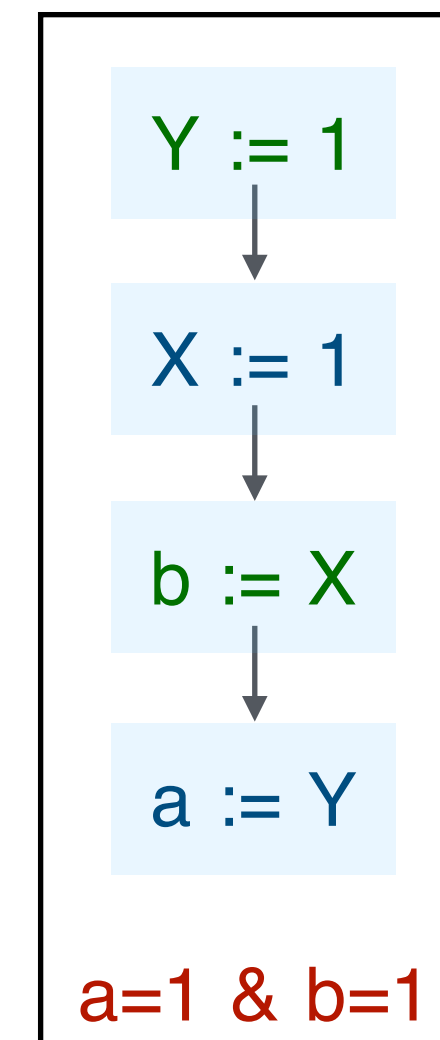
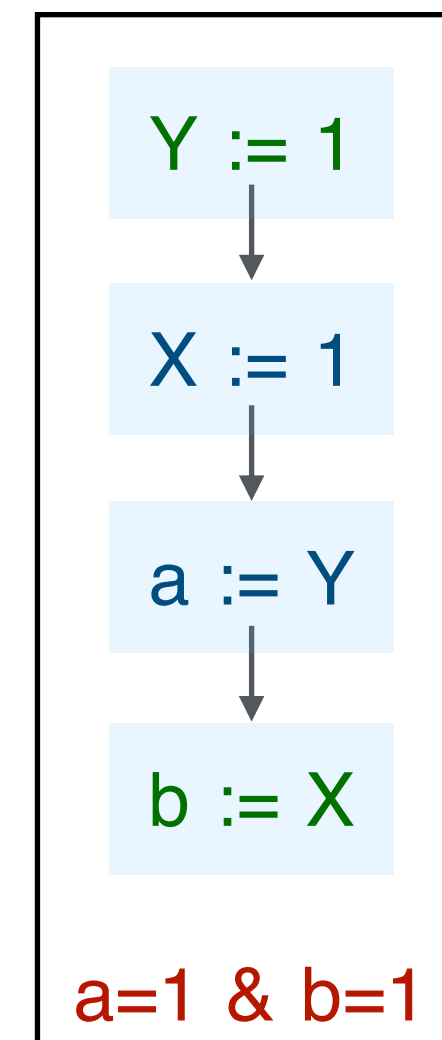
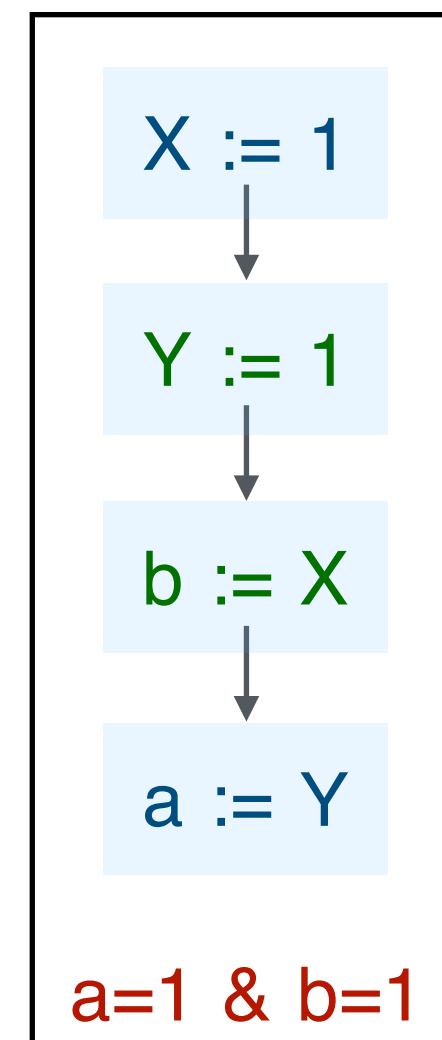
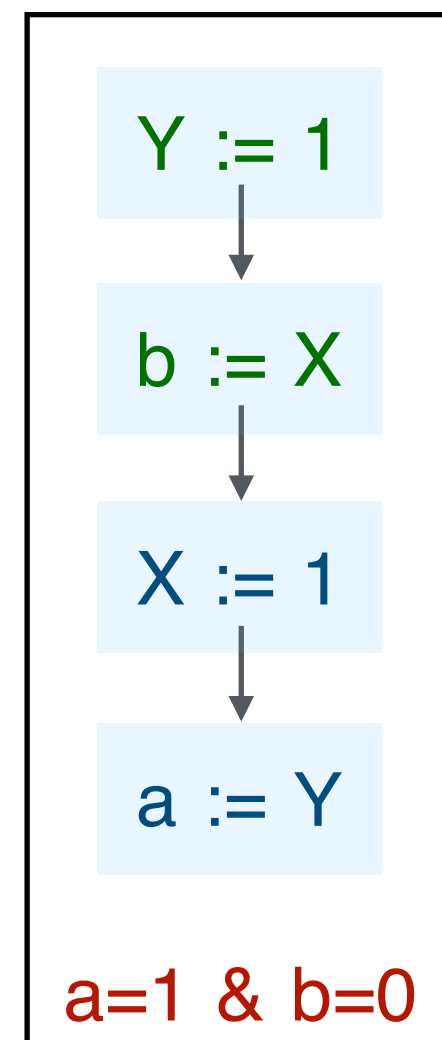
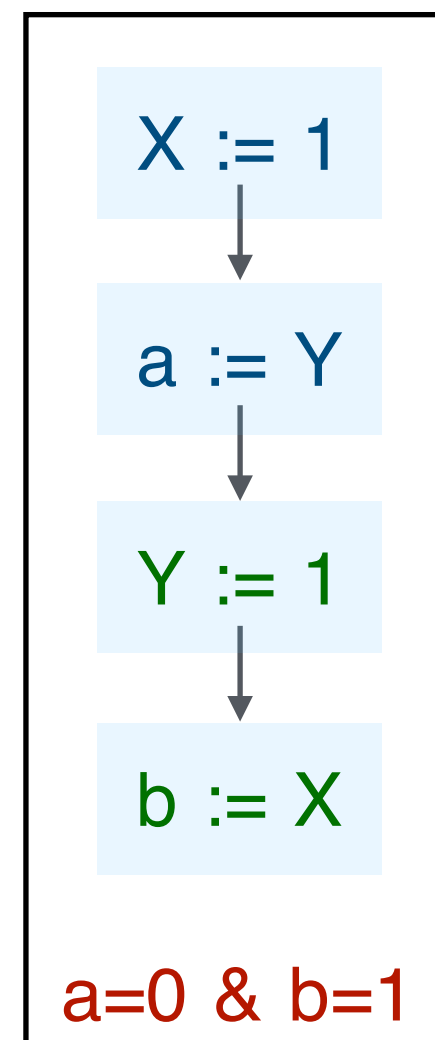
```
wants_to_enter[1] ← true
while (wants_to_enter[0]) {
  if (turn ≠ 1) {
    wants_to_enter[1] ← false
    while (turn ≠ 1) { // busy wait }
    wants_to_enter[1] ← true
  }
  // critical section
  ...
  turn ← 0
  wants_to_enter[1] ← false
  // remainder section
```

# Example

initially,  $X=Y=0$

```
X := 1
a := Y // 0
```

```
Y := 1
b := X // 0
```



# Demo

```
int X, Y, a, b;

void thread1() {
    X = 1;
    a = Y;
}

void thread2() {
    Y = 1;
    b = X;
}
```

```
int main () {
    int cnt = 0;

    do {
        X = 0; Y = 0;

        thread first(thread1);
        thread second(thread2);

        first.join();
        second.join();
        cnt++;

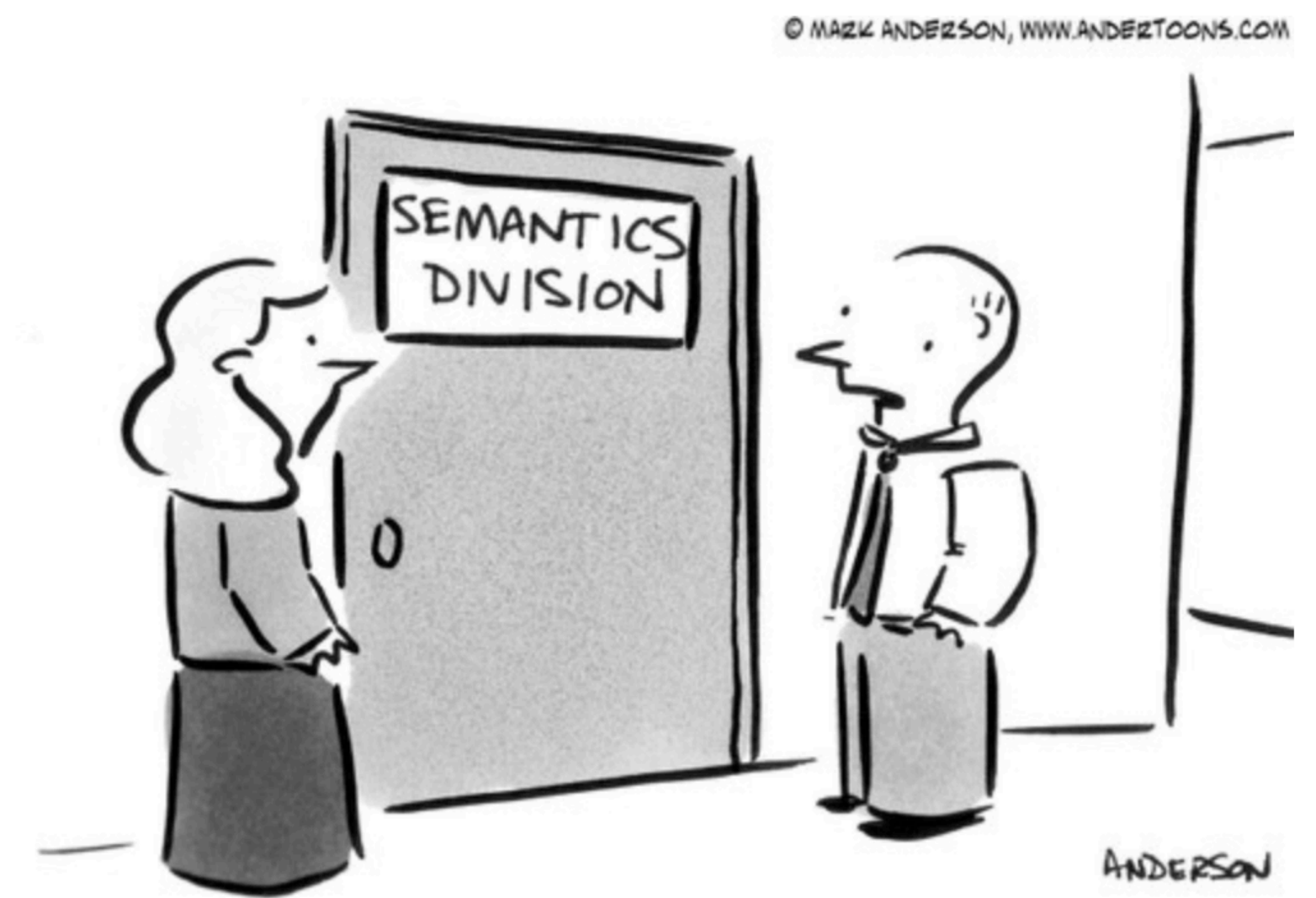
    } while (a != 0 || b != 0);

    printf("%d\n", cnt);
    return 0;
}
```

# How come airplanes don't crash?

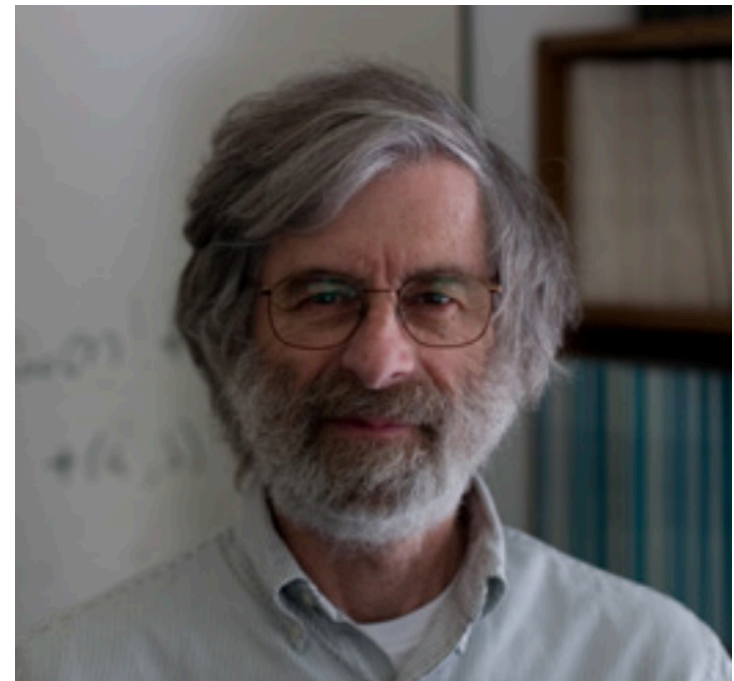
- There are ways to demand strong semantics when we need it
- We often don't need strong semantics in its full power

*Before programming/verification,  
we need semantics*

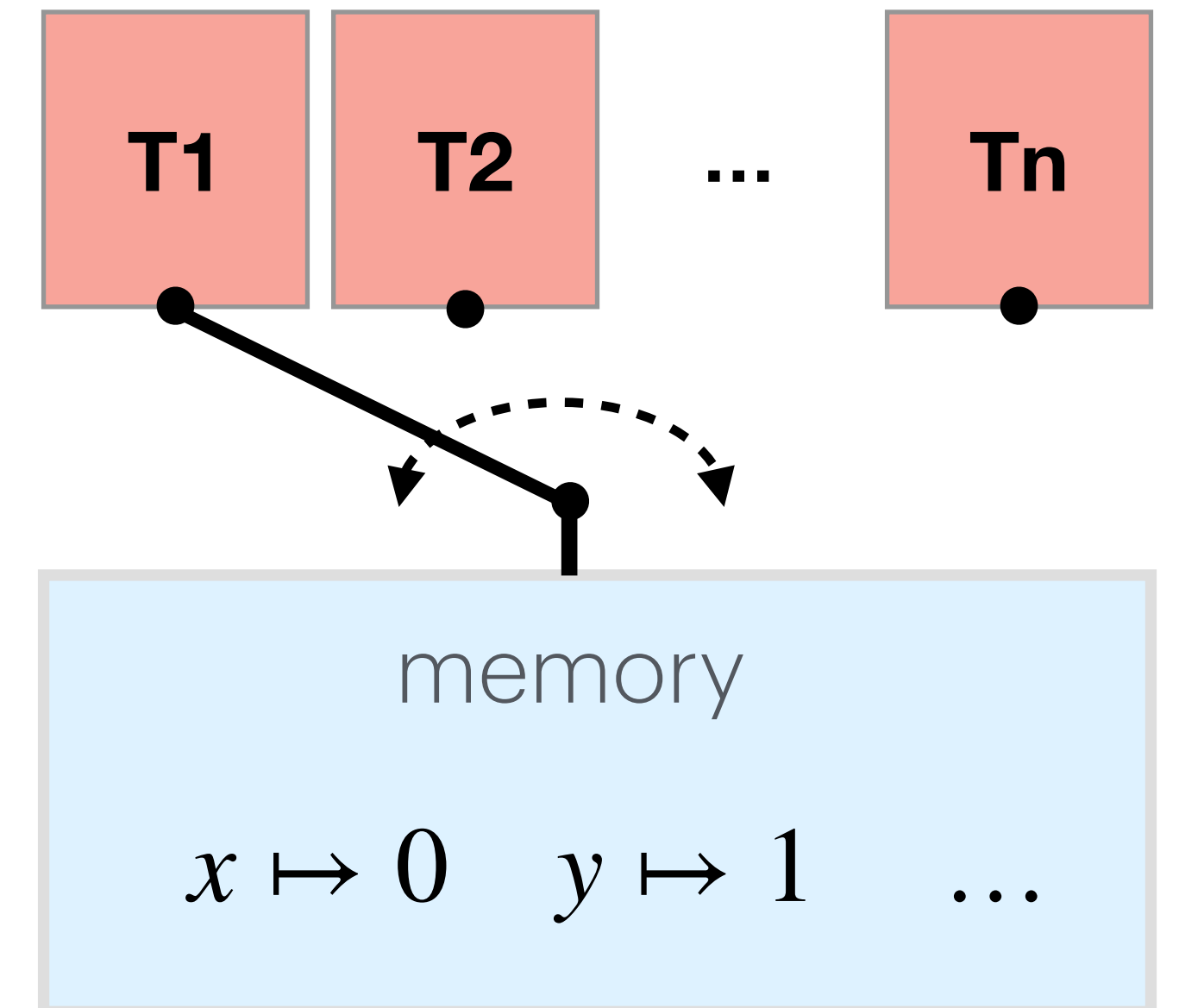


"We're really more of a department."

# Sequential consistency (SC)



*...the result of any execution is the same as if the operations of all the processors were executed in some sequential order, and the operations of each individual processor appear in this sequence in the order specified by its program...*



Lamport. 1979. **How to Make a Multiprocessor Computer That Correctly Executes Multiprocess Programs.** IEEE Trans. Comput. <https://doi.org/10.1109/TC.1979.1675439>

*The requirements needed to guarantee sequential consistency rule out some techniques which can be used to speed up individual sequential processors. For some applications, achieving sequential consistency may not be worth the price of slowing down the processors. In this case, one must be aware that conventional methods for designing multiprocess algorithms cannot be relied upon to produce correctly executing programs. Protocols for synchronizing the processors must be designed at the lowest level of the machine instruction code, and verifying their correctness becomes a monumental task.*

Lamport. 1979. **How to Make a Multiprocessor Computer That Correctly Executes Multiprocess Programs.** IEEE Trans. Comput. <https://doi.org/10.1109/TC.1979.1675439>

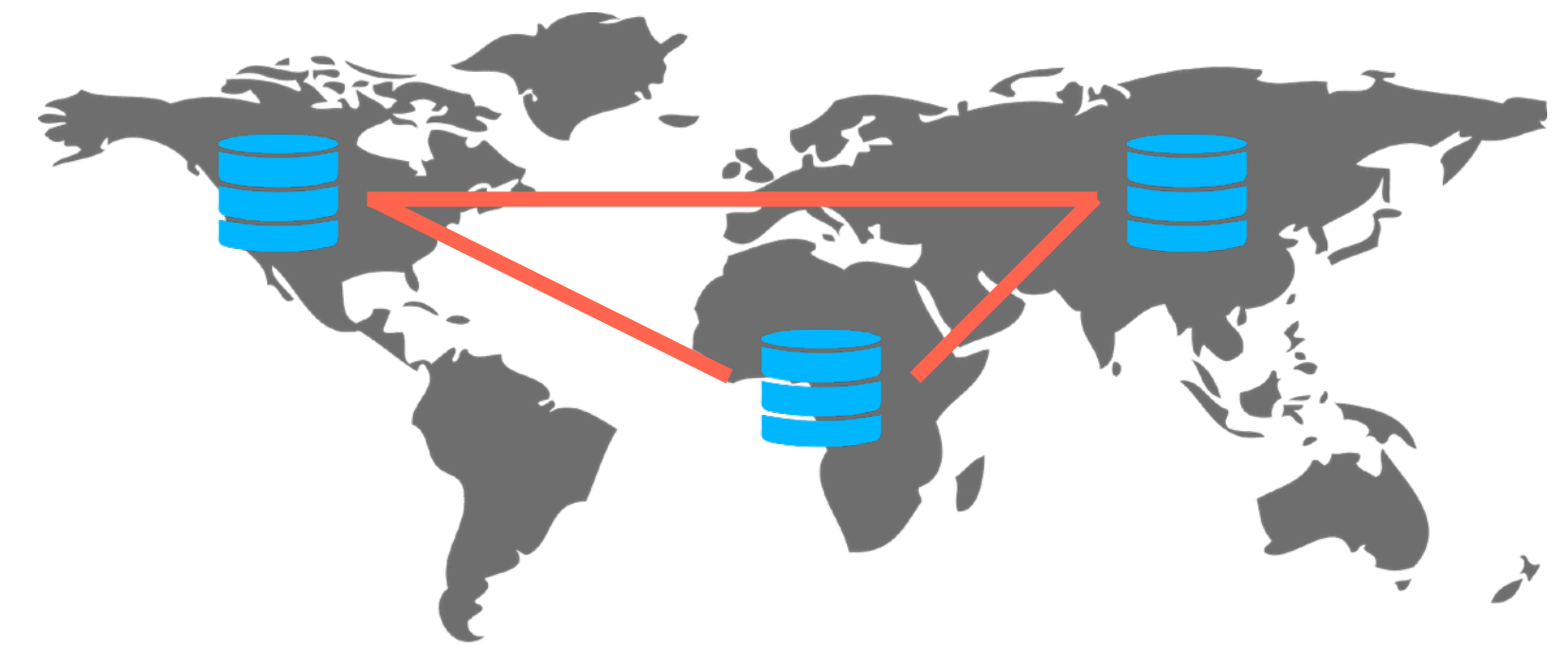
# SC is unrealistic

- for better **performance/scalability** shared-memory implementations perform various optimizations:
  - local store buffers
  - out-of-order execution
  - hierarchies of caches
  - ...
- **Compilers** further stir the pot by performing thread-local program optimizations
- These optimizations are:
  - unobservable in sequential programs
  - but can be observed by concurrent code!

# Weak consistency in distributed systems

```
send(X = 1)  
get(Y) // 0
```

```
send(Y = 1)  
get(X) // 0
```



```
Email := "dear bob, ..."  
Sms := "check your email"
```

```
a := Sms // "check your email"  
b := Email // "no new email"
```



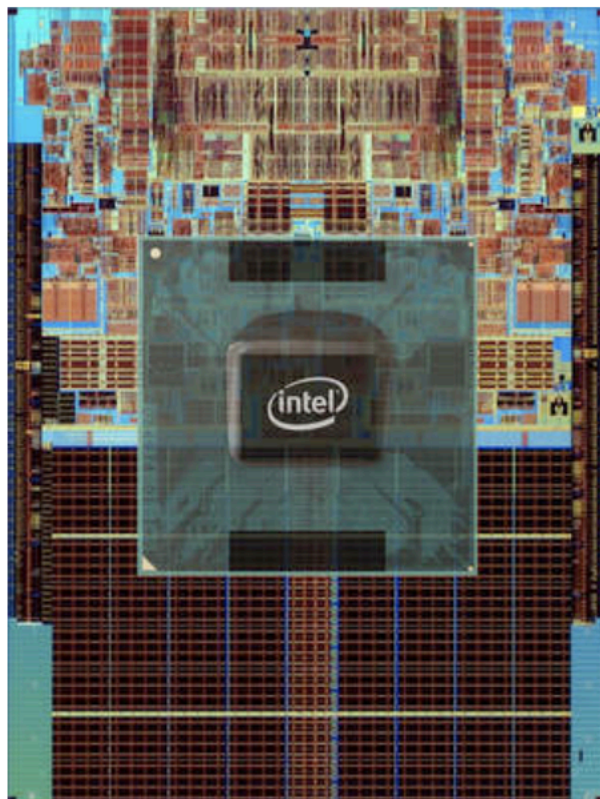
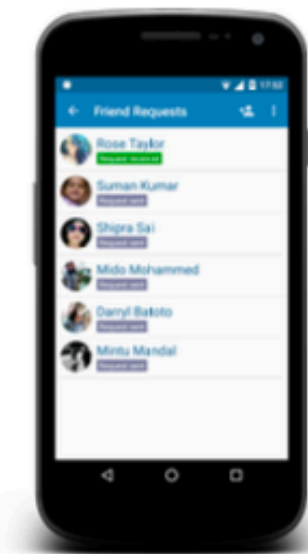


# Weak memory models

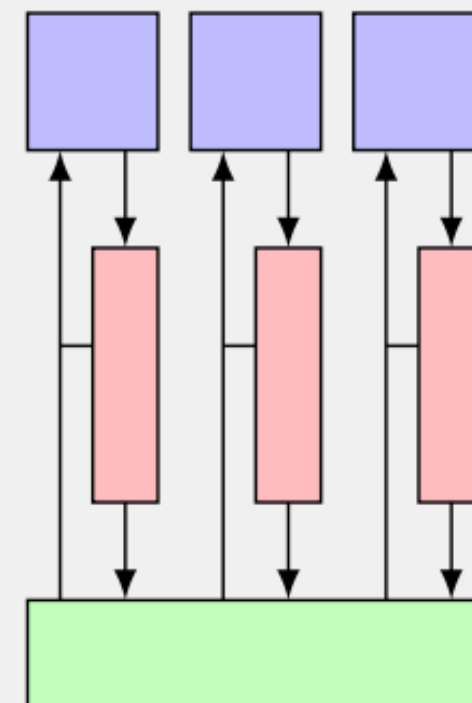
- A formal *interface* between the user and the implementation:
  - What are the *possible behaviors* of a concurrent program?
  - More concretely, *what values each read may return?*
- A *weak memory model* (WMM) allows all outcomes allowed by SC *and more*


# Hardware memory models

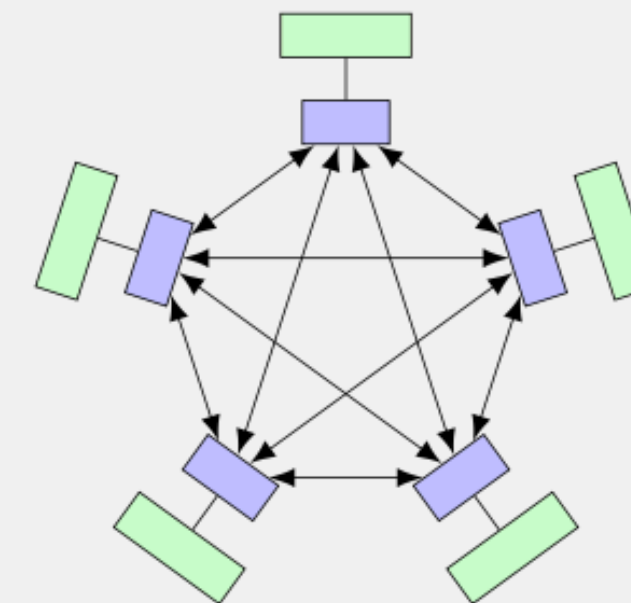
- Each architecture has its own WMM: x86-TSO, ARM, Power, RISC-V...
- Often: subtle differences
- None of them is SC




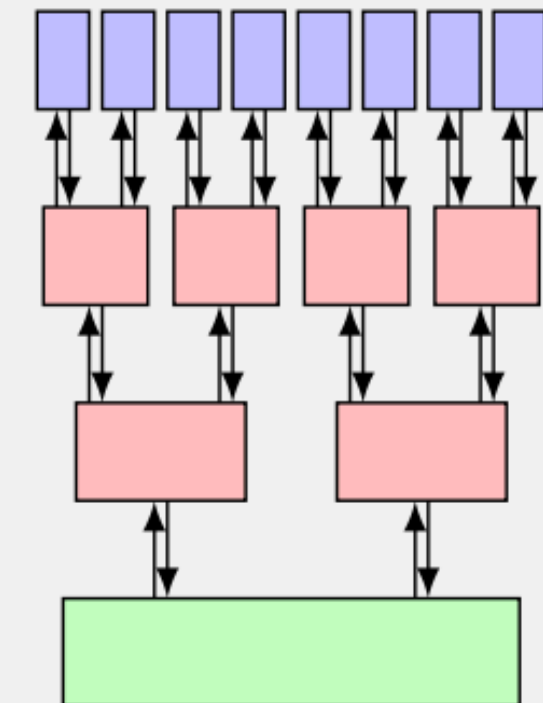
**x86-TSO**    
(2010)



**POWER**   
(2011)



**ARMv8**   
(2016)

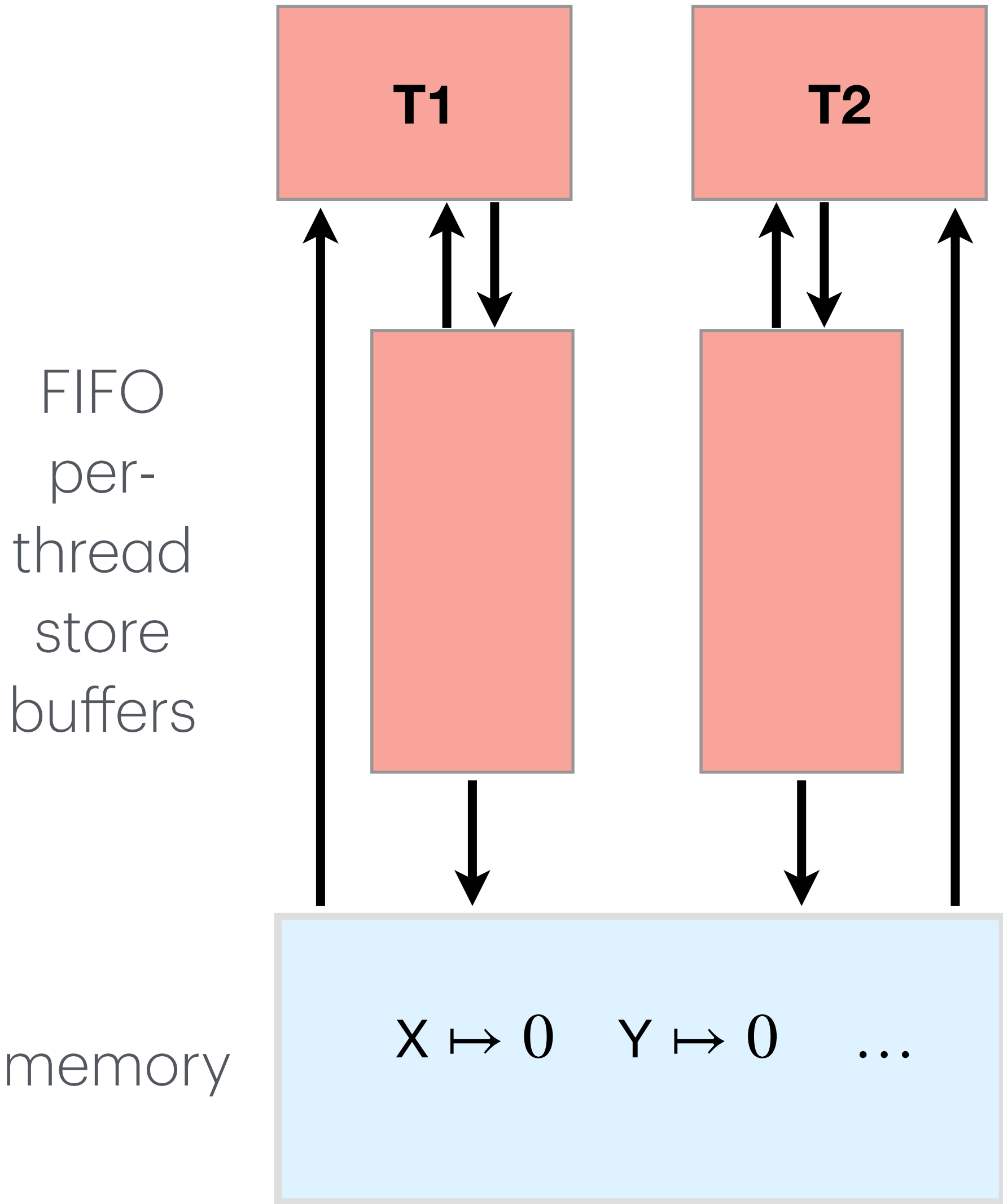




# x86-TSO

<code>X := 1</code> <code>a := Y // 0</code>	<code>Y := 1</code> <code>b := X // 0</code>
---	---

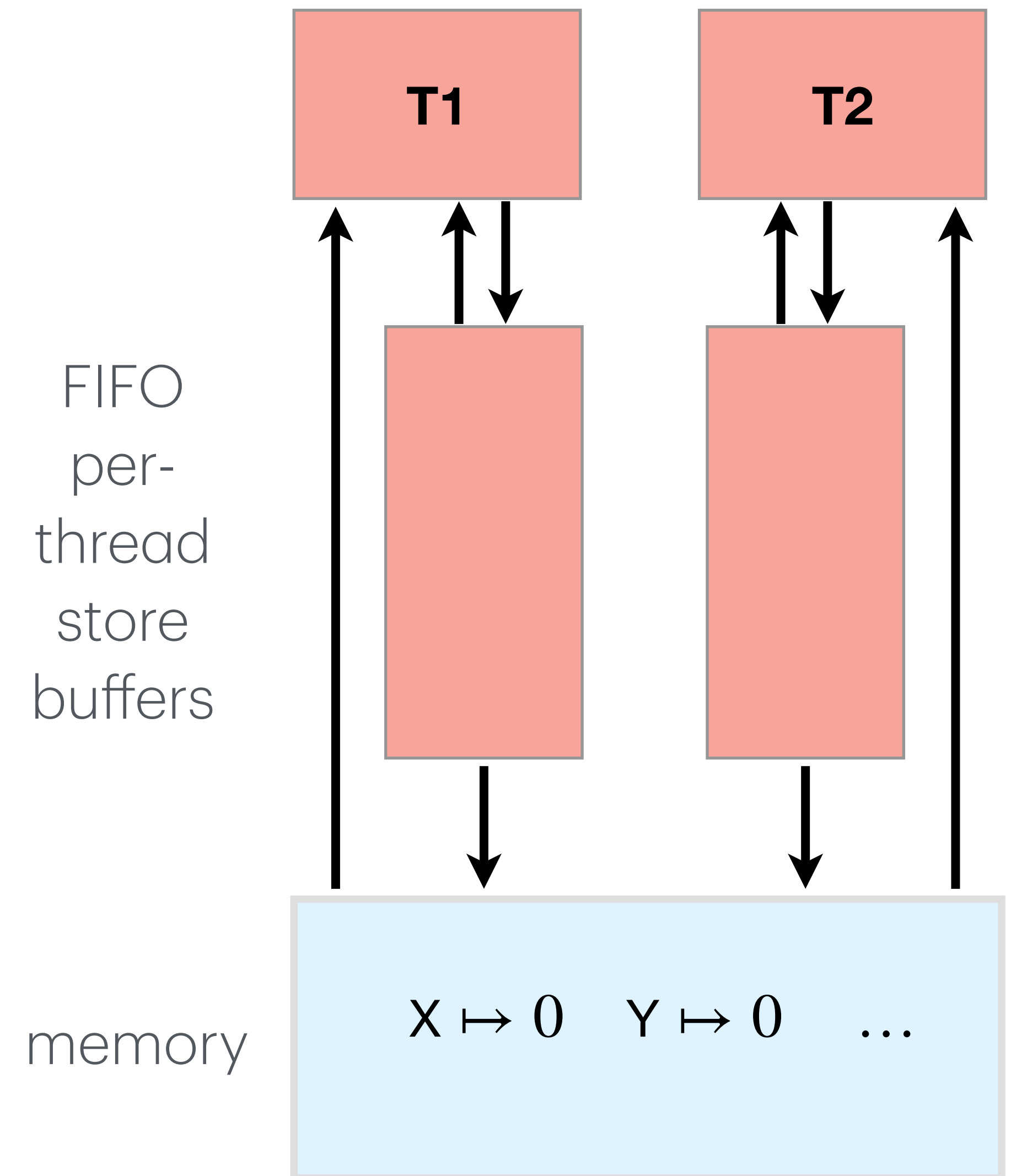
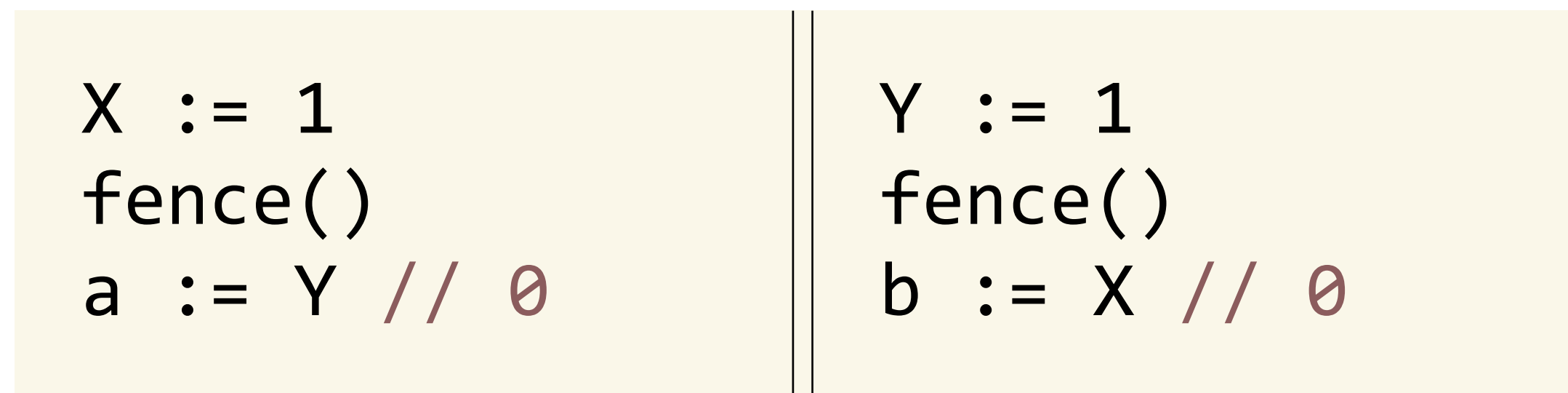
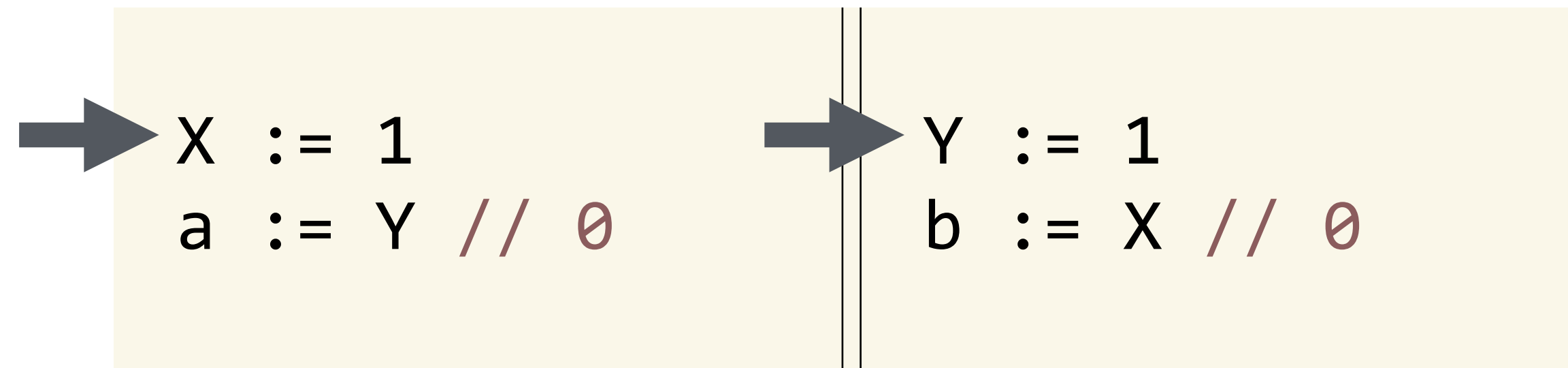
<code>X := 1</code> <code>fence()</code> <code>a := Y // 0</code>	<code>Y := 1</code> <code>fence()</code> <code>b := X // 0</code>
---	---



Sewell, Sarkar, Owens, Zappa Nardelli, Myreen: **x86-TSO: a rigorous and usable programmer's model for x86 multiprocessors.** Commun. ACM 53(7) 2010. <https://doi.org/10.1145/1785414.1785443>



# x86-TSO

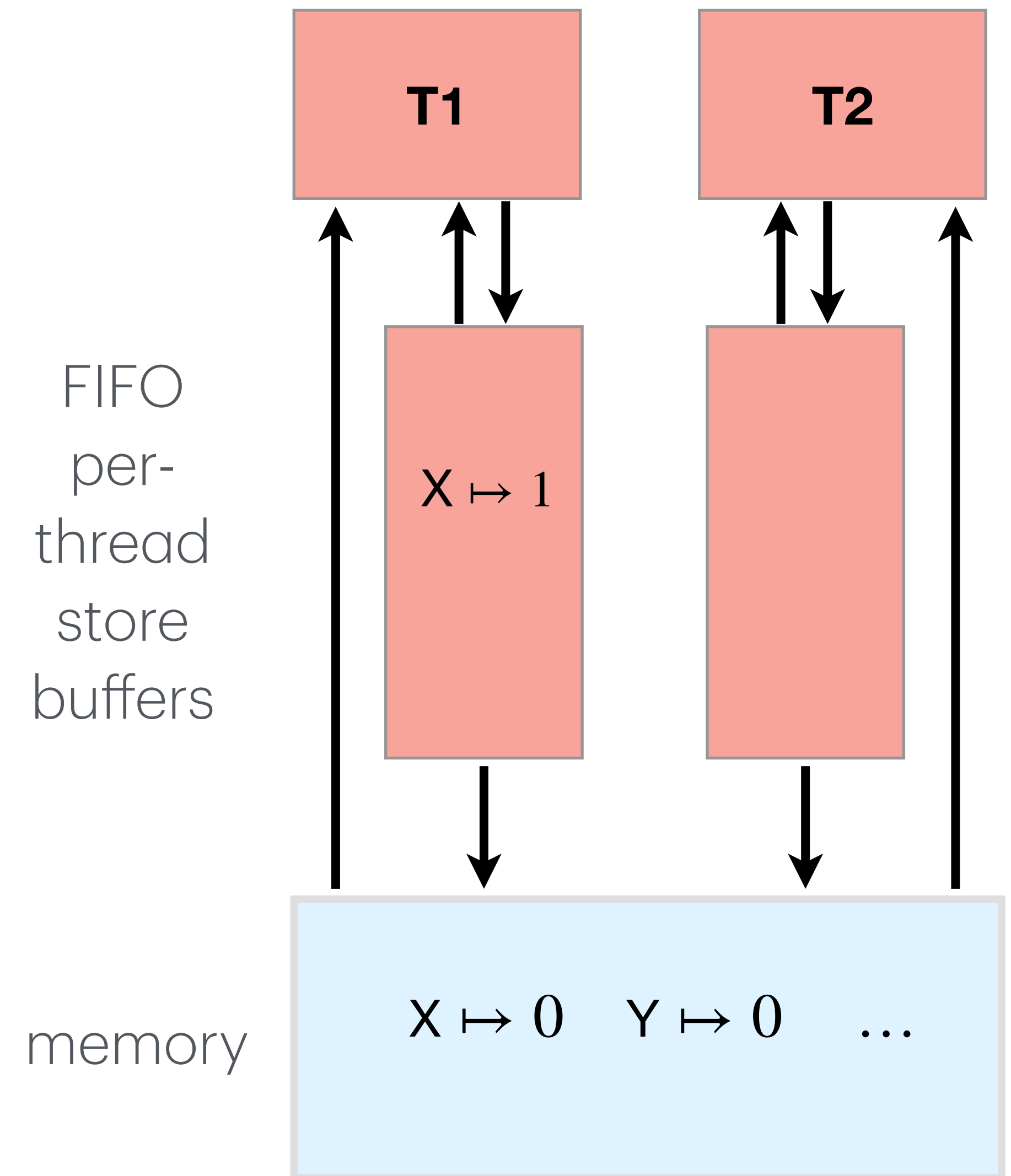
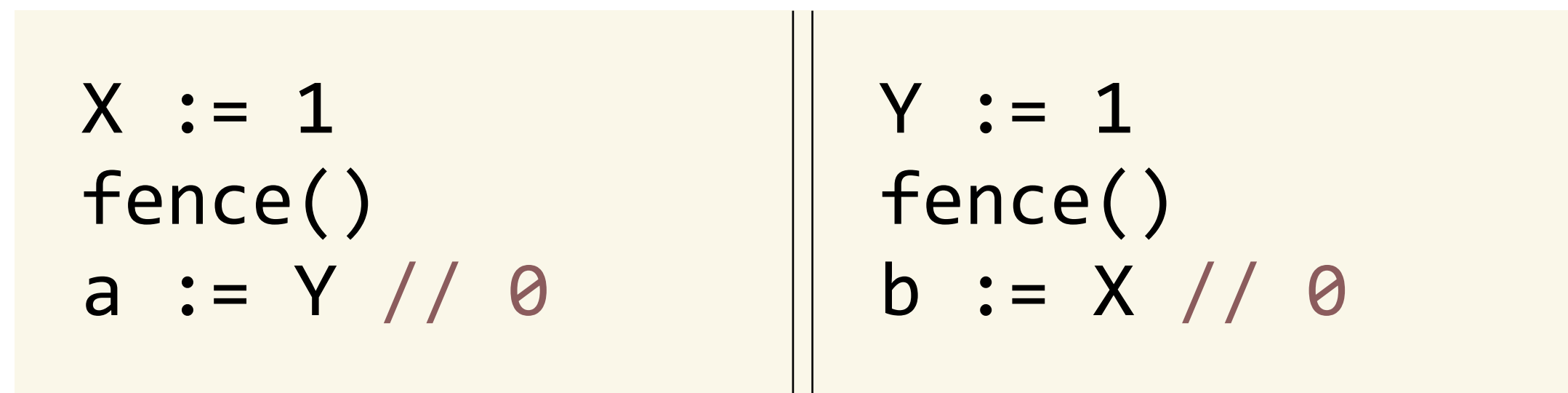
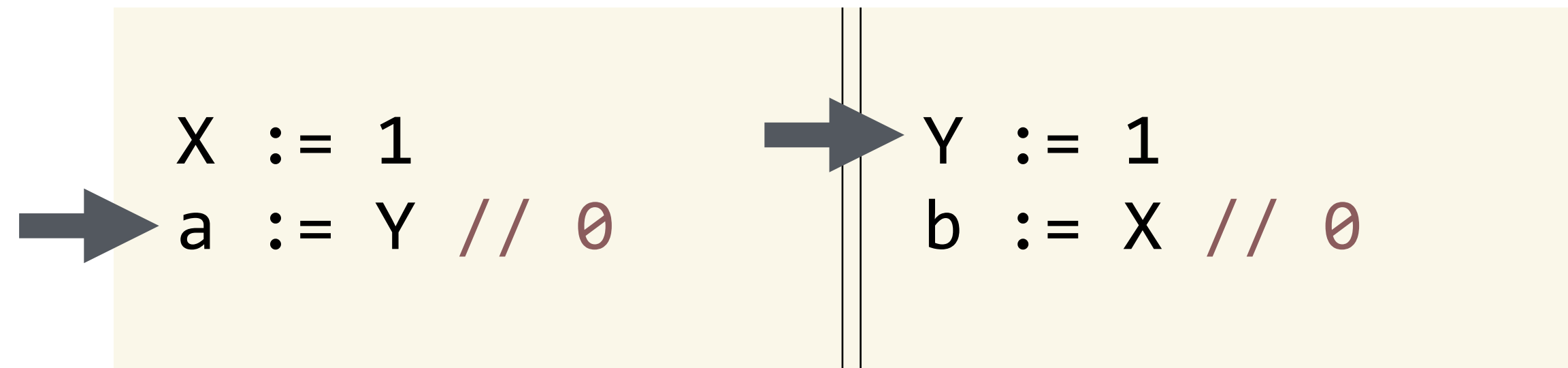


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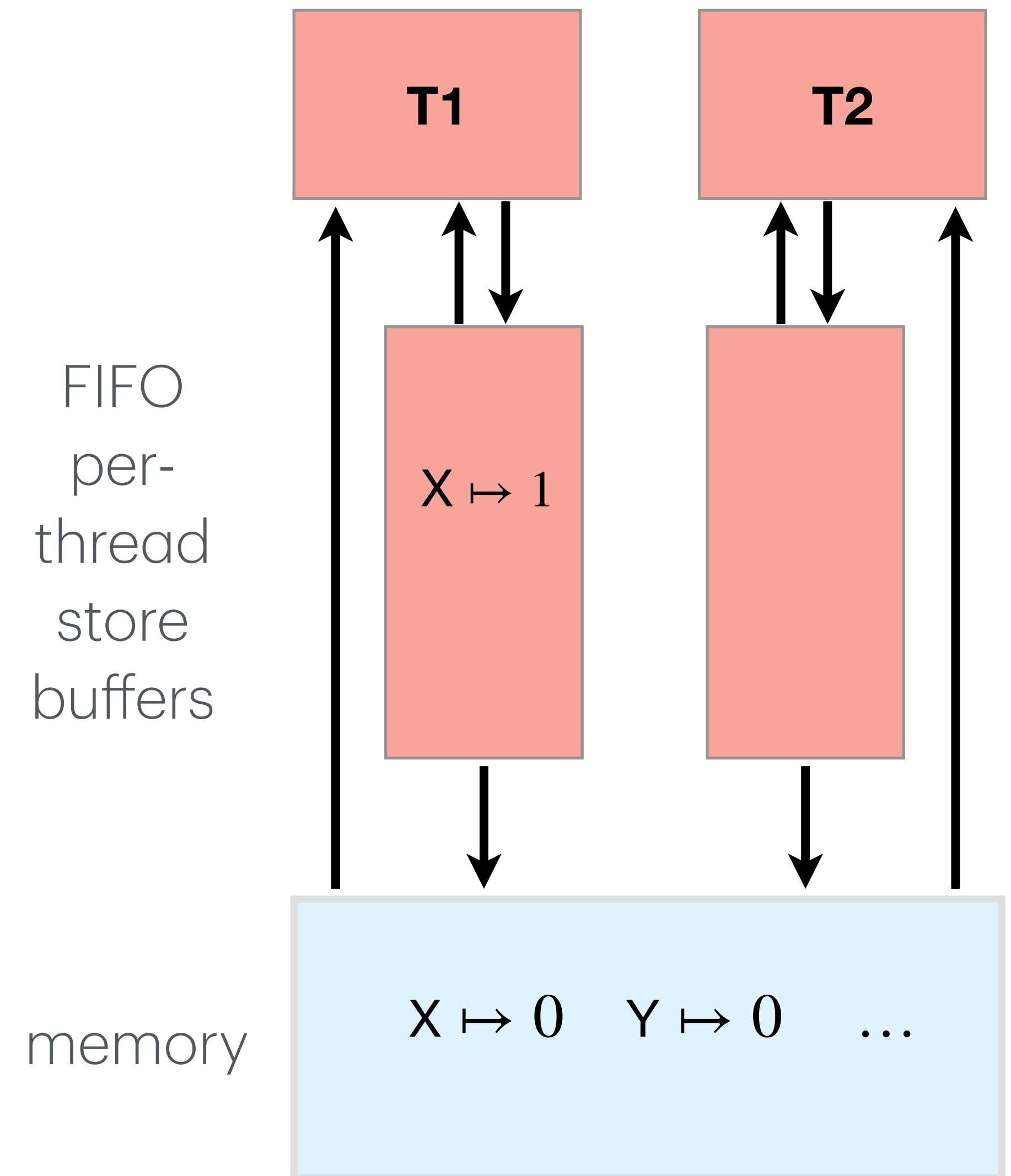
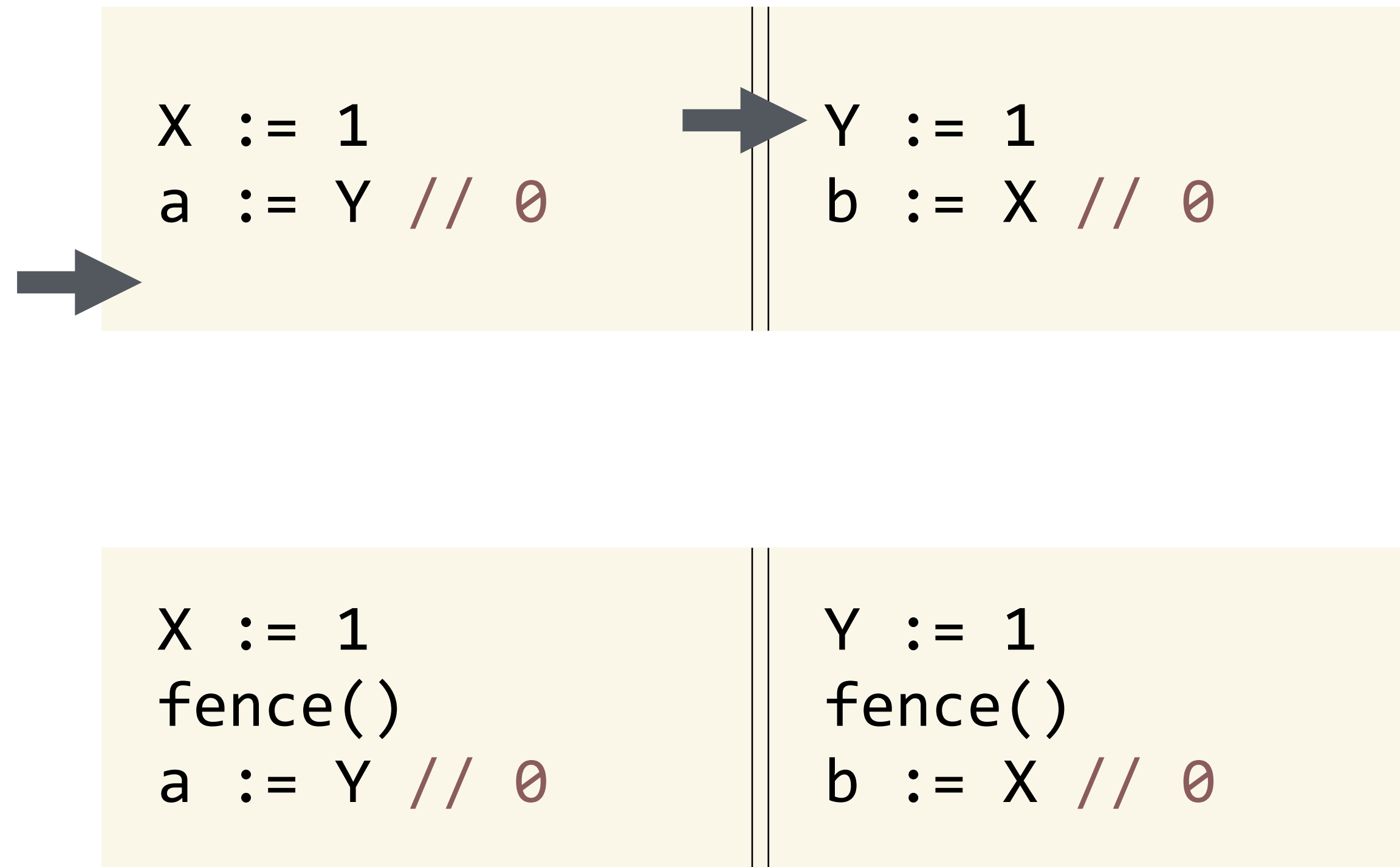


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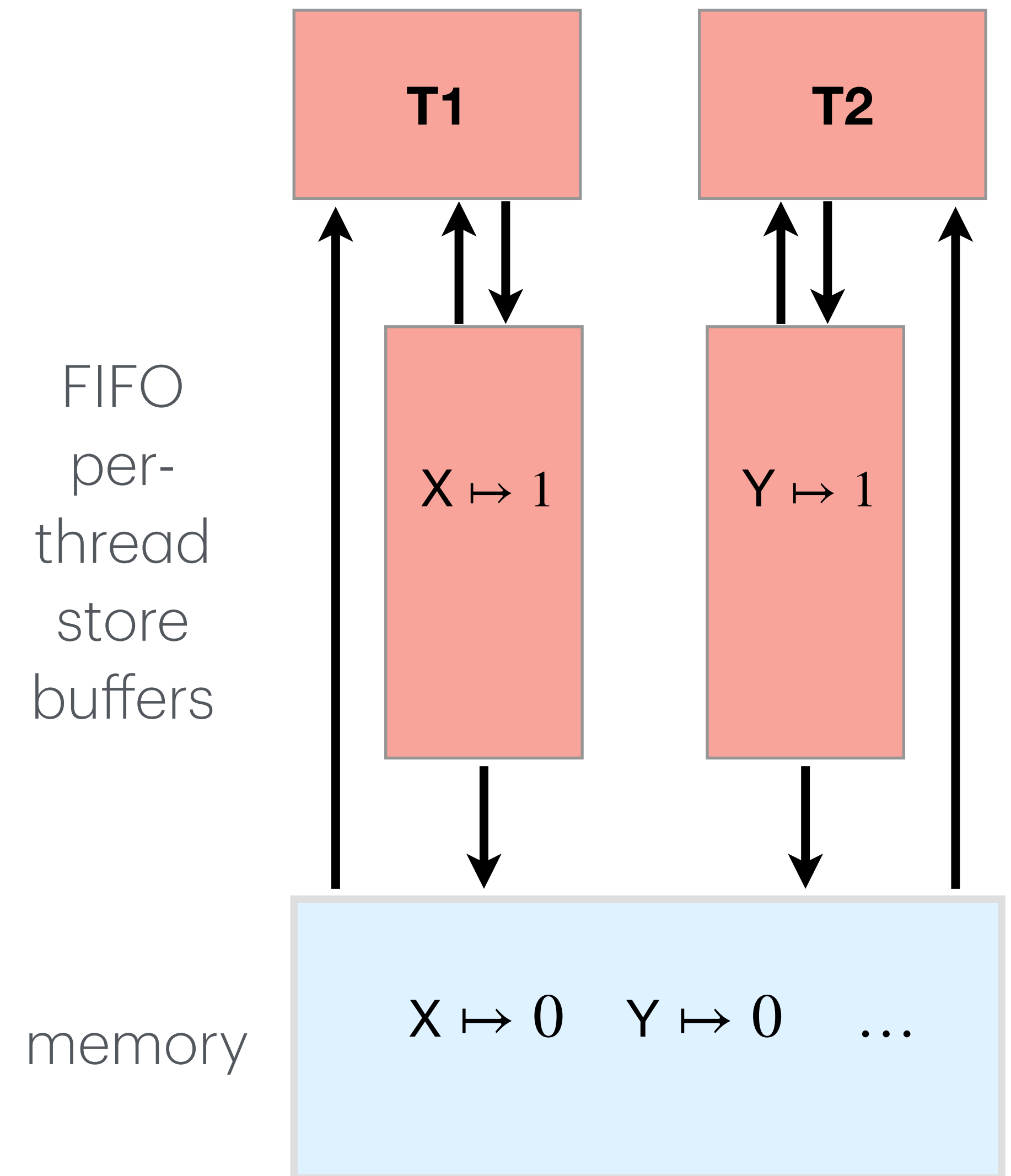
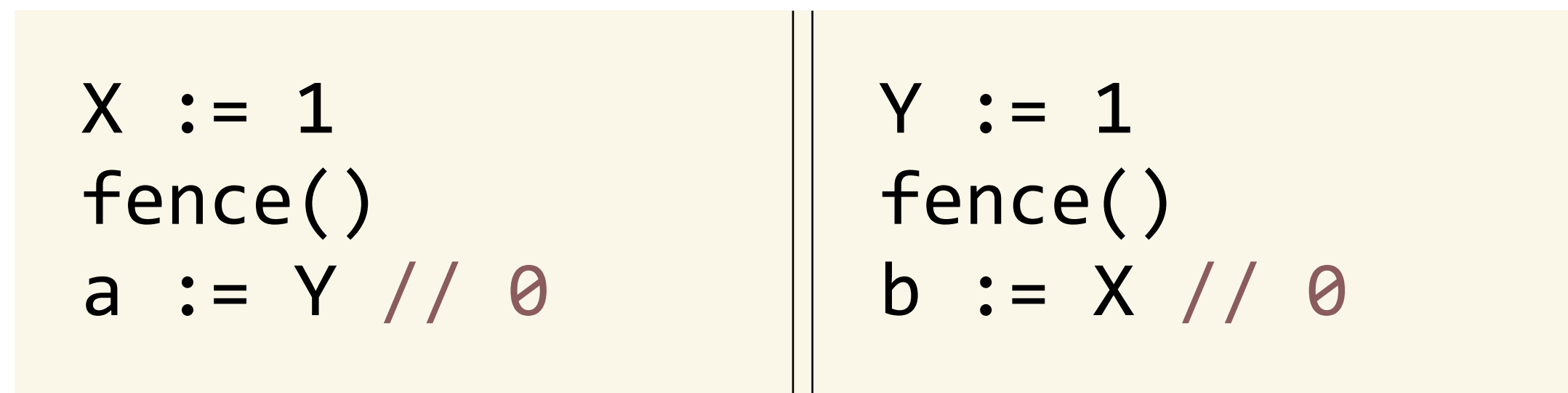
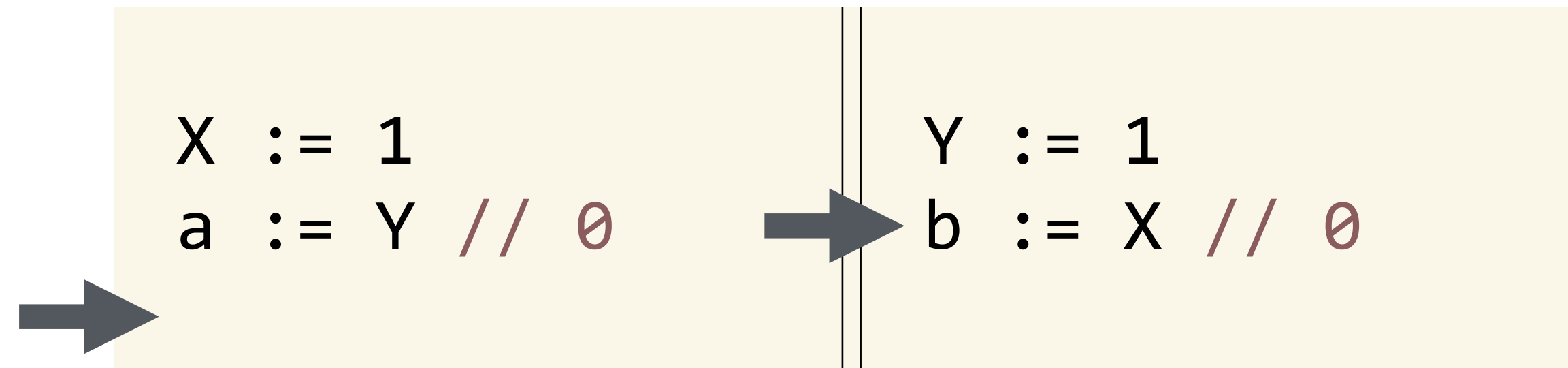
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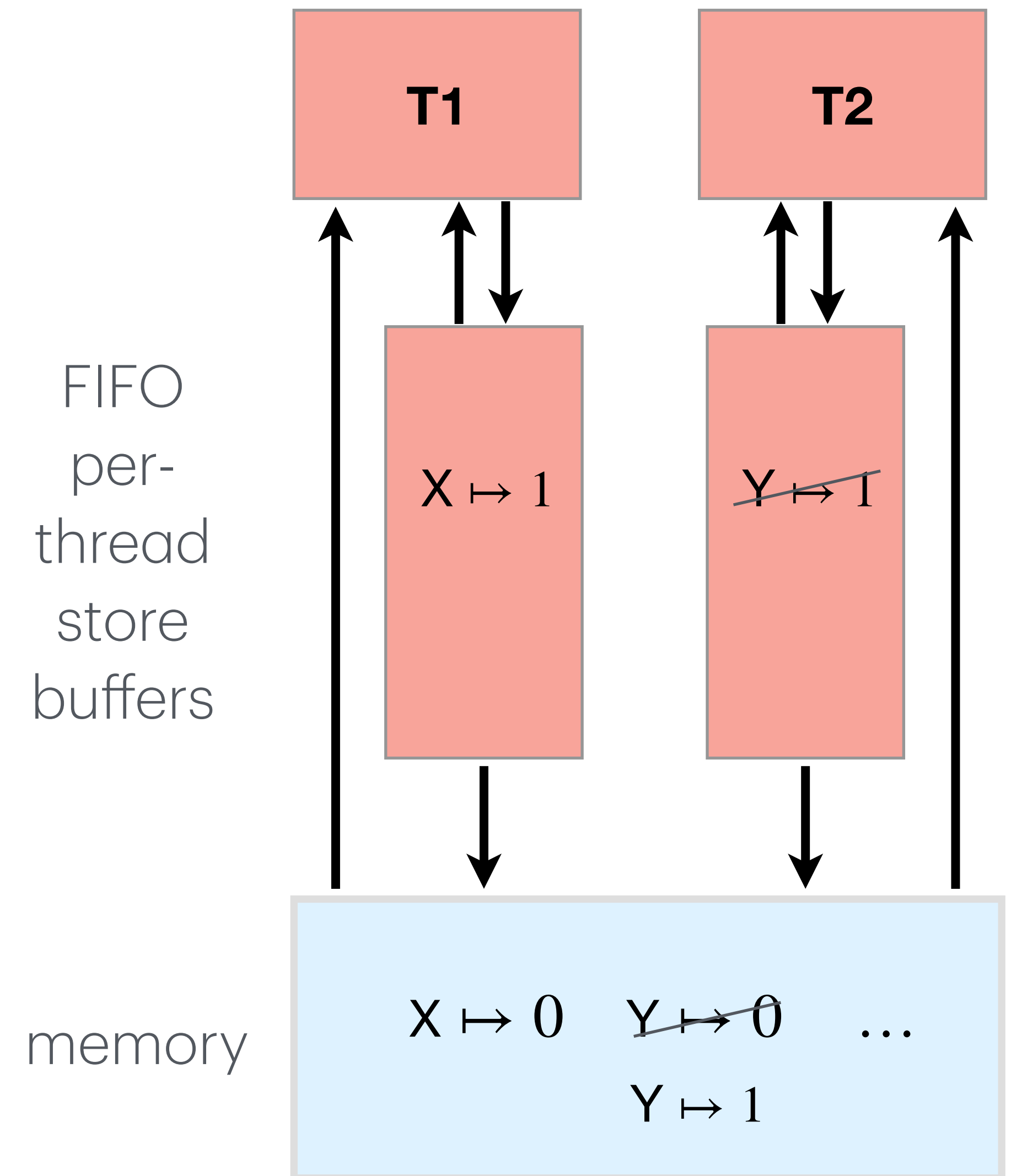
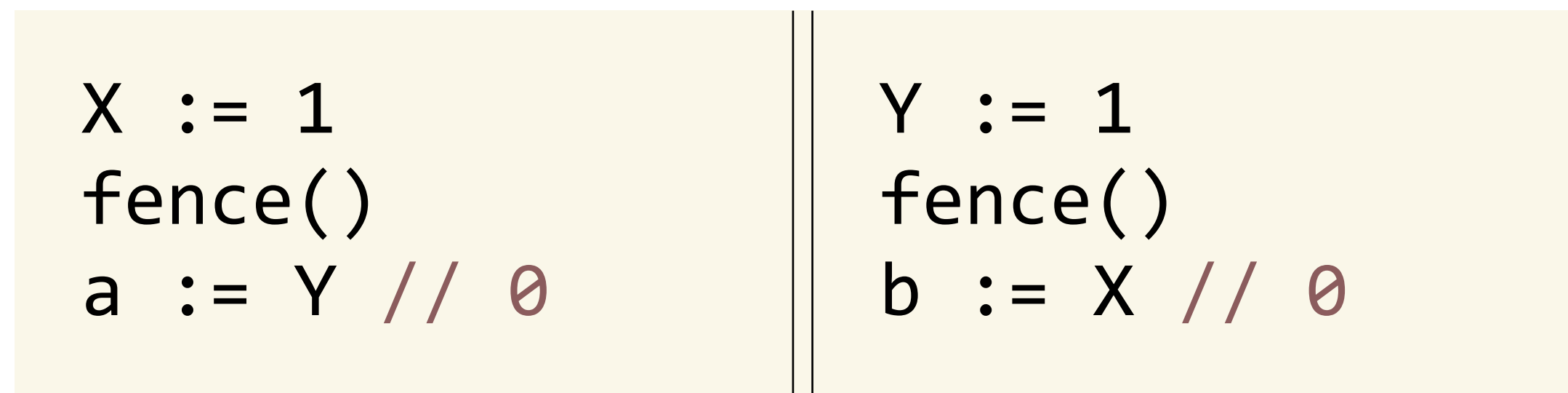
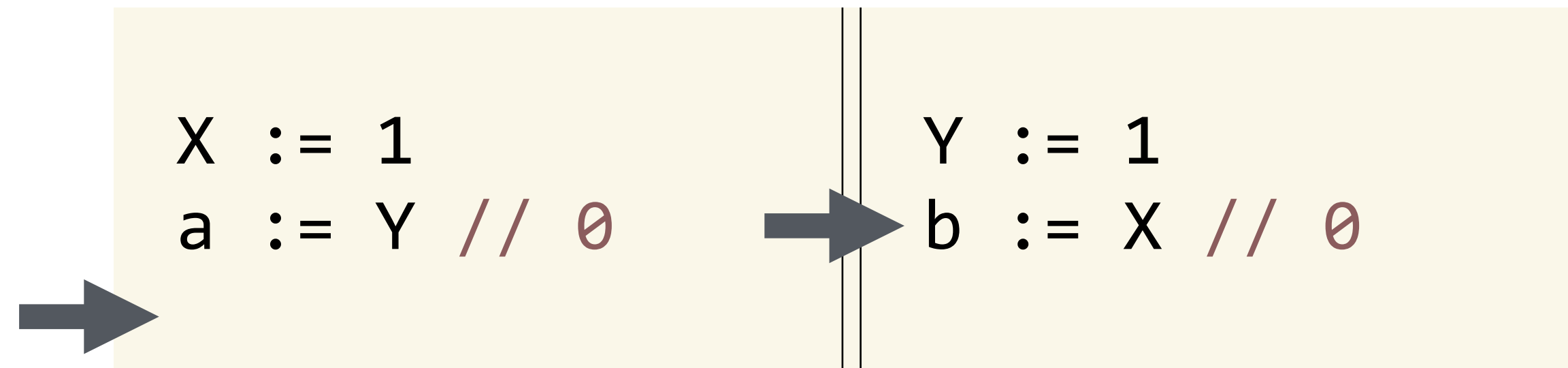


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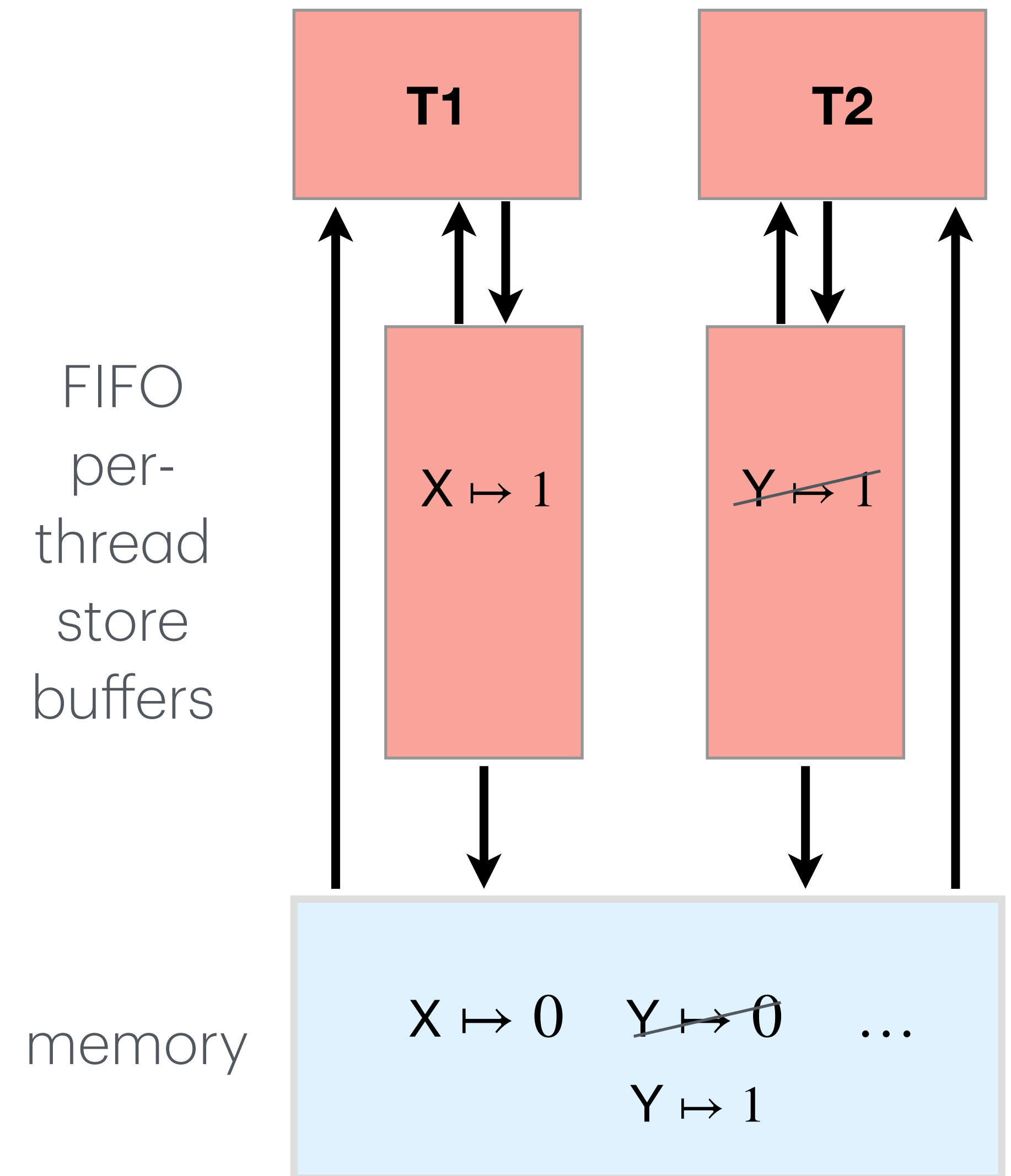
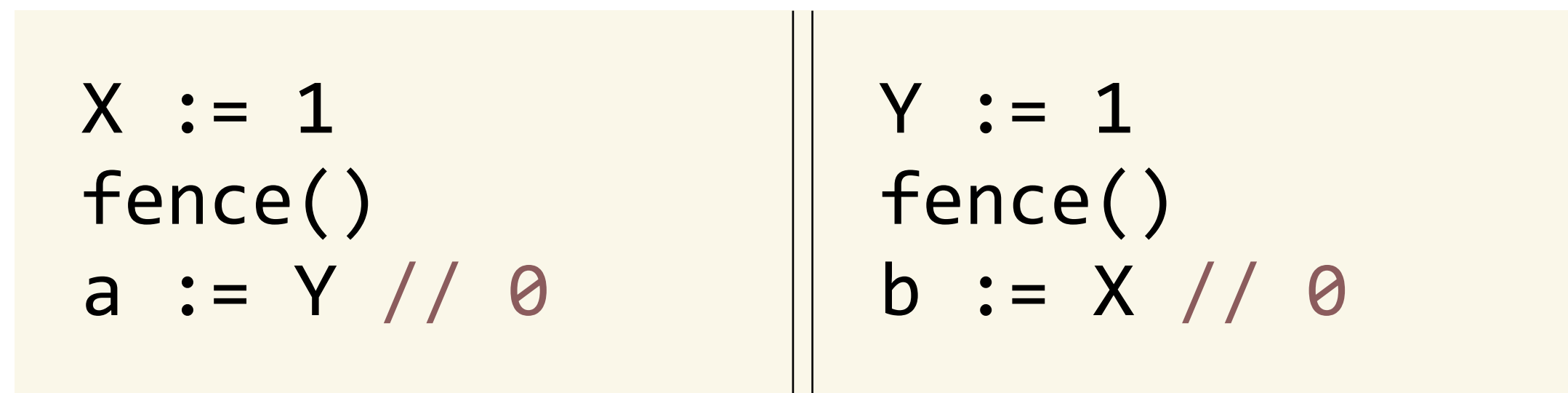
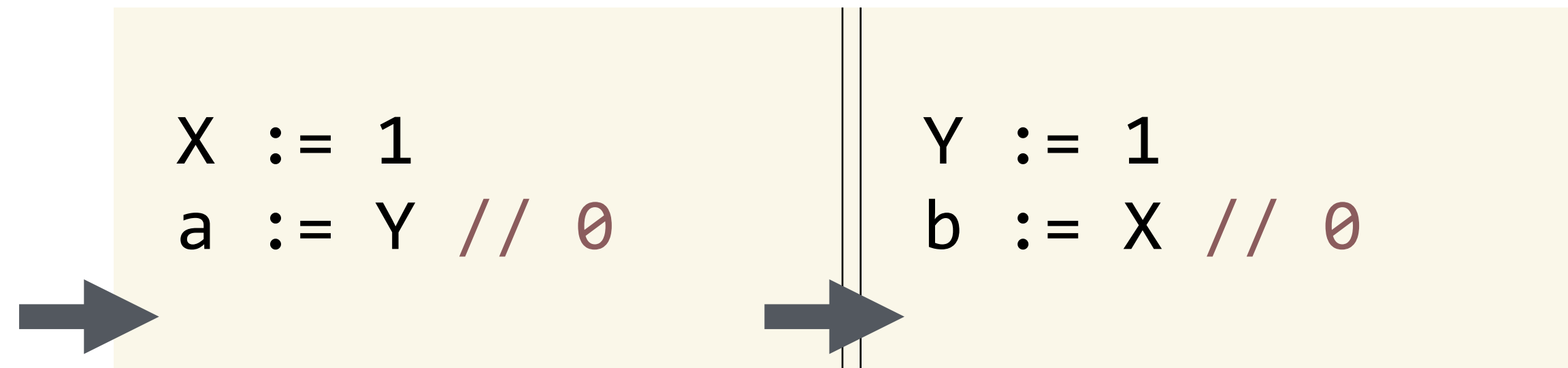
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# x86-TSO



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Commun. ACM 53(7) 2010. <https://doi.org/10.1145/1785414.1785443>

# WMM = out-of-order execution?

C1 / C2	Store	Load
Store	N	Y
Load	N	N

```

X := 1      |      Y := 1
a := Y // 0 |      b := X // 0
    
```



C1 / C2	Store	Load
Store	Y	Y
Load	Y	Y

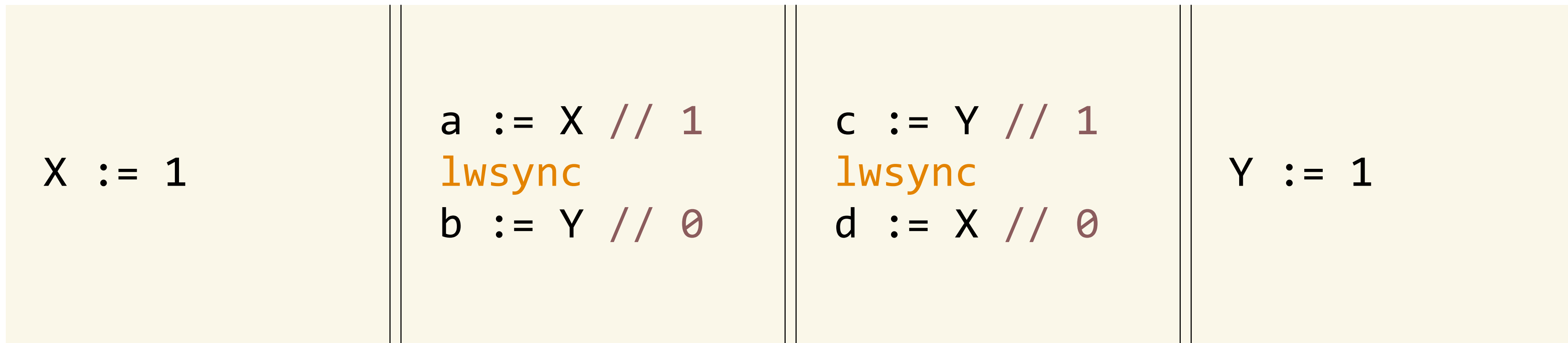
```

a := X // 1 |      b := Y // 1
Y := 1      |      X := b
    
```



possible reordering for *independent* accesses

# WMM ≠ out-of-order execution



- Because of the `lwsync` fences, no intra-process reorderings are possible
- The threads may still observe the writes in different orders



# WMM = hardware models?



# Read from untaken branch

```
a := X
```

```
Y := a
```

```
b := Y
```

```
if (b = 42) then
```

```
  c := 1
```

```
else
```

```
  c := 2
```

```
  b := 42
```

```
X := b
```

Can this program end with `c = 1` ?

# Read from untaken branch

```
a := X
```

```
Y := a
```

```
b := Y
```

```
if (b = 42) then
```

```
  c := 1
```

```
else
```

```
  c := 2
```

```
  b := 42
```

```
X := b 42
```

Can this program end with `c = 1` ?

# Read from untaken branch

```
a := X
```

```
Y := a
```

```
b := Y
```

```
if (b = 42) then
```

```
  c := 1
```

```
else
```

```
  c := 2
```

```
  b := 42
```

```
X := b 42
```

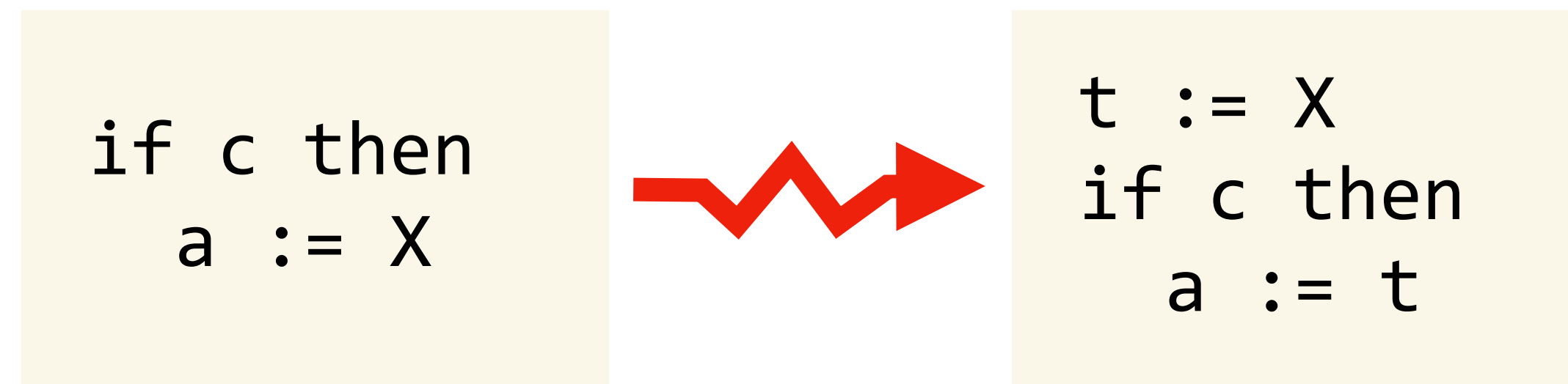
Can this program end with `c = 1` ?

# Tricky combinations

- Repeated read elimination over a lock:



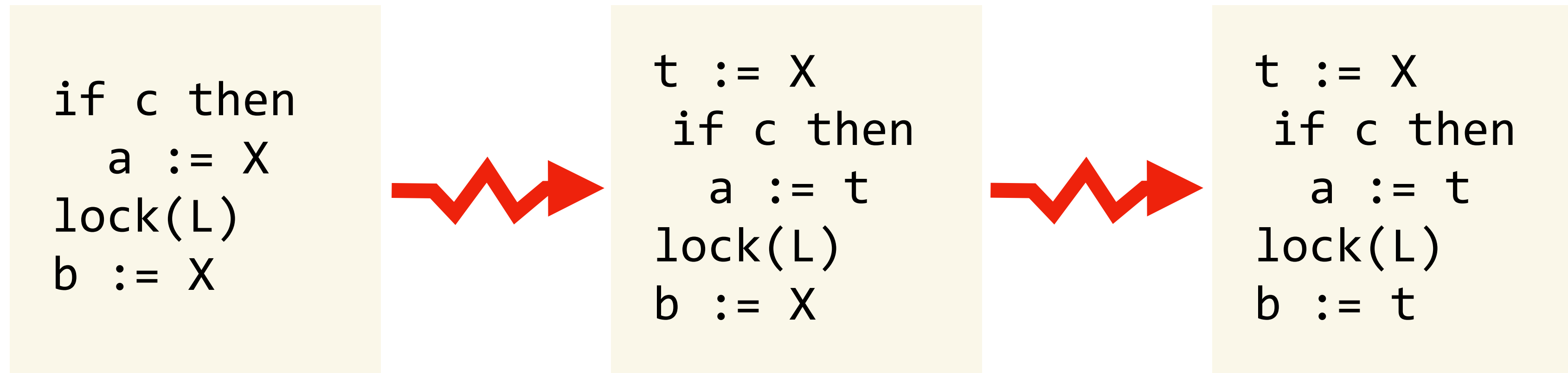
- Read hoisting (**t** is a fresh temporary):





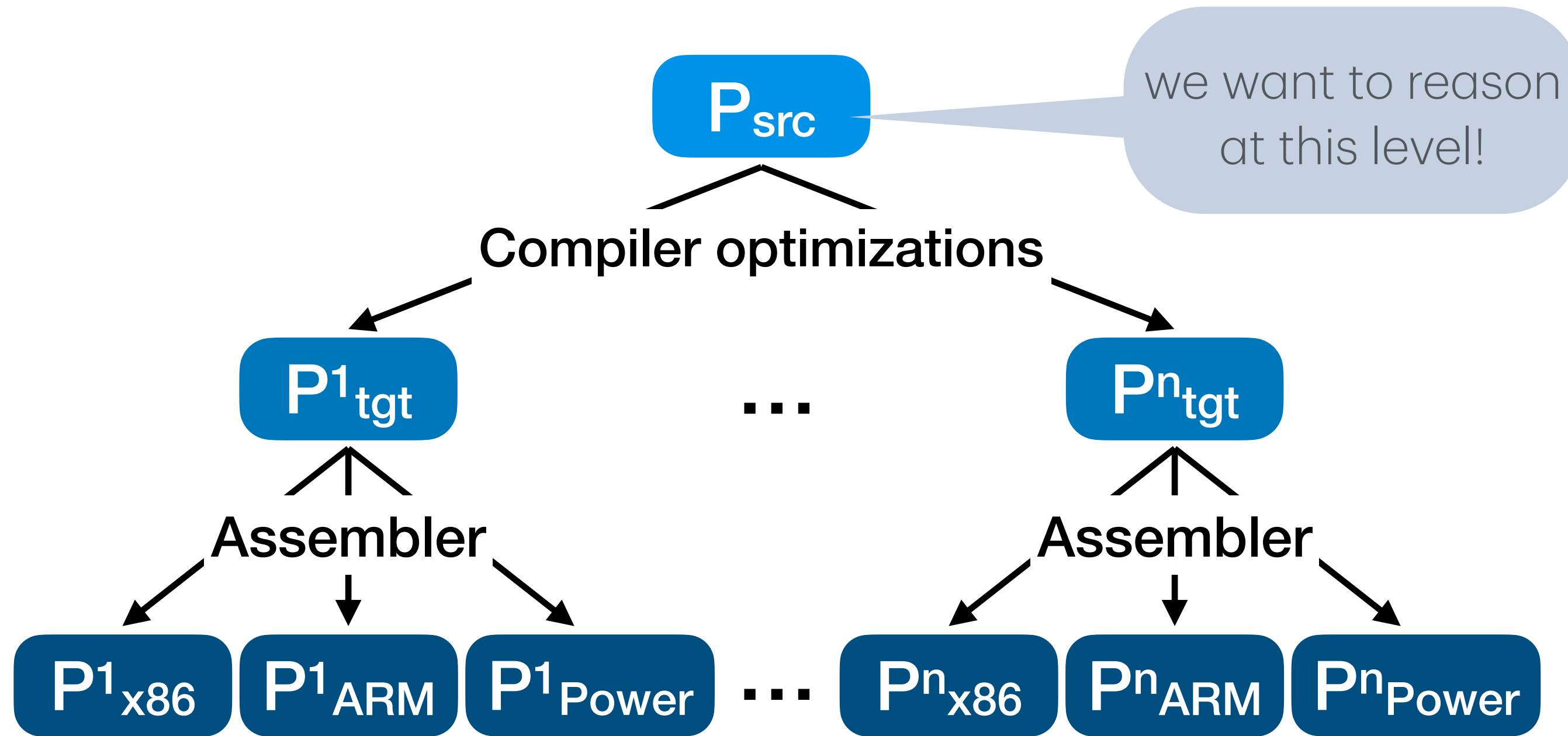
# Allowing both is wrong!

- The combination of the two is unsafe:

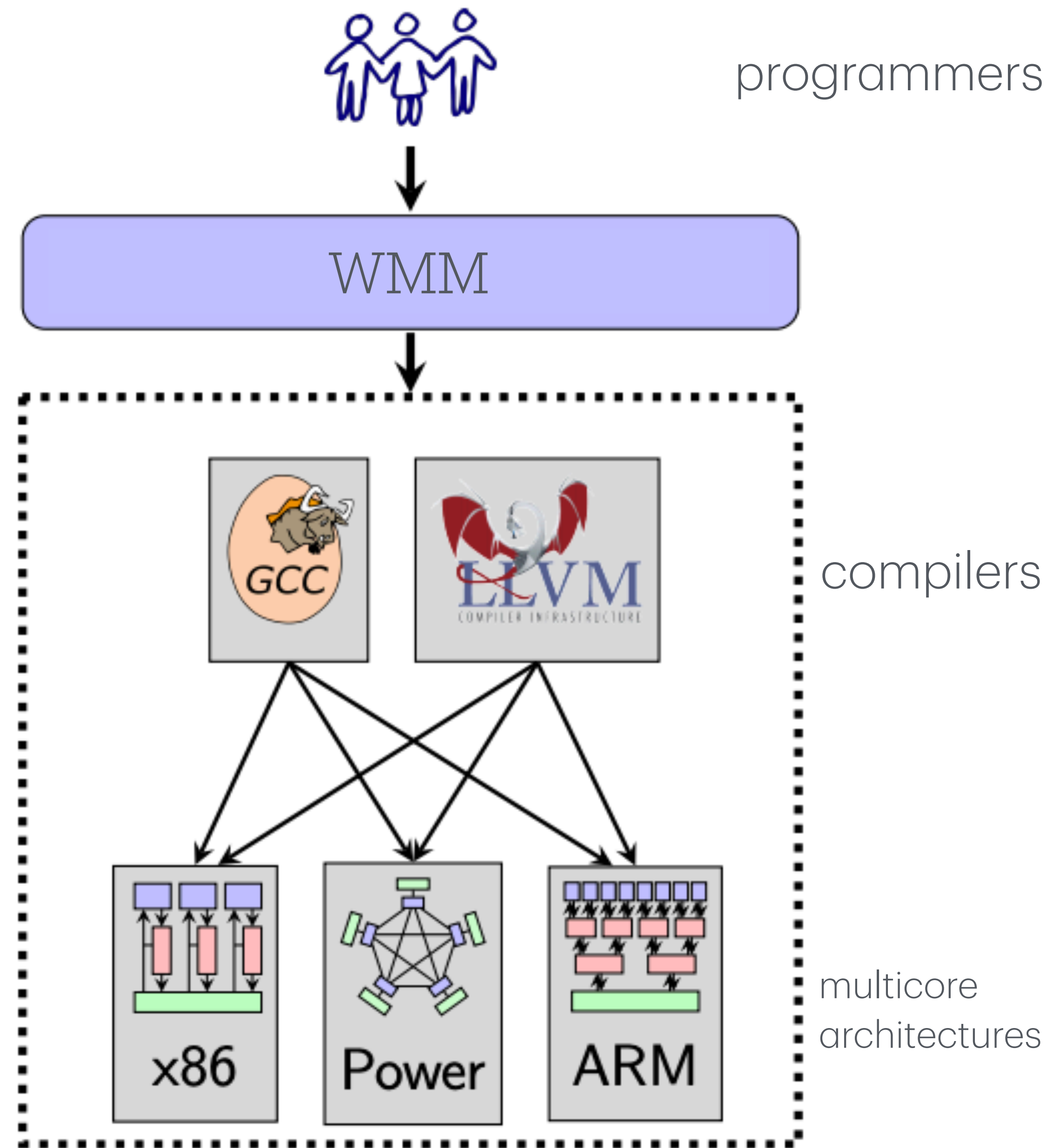
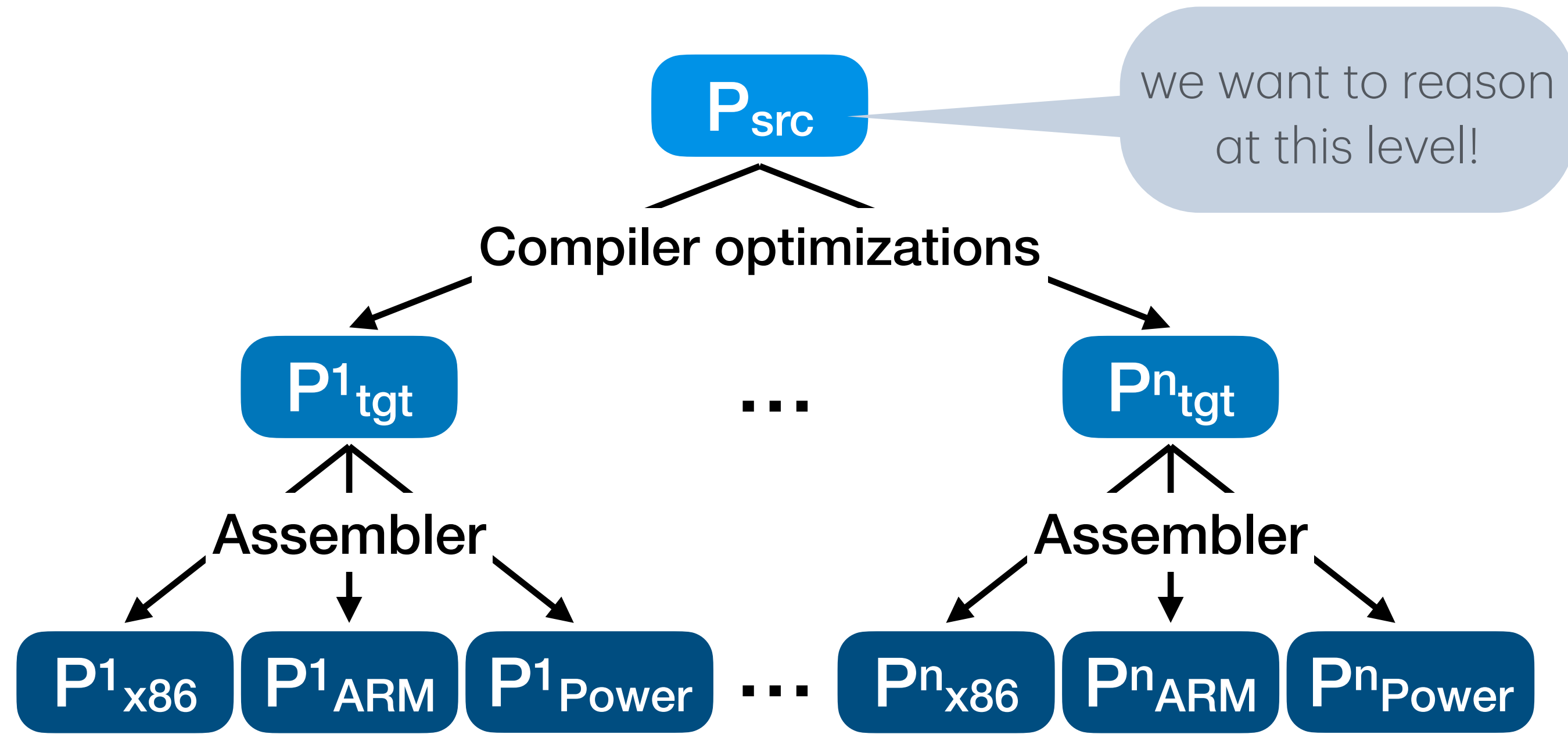


- When **c** is false, **X** is moved out of the critical region!
- We have to **forbid** one of the transformations:
  - C forbids load hoisting
  - LLVM forbids repeated read elimination over a lock

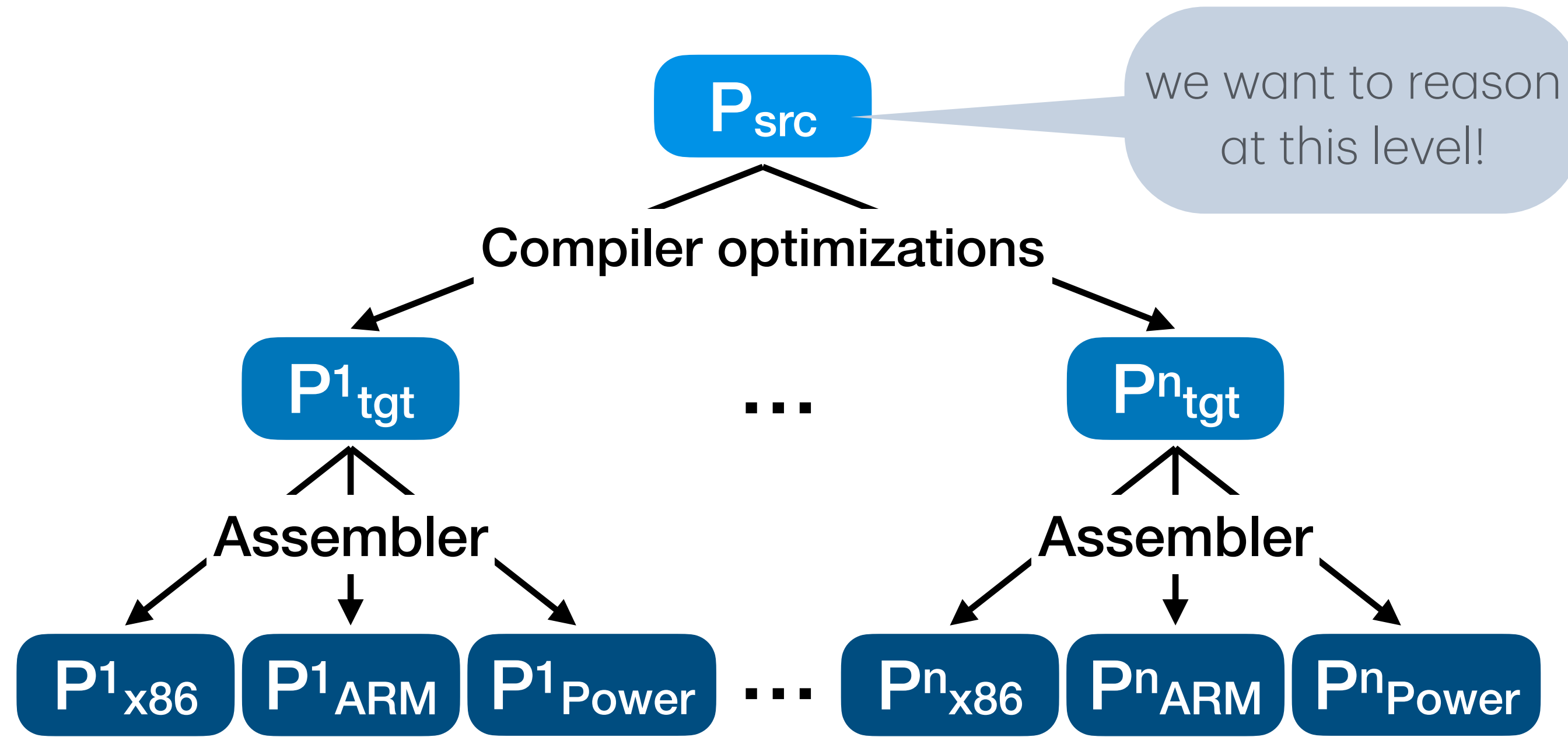
# A WMM for a PL



# A WMM for a PL



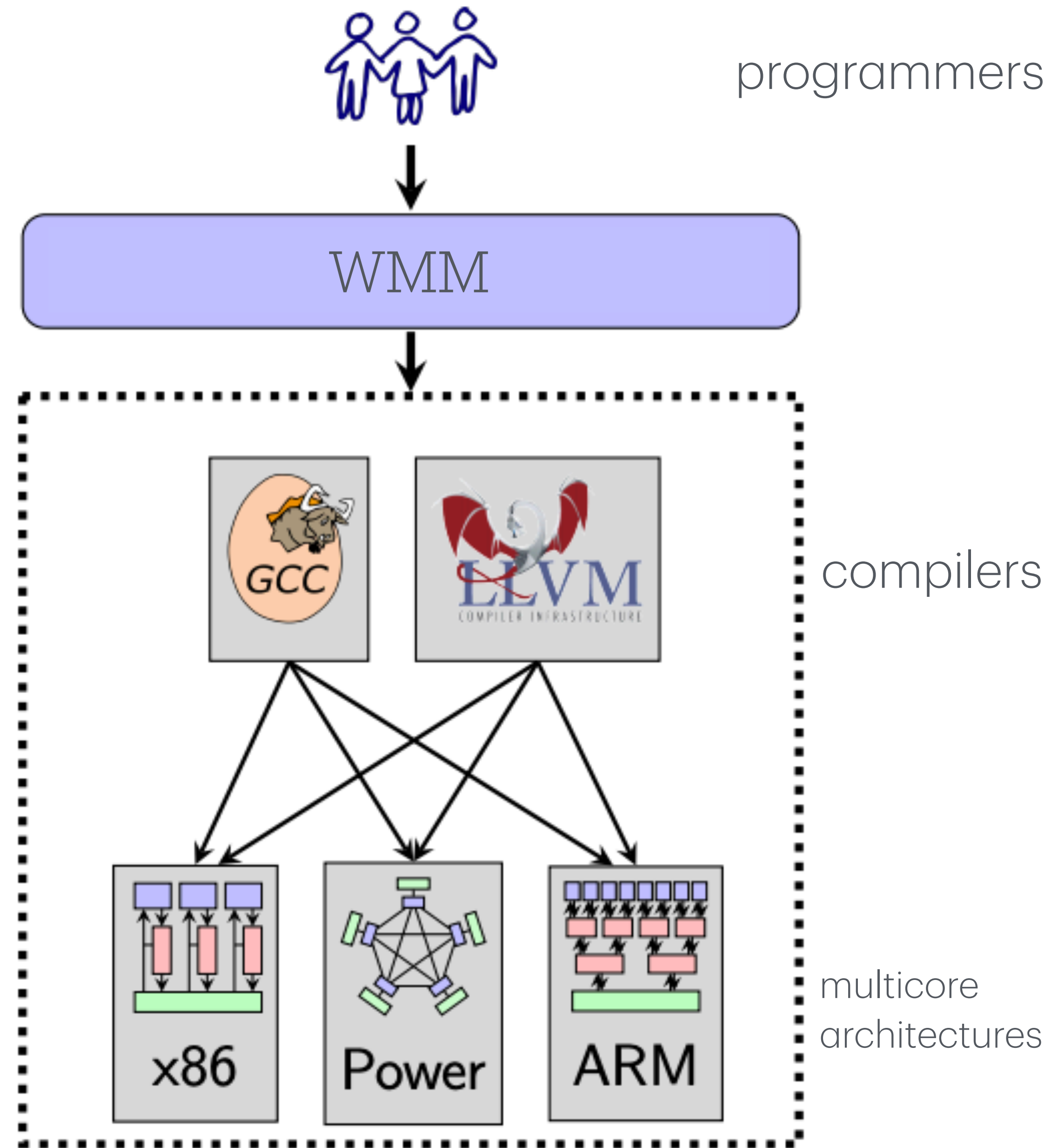
# A WMM for a PL



we want to reason at this level!

- C, C++
- Java
- OCaml
- JavaScript
- WebAssembly

- Linux kernel
- Rust
- LLVM
- ...



**Solving the memory model problem will require an ambitious and cross-disciplinary research direction.**

BY SARITA V. ADVE AND HANS-J. BOEHM

# Memory Models: A Case for Rethinking Parallel Languages and Hardware

Commun. ACM 53, 8 (August 2010).  
<https://doi.org/10.1145/1582716.1582718>

## The Problem of Programming Language Concurrency Semantics

Mark Batty, Kayvan Memarian, Kyndylan Nienhuis, Jean Pichon-Pharabod, and Peter Sewell

University of Cambridge

“Disturbingly, 40+ years after the first relaxed-memory hardware was introduced (the IBM 370/158MP), the field still *does not have a credible proposal for the concurrency semantics* of any general-purpose high-level language that includes high performance shared-memory concurrency primitives. This is a *major open problem* for programming language semantics.”

ESOP 2015. [https://doi.org/10.1007/978-3-662-46669-8\\_12](https://doi.org/10.1007/978-3-662-46669-8_12)


# Embracing weak consistency

- Not only a **threat**, but also an **opportunity**:
  - More **scalable** algorithms
  - Many (most?) concurrent idioms/algorithms **do not need SC**
  - Better understanding of our **algorithms**
  - Better understanding of **concurrency**
  - **Local** reasoning and more scalable verification
  
- Open research problems!



# The C11 memory model

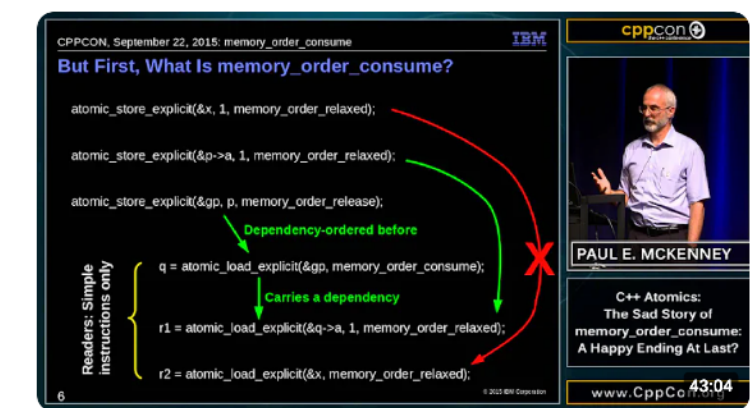
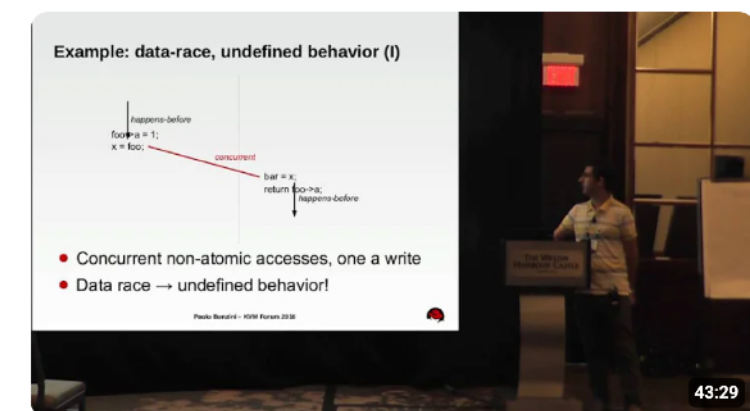
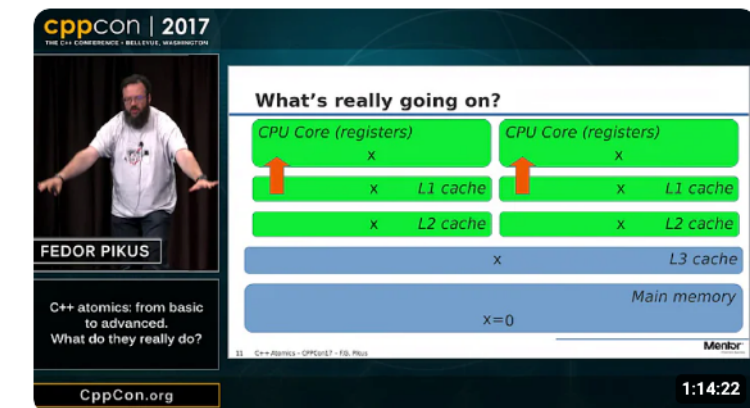
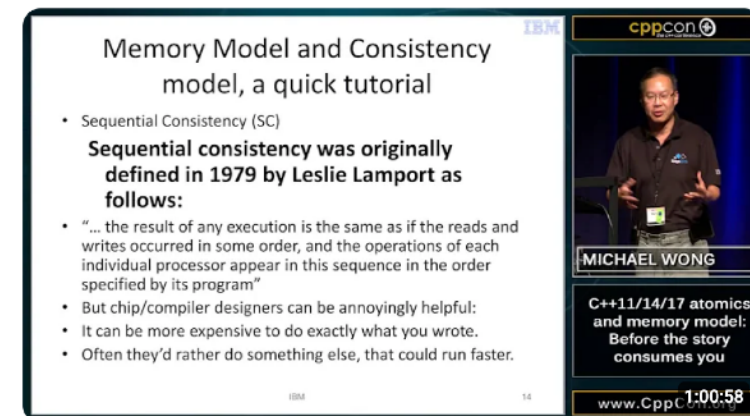
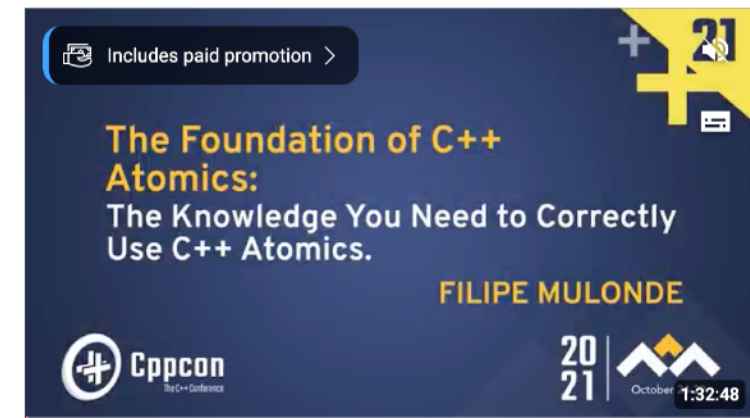
# The C/C++11 memory model

- In C/C++11 threads were made a part of the language specification
- A careful and sophisticated *declarative* weak memory model was published:
  - a result of several years of effort (starting around 2004)
  - building on the experience with the Java memory model
- Main design principles:
  - Tell *non-expert programmers* to avoid *data races* and provide strong semantics for them
    - Leave the semantics of *data races* completely **undefined** (“**catch-fire**”) 
    - This way we can allow more flexible implementations and simpler model
  - Give *experts* a way to write very carefully crafted, but portable, synchronization code that approaches the performance of assembly code



# Some resources

- For language lawyers:
  - <http://www.open-std.org/jtc1/sc22/wg14/www/docs/n1548.pdf>
- A popularized history:
  - Boehm, Adve: **You Don't Know Jack about Shared Variables or Memory Models**. Commun. ACM 55.2 (Feb. 2012). <https://doi.org/10.1145/2076450.2076465>
- Formal treatment:
  - Batty, Owens, Sarkar, Sewell, Weber: **Mathematizing C++ Concurrency**. POPL 2011. <http://doi.acm.org/10.1145/1926385.1926394>
  - L, Vafeiadis, Kang, Hur, Dreyer: **Repairing Sequential Consistency in C/C++11**. PLDI 2017. <https://doi.org/10.1145/3140587.3062352>



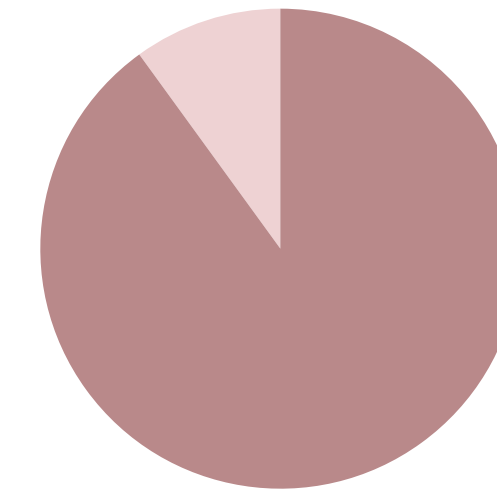
# Why focus on C11?

- The world is programmed in C/C++.
- C11 is a **prototype PL memory model**: a solid starting point for other languages: LLVM, Java 9, WebAssembly, Rust, JavaScript...
- **Architecture vendors** aim to efficiently implement C11
- One of the most **well-studied** weak memory models: correctness, programmability guarantees, algorithms, verification,...

# Main ingredients

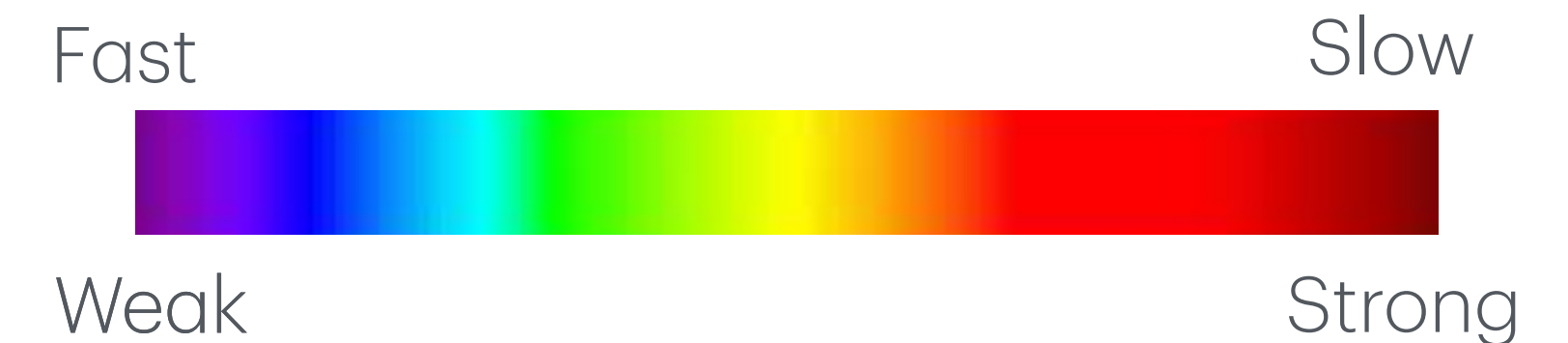
non-expert mode

- **Non-atomic** memory accesses (reads/writes):
  - ordinary accesses for **data manipulations**
  - the majority of accesses in a typical program
  - insensitive to access granularity
- **Locks**
  - used to avoid data-races



expert mode

- **Atomic** memory accesses (reads/writes/RMWs)
  - used for **synchronization**
- **Fences** for fine-tuned synchronization patterns



relaxed  release/acquire  sc  
memory orders

# Syntax examples (atomics)

- In C:

- annotate the type, and then all accesses default to SC memory order:

```
Atomic(Node *) top;
```

- Annotate an access:

```
t = atomic_load_explicit(top, memory_order_acquire);
```

- CAS in C++:

```
atomic_compare_exchange_weak_explicit(&head, &new_node->next, new_node,  
memory_order_release, memory_order_relaxed);
```

# Examples with non-atomics and locks

```
lock(L)
X := 1
unlock(L)
```

```
lock(L)
a := X // 2
unlock(L)
```

```
a := X // 1
Y := 1
```

```
b := Y // 1
X := 1
```

```
a := X // 1
if a = 1 then
  Y := 1
```

```
b := Y // 1
if b = 1 then
  X := 1
```

```
X := 1
lock(L)
Y := 1
unlock(L)
```

```
lock(L)
a := Y // 1
unlock(L)
if (a = 1) then
  b := X // 0
```

Which of these programs are *race-free*? How are *data races* defined?

What are the guarantees for *race-free* programs?

# The full model...

$$\begin{aligned}
[-] &: \text{CExp} \rightarrow \mathbb{P}(\langle \text{res} : \text{Val} \cup \{\perp\}, \mathcal{A} : \mathbb{P}(\text{AName}), \text{lab} : \mathcal{A} \rightarrow \text{Act}, \text{sb} : \mathbb{P}(\mathcal{A} \times \mathcal{A}), \text{fst} : \mathcal{A}, \text{lst} : \mathcal{A} \rangle) \\
[v] &\stackrel{\text{def}}{=} \{\langle v, \{a\}, \text{lab}, \emptyset, a, a \rangle \mid a \in \text{AName} \wedge \text{lab}(a) = \text{skip}\} \\
[\text{alloc}()] &\stackrel{\text{def}}{=} \{\langle \ell, \{a\}, \text{lab}, \emptyset, a, a \rangle \mid a \in \text{AName} \wedge \ell \in \text{Loc} \wedge \text{lab}(a) = \text{A}(\ell)\} \\
[[v]_Z := v'] &\stackrel{\text{def}}{=} \{\langle v', \{a\}, \text{lab}, \emptyset, a, a \rangle \mid a \in \text{AName} \wedge \text{lab}(a) = \text{W}_Z(v, v')\} \\
[[v]_Z] &\stackrel{\text{def}}{=} \{\langle v', \{a\}, \text{lab}, \emptyset, a, a \rangle \mid a \in \text{AName} \wedge v' \in \text{Val} \wedge \text{lab}(a) = \text{R}_Z(v, v')\} \\
[\text{CAS}_{X,Y}(v, v_o, v_n)] &\stackrel{\text{def}}{=} \{\langle v', \{a\}, \text{lab}, \emptyset, a, a \rangle \mid a \in \text{AName} \wedge v' \in \text{Val} \wedge v' \neq v_o \wedge \text{lab}(a) = \text{R}_Y(v, v')\} \\
&\quad \cup \{\langle v_o, \{a\}, \text{lab}, \emptyset, a, a \rangle \mid a \in \text{AName} \wedge \text{lab}(a) = \text{RMW}_X(v, v_o, v_n)\} \\
[\text{let } x = E_1 \text{ in } E_2] &\stackrel{\text{def}}{=} \{\langle \perp, \mathcal{A}_1, \text{lab}_1, \text{sb}_1, \text{fst}_1, \text{lst}_1 \rangle \mid \langle \perp, \mathcal{A}_1, \text{lab}_1, \text{sb}_1, \text{fst}_1, \text{lst}_1 \rangle \in [[E_1]]\} \\
&\quad \cup \{\langle \text{res}_2, \mathcal{A}_1 \uplus \mathcal{A}_2, \text{lab}_1 \cup \text{lab}_2, \text{sb}_1 \cup \text{sb}_2 \cup \{(lst_1, \text{fst}_2)\}, \text{fst}_1, \text{lst}_2 \rangle \mid \\
&\quad \langle v_1, \mathcal{A}_1, \text{lab}_1, \text{sb}_1, \text{fst}_1, \text{lst}_1 \rangle \in [[E_1]] \wedge \langle \text{res}_2, \mathcal{A}_2, \text{lab}_2, \text{sb}_2, \text{fst}_2, \text{lst}_2 \rangle \in [[E_2[v_1/x]]]\} \\
[\text{repeat } E \text{ end}] &\stackrel{\text{def}}{=} \{\langle \text{res}_N, \biguplus_{i \in [1..N]} \mathcal{A}_i, \bigcup_{i \in [1..N]} \text{lab}_i, \bigcup_{i \in [1..N]} \text{sb}_i \cup \{(lst_1, \text{fst}_2), \dots, (lst_{N-1}, \text{fst}_N)\}, \text{fst}_1, \text{lst}_N \rangle \mid \\
&\quad \forall i. \langle \text{res}_i, \mathcal{A}_i, \text{lab}_i, \text{sb}_i, \text{fst}_i, \text{lst}_i \rangle \in [[E]] \wedge (i \neq N \implies \text{res}_i = 0) \wedge \text{res}_N \neq 0\} \\
[[E_1] \parallel [E_2]] &\stackrel{\text{def}}{=} \{\langle \text{combine}(\text{res}_1, \text{res}_2), \mathcal{A}_1 \uplus \mathcal{A}_2 \uplus \{a_{\text{fork}}, a_{\text{join}}\}, \text{lab}_1 \cup \text{lab}_2 \cup \{a_{\text{fork}} \mapsto \text{skip}, a_{\text{join}} \mapsto \text{skip}\}, \\
&\quad \text{sb}_1 \cup \text{sb}_2 \cup \{(a_{\text{fork}}, \text{fst}_1), (a_{\text{fork}}, \text{fst}_2), (lst_1, a_{\text{join}}), (lst_2, a_{\text{join}})\}, a_{\text{fork}}, a_{\text{join}} \rangle \mid \\
&\quad \langle \text{res}_1, \mathcal{A}_1, \text{sb}_1, \text{fst}_1, \text{lst}_1 \rangle \in [[E_1]] \wedge \langle \text{res}_2, \mathcal{A}_2, \text{sb}_2, \text{fst}_2, \text{lst}_2 \rangle \in [[E_2]] \wedge a_{\text{fork}}, a_{\text{join}} \in \text{AName}\}
\end{aligned}$$

Figure 2. Semantics of closed program expressions.

$$\begin{aligned}
&\nexists x. \text{hb}(x, x) && (\text{IrreflexiveHB}) \\
&\forall \ell. \text{totalorder}(\{a \in \mathcal{A} \mid \text{iswrite}_\ell(a)\}, \text{mo}) \wedge \text{hb}_\ell \subseteq \text{mo} && (\text{ConsistentMO}) \\
&\text{totalorder}(\{a \in \mathcal{A} \mid \text{isSeqCst}(a)\}, \text{sc}) \wedge \text{hb}_{\text{SeqCst}} \subseteq \text{sc} \wedge \text{mo}_{\text{SeqCst}} \subseteq \text{sc} && (\text{ConsistentSC}) \\
&\forall b. \text{rf}(b) \neq \perp \iff \exists \ell, a. \text{iswrite}_\ell(a) \wedge \text{isread}_\ell(b) \wedge \text{hb}(a, b) && (\text{ConsistentRFdom}) \\
&\forall a, b. \text{rf}(b) = a \implies \exists \ell, v. \text{iswrite}_{\ell, v}(a) \wedge \text{isread}_{\ell, v}(b) \wedge \neg \text{hb}(b, a) && (\text{ConsistentRF}) \\
&\forall a, b. \text{rf}(b) = a \wedge (\text{mode}(a) = \text{na} \vee \text{mode}(b) = \text{na}) \implies \text{hb}(a, b) && (\text{ConsistentRFna}) \\
&\forall a, b. \text{rf}(b) = a \wedge \text{isSeqCst}(b) \implies \text{isc}(a, b) \vee \neg \text{isSeqCst}(a) \wedge (\forall x. \text{isc}(x, b) \implies \neg \text{hb}(a, x)) && (\text{RestrSCReads}) \\
&\nexists a, b. \text{hb}(a, b) \wedge \text{mo}(\text{rf}(b), \text{rf}(a)) \wedge \text{locs}(a) = \text{locs}(b) && (\text{CoherentRR}) \\
&\nexists a, b. \text{hb}(a, b) \wedge \text{mo}(\text{rf}(b), a) \wedge \text{iswrite}(a) \wedge \text{locs}(a) = \text{locs}(b) && (\text{CoherentWR}) \\
&\nexists a, b. \text{hb}(a, b) \wedge \text{mo}(b, \text{rf}(a)) \wedge \text{iswrite}(b) \wedge \text{locs}(a) = \text{locs}(b) && (\text{CoherentRW}) \\
&\forall a. \text{isrmw}(a) \wedge \text{rf}(a) \neq \perp \implies \text{mo}(\text{rf}(a), a) \wedge \nexists c. \text{mo}(\text{rf}(a), c) \wedge \text{mo}(c, a) && (\text{AtomicRMW}) \\
&\forall a, b, \ell. \text{lab}(a) = \text{lab}(b) = \text{A}(\ell) \implies a = b && (\text{ConsistentAlloc})
\end{aligned}$$

where  $\text{iswrite}_{\ell, v}(a) \stackrel{\text{def}}{=} \exists X, v_{\text{old}}. \text{lab}(a) \in \{\text{W}_X(\ell, v), \text{RMW}_X(\ell, v_{\text{old}}, v)\}$   $\text{iswrite}_\ell(a) \stackrel{\text{def}}{=} \exists v. \text{iswrite}_{\ell, v}(a)$   
 $\text{isread}_{\ell, v}(a) \stackrel{\text{def}}{=} \exists X, v_{\text{new}}. \text{lab}(a) \in \{\text{R}_X(\ell, v), \text{RMW}_X(\ell, v, v_{\text{new}})\}$  etc.  
 $\text{rsElem}(a, b) \stackrel{\text{def}}{=} \text{sameThread}(a, b) \vee \text{isrmw}(b)$   
 $\text{rseq}(a) \stackrel{\text{def}}{=} \{a\} \cup \{b \mid \text{rsElem}(a, b) \wedge \text{mo}(a, b) \wedge (\forall c. \text{mo}(a, c) \wedge \text{mo}(c, b) \implies \text{rsElem}(a, c))\}$   
 $\text{sw} \stackrel{\text{def}}{=} \{(a, b) \mid \text{mode}(a) \in \{\text{rel}, \text{rel\_acq}, \text{sc}\} \wedge \text{mode}(b) \in \{\text{acq}, \text{rel\_acq}, \text{sc}\} \wedge \text{rf}(b) \in \text{rseq}(a)\}$   
 $\text{hb} \stackrel{\text{def}}{=} (\text{sb} \cup \text{sw})^+$   
 $\text{hb}_\ell \stackrel{\text{def}}{=} \{(a, b) \in \text{hb} \mid \text{iswrite}_\ell(a) \wedge \text{iswrite}_\ell(b)\}$   
 $X_{\text{SeqCst}} \stackrel{\text{def}}{=} \{(a, b) \in X \mid \text{isSeqCst}(a) \wedge \text{isSeqCst}(b)\}$   
 $\text{isc}(a, b) \stackrel{\text{def}}{=} \text{iswrite}_{\text{locs}(b)}(a) \wedge \text{sc}(a, b) \wedge \nexists c. \text{sc}(a, c) \wedge \text{sc}(c, b) \wedge \text{iswrite}_{\text{locs}(b)}(c)$

Figure 3. Axioms satisfied by consistent C11 executions,  $\text{Consistent}(\mathcal{A}, \text{lab}, \text{sb}, \text{rf}, \text{mo}, \text{sc})$ .

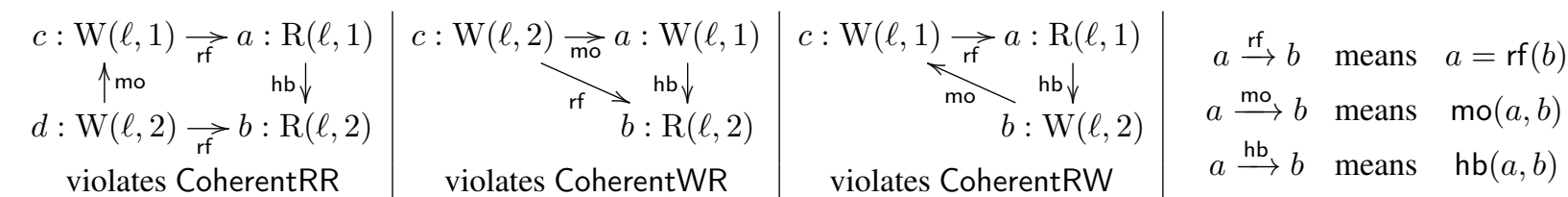


Figure 4. Sample executions violating coherency conditions (Batty et al. 2011).

$$\begin{aligned}
\text{rb} &\triangleq \text{rf}^{-1}; \text{mo} && (\text{reads-before}) \\
\text{eco} &\triangleq (\text{rf} \cup \text{mo} \cup \text{rb})^+ && (\text{extended coherence order}) \\
\text{rs} &\triangleq [\text{W}]; \text{sb}_{\text{loc}}^?; [\text{W}^{\text{relx}}]; (\text{rf}; \text{rmw})^* && (\text{release sequence}) \\
\text{sw} &\triangleq [\text{E}^{\text{rel}}]; ([\text{F}]; \text{sb})^?; \text{rs}; \text{rf}; && (\text{synchronizes with}) \\
&\quad [\text{R}^{\text{relx}}]; (\text{sb}; [\text{F}])^?; [\text{E}^{\text{acq}}] \\
\text{hb} &\triangleq (\text{sb} \cup \text{sw})^+ && (\text{happens-before})
\end{aligned}$$

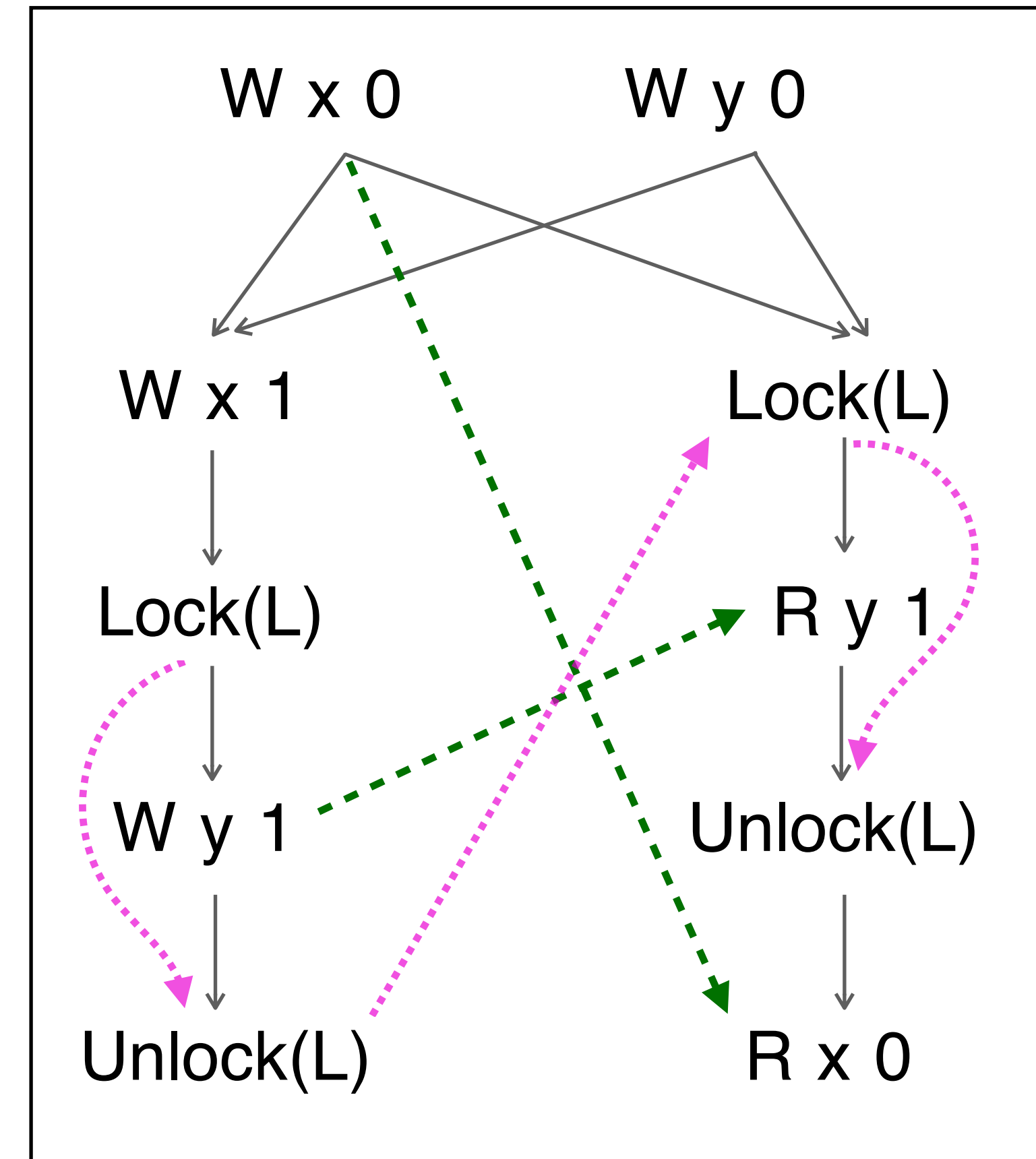
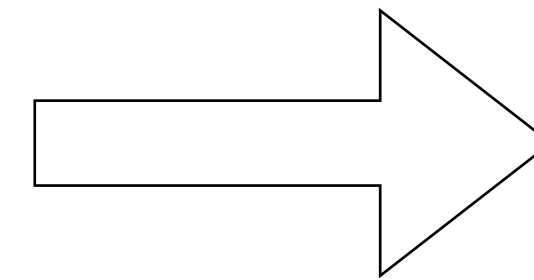
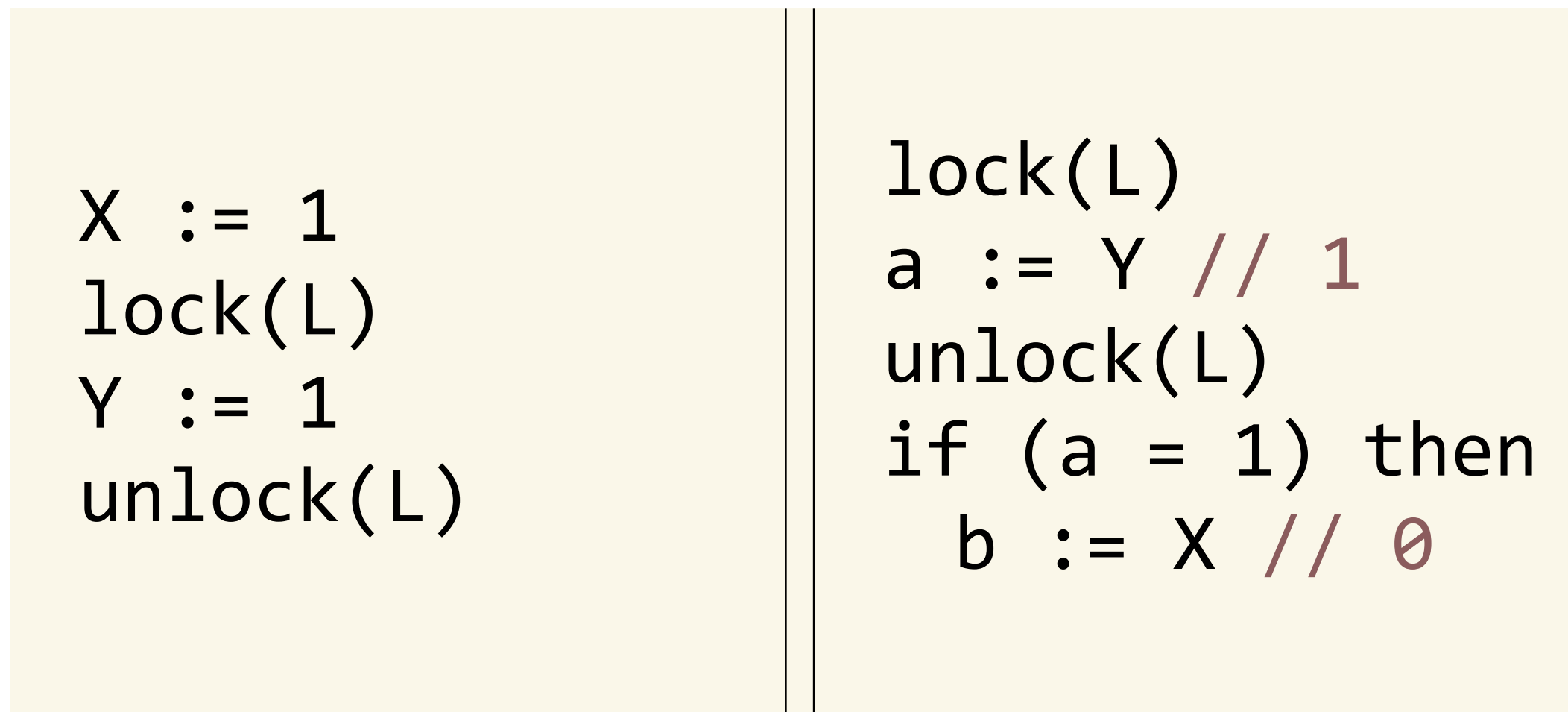
$$\begin{aligned}
\text{sb}_{\neq \text{loc}} &\triangleq \text{sb} \setminus \text{sb}_{\text{loc}} \\
\text{scb} &\triangleq \text{sb} \cup \text{sb}_{\neq \text{loc}}; \text{hb}; \text{sb}_{\neq \text{loc}} \cup \text{hb}_{\text{loc}} \cup \text{mo} \cup \text{rb} \\
\text{psc}_{\text{base}} &\triangleq ([\text{E}^{\text{sc}}] \cup [\text{F}^{\text{sc}}]; \text{hb}^?); \text{scb}; ([\text{E}^{\text{sc}}] \cup \text{hb}^?; [\text{F}^{\text{sc}}]) \\
\text{psc}_{\text{F}} &\triangleq [\text{F}^{\text{sc}}]; (\text{hb} \cup \text{hb}; \text{eco}; \text{hb}); [\text{F}^{\text{sc}}] \\
\text{psc} &\triangleq \text{psc}_{\text{base}} \cup \text{psc}_{\text{F}}
\end{aligned}$$

**Definition 1.** An execution  $G$  is called *RC11-consistent* if it is complete and the following hold:

- $\text{hb}; \text{eco}^?$  is irreflexive. (COHERENCE)
- $\text{rmw} \cap (\text{rb}; \text{mo}) = \emptyset$ . (ATOMICITY)
- $\text{psc}$  is acyclic. (SC)
- $\text{sb} \cup \text{rf}$  is acyclic. (NO-THIN-AIR)

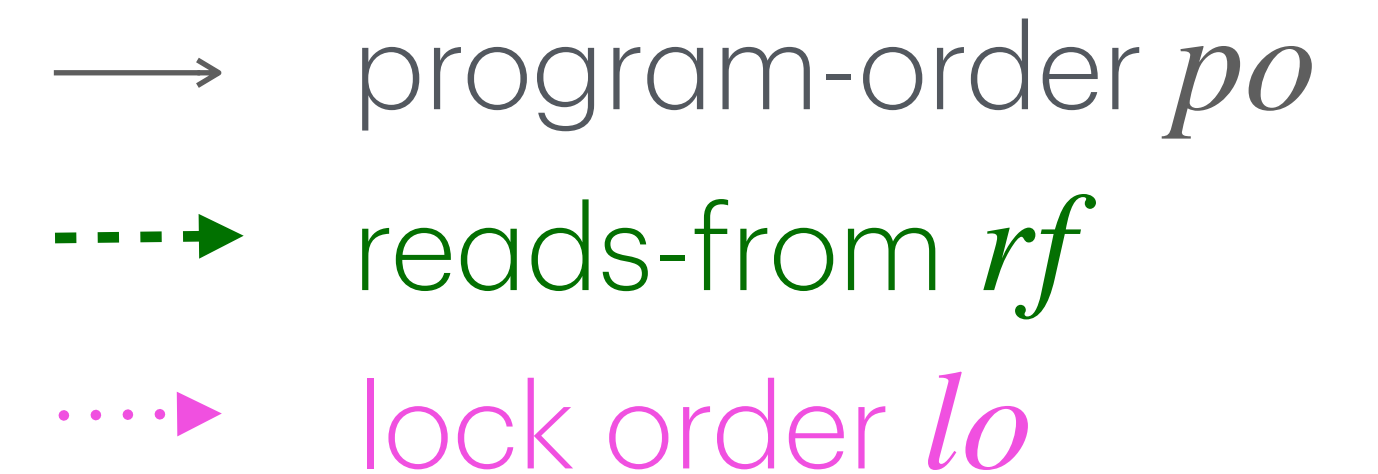
# Declarative memory models

- Possible program *behaviors* are represented by **directed graphs**

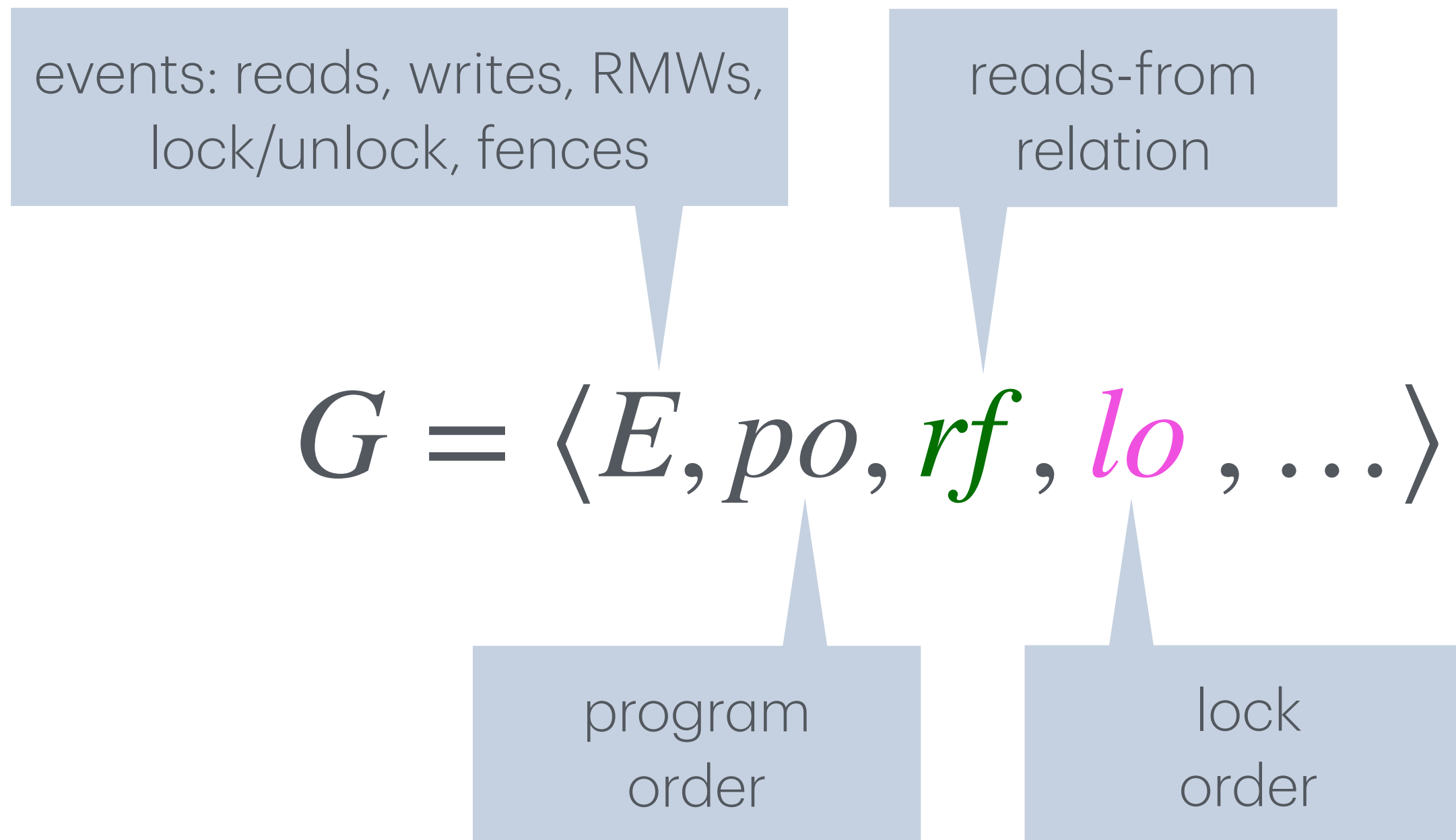


- The model defines:

- consistent** execution graphs
- racy** execution graphs



# Execution graphs



- There is a standard translation:

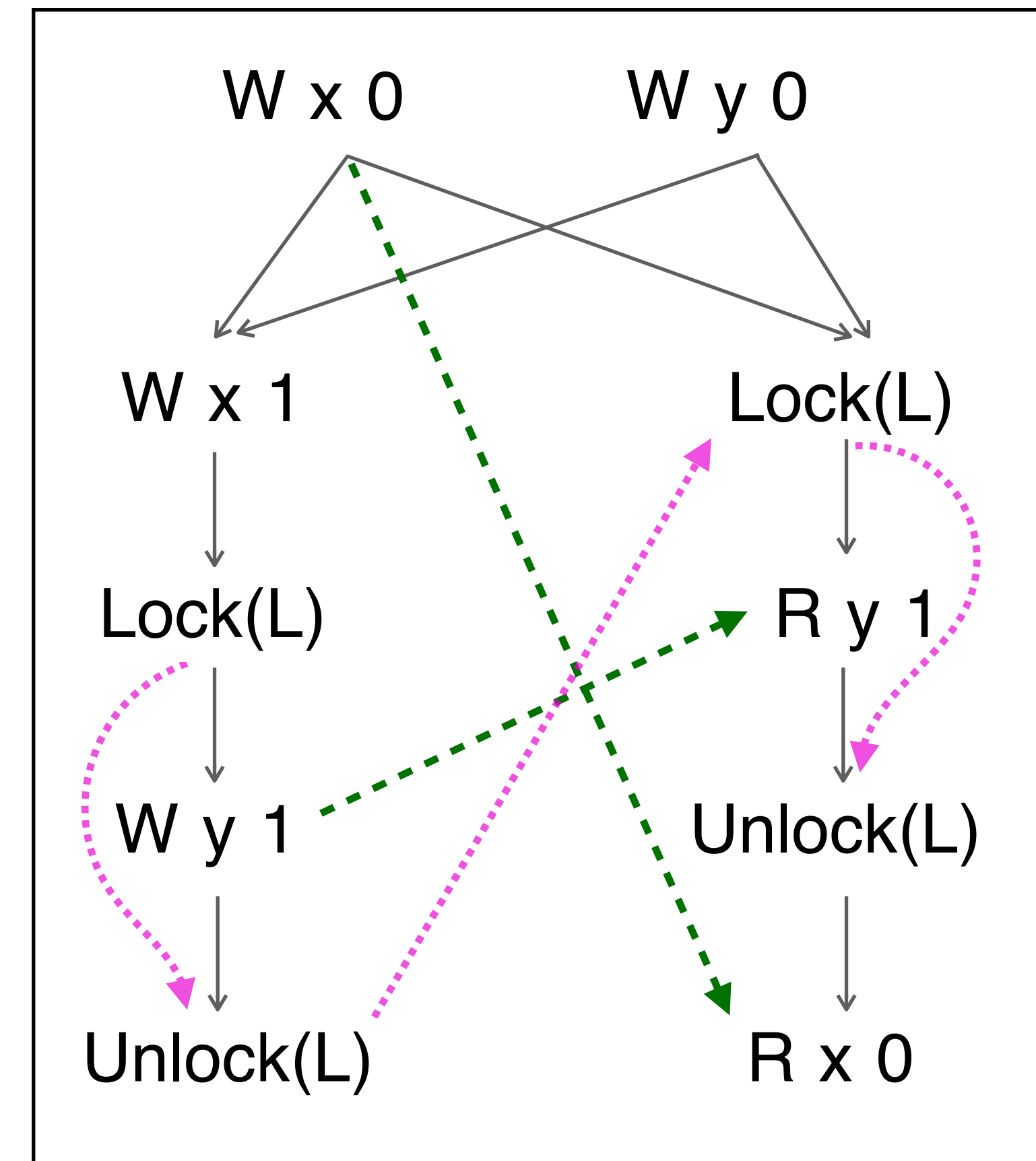
**program**  $\mapsto$  **set of candidate execution graphs**

- Read values are not constrained at this stage
- Except for *po*, relations are existentially quantified



# Well-formedness

- Program order *po*: partial order, per-thread total, initialization before everything
- reads-from *rf*: from a writing event to a reading event, value and location should match, every read reads from some write, an RMW cannot read from itself
- lock order *lo*: among lock and unlock events of the same lock, partial order, per-lock total, properly interleaved



- program-order *po*
- - - → reads-from *rf*
- · · → lock order *lo*

# Happens-before

- The most central derived relation:

$$hb = (po \cup lo \cup \dots)^+$$

transitive  
closure

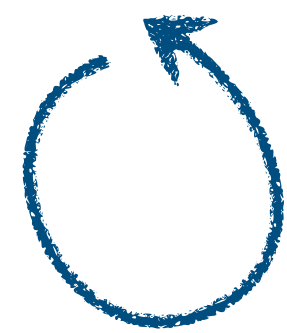
more to be  
added later

- Intuitively represents “knowledge”, “synchronization”, “causality”

# Execution-graph consistency

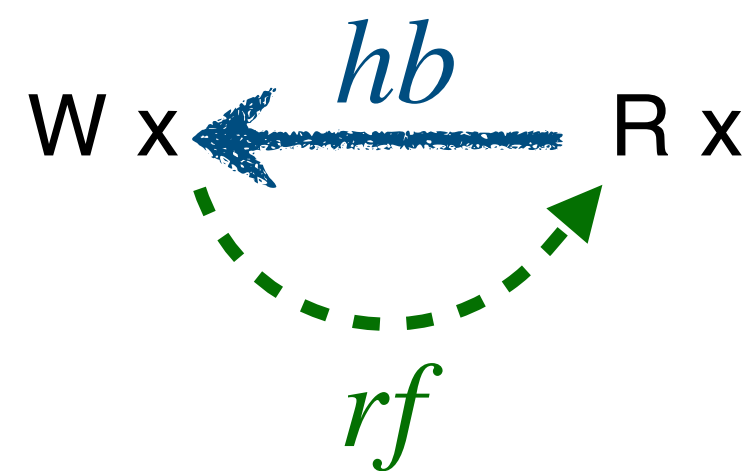
assuming only non-atomics & locks

The following patterns should never occur:



*hb*

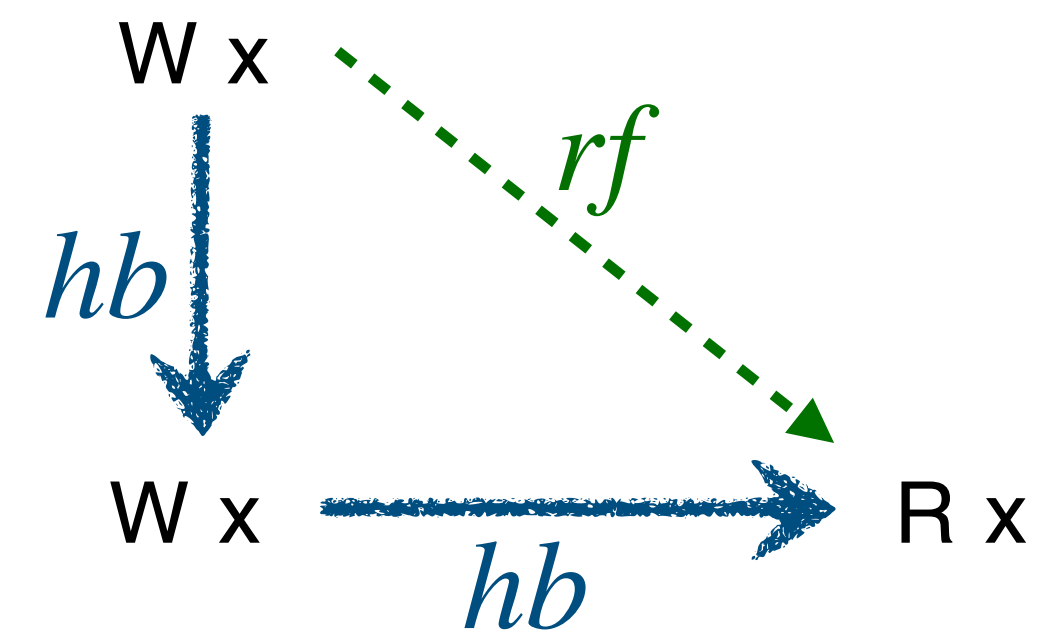
*hb* should be irreflexive



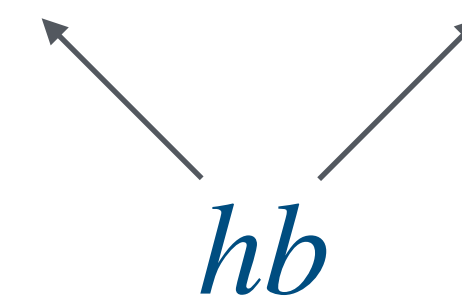
no reads from later writes



*hb*



a thread may not read from a write if it is aware of a later write to the same variable



*hb*

```

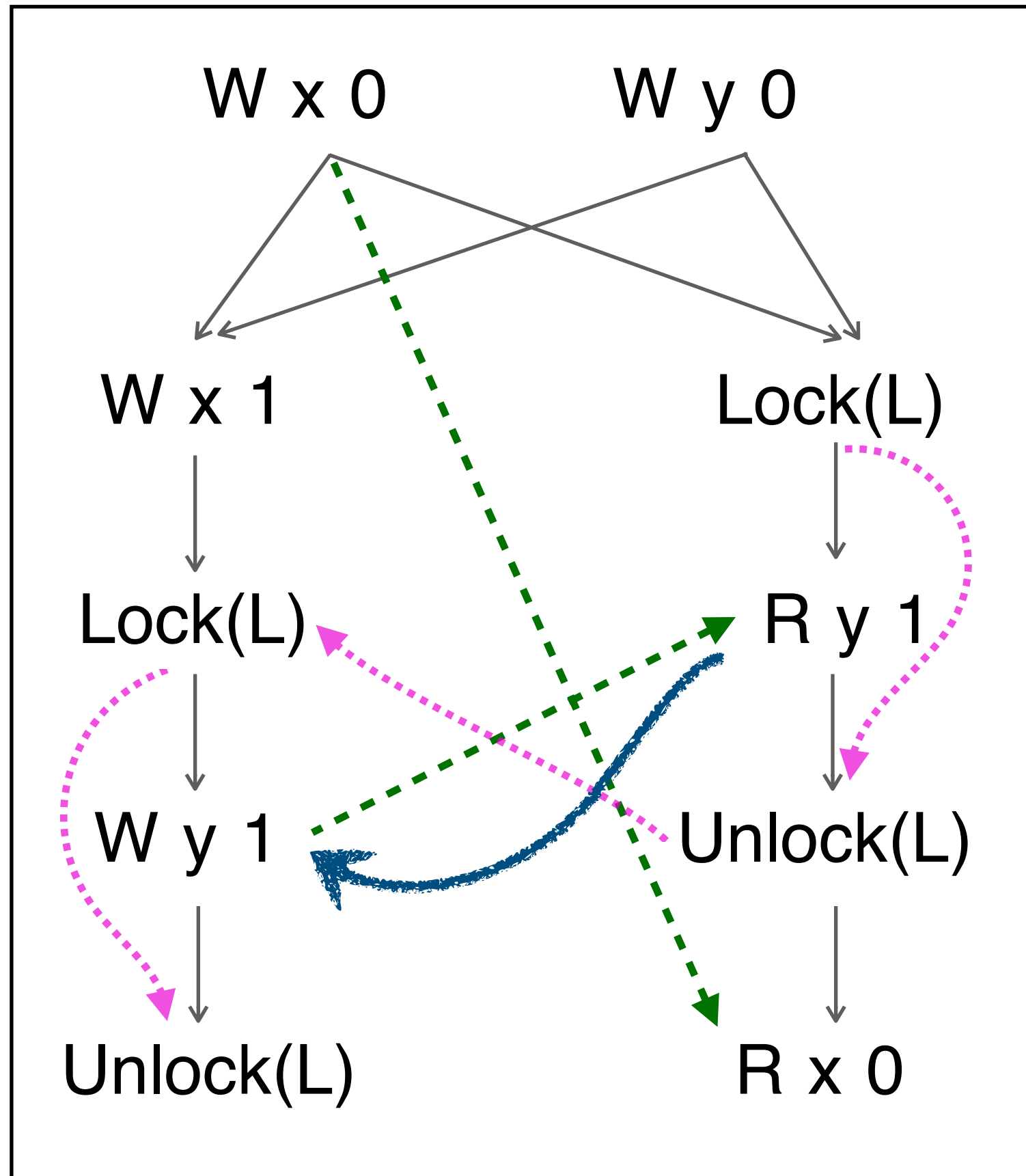
X := 1
lock(L)
Y := 1
unlock(L)

```

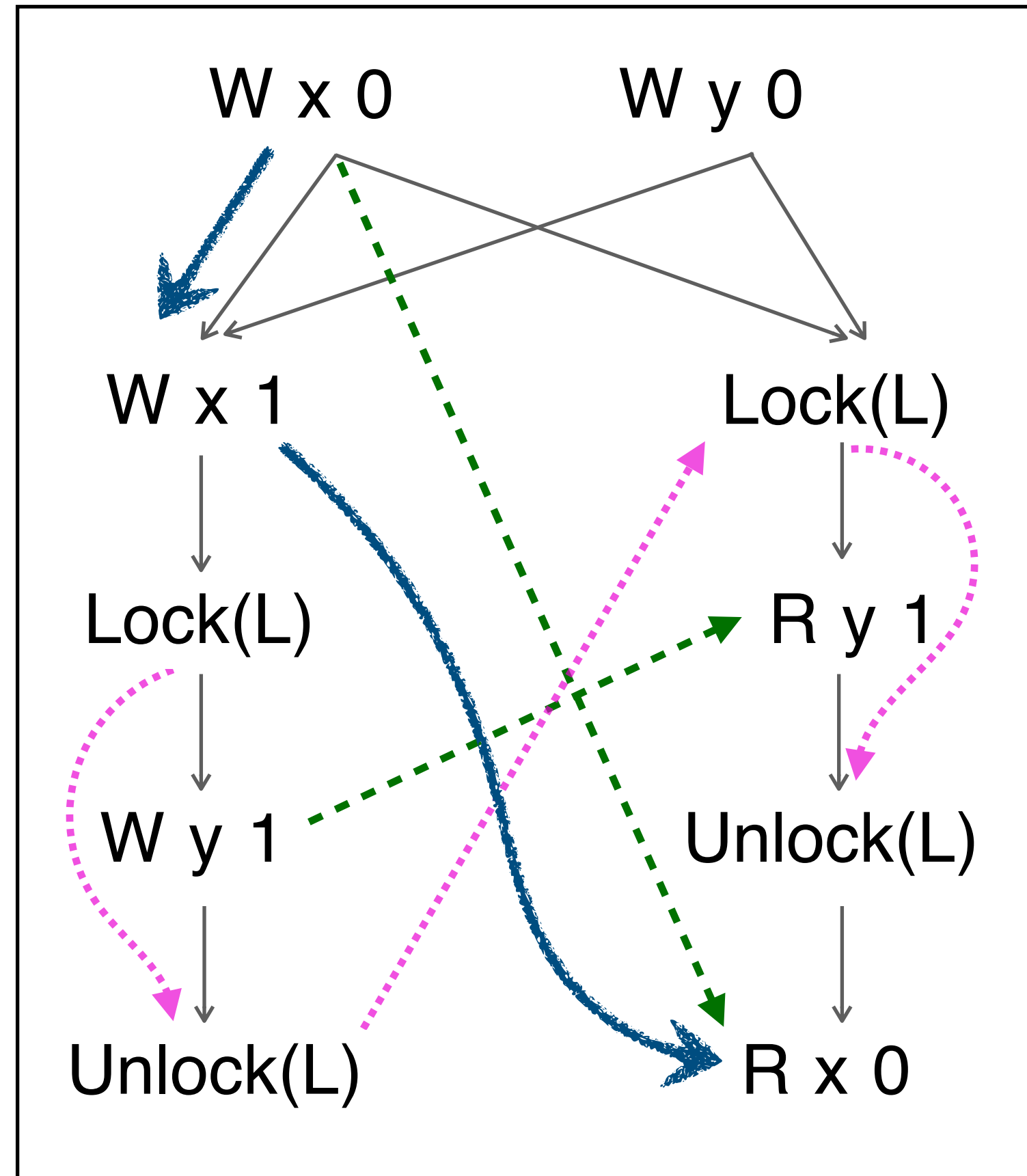
```

lock(L)
a := Y
unlock(L)
if (a = 1) then
  b := X

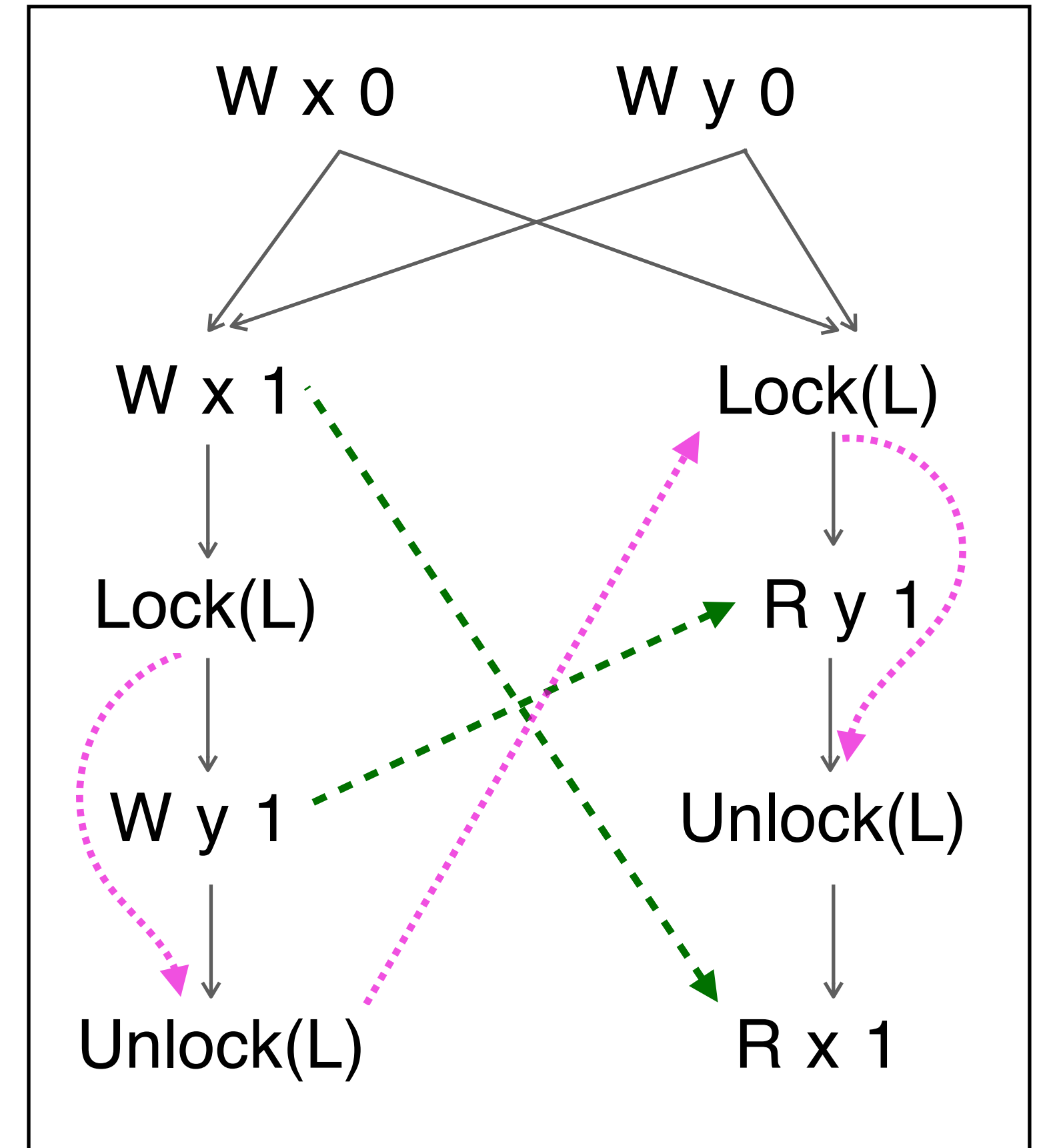
```



inconsistent



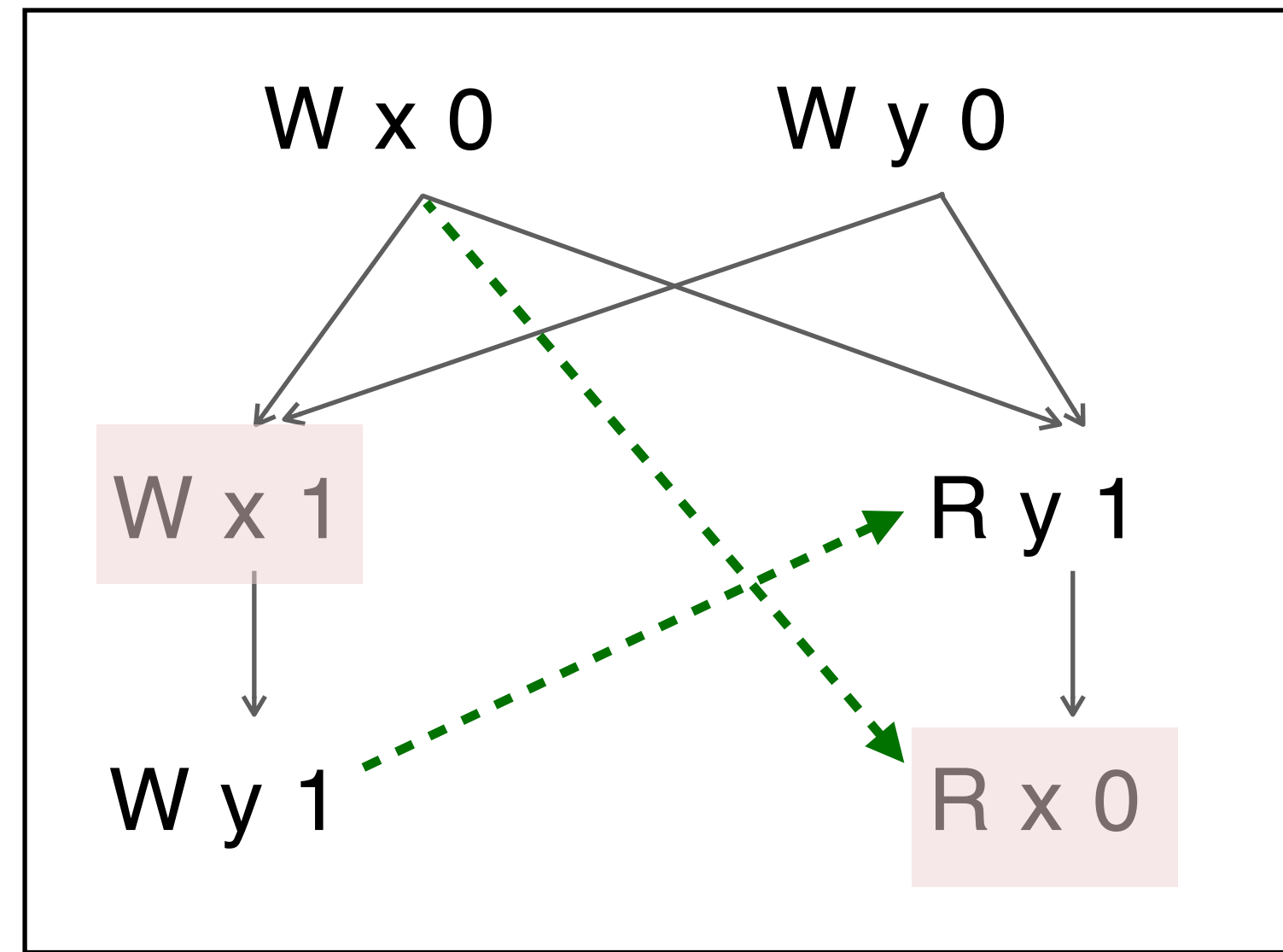
inconsistent



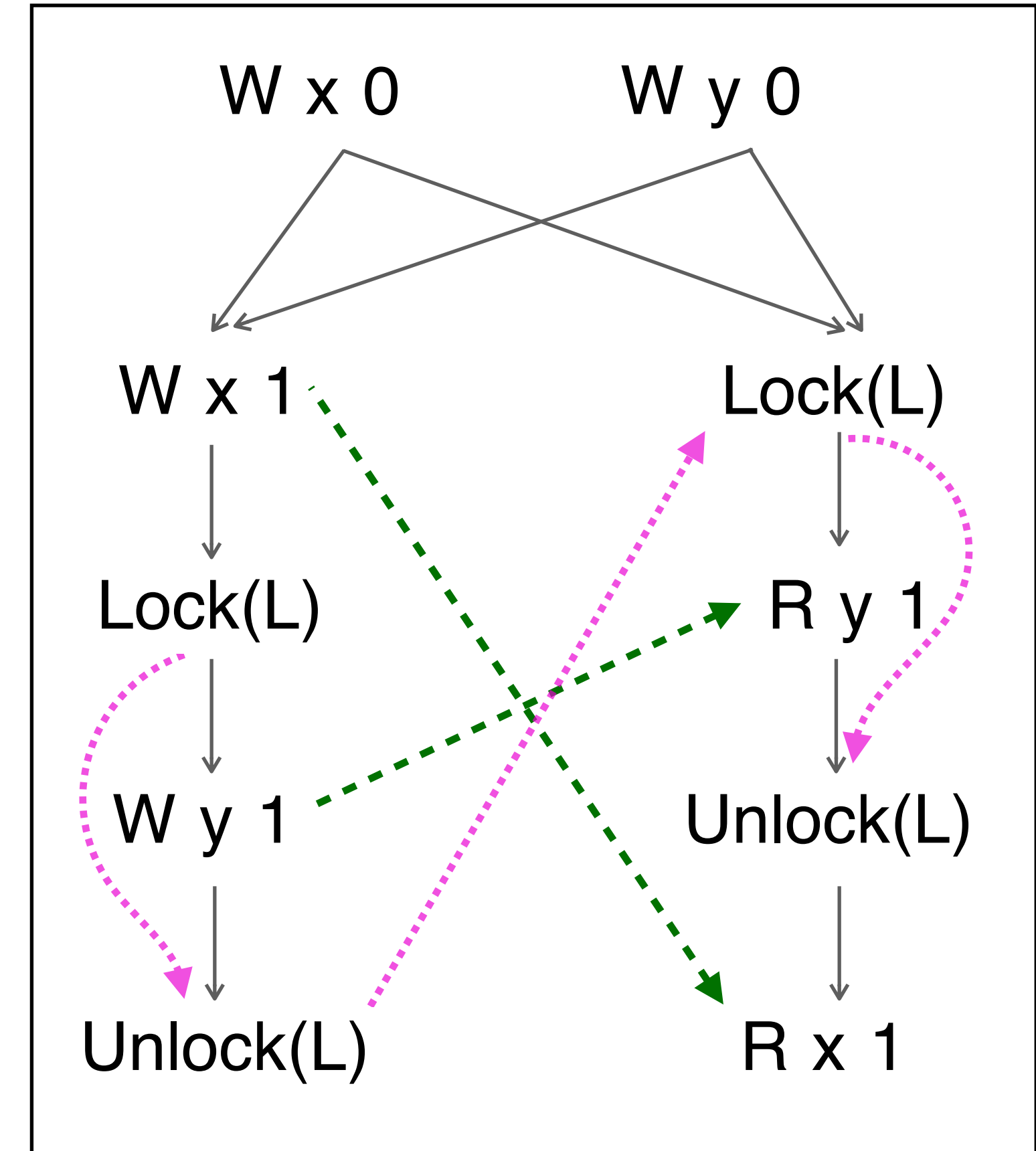
consistent

# Data-races

- Two events are *conflicting*:
  - access the **same location**
  - at least one is a **write**



racy



not racy

- A *data-race* = two **conflicting** events:
  - at least one is **non-atomic**
  - unordered by *hb*

# Allowed behaviors

A behavior of a program is *allowed* if *one* of the following holds:

- It is obtained by some *consistent execution graph* of the program
  - Some consistent execution graph of the program has a *data race*
- 
- Side note: *What is a behavior?*
    - Often taken to be the final values of the local variables
    - But, there are other options...



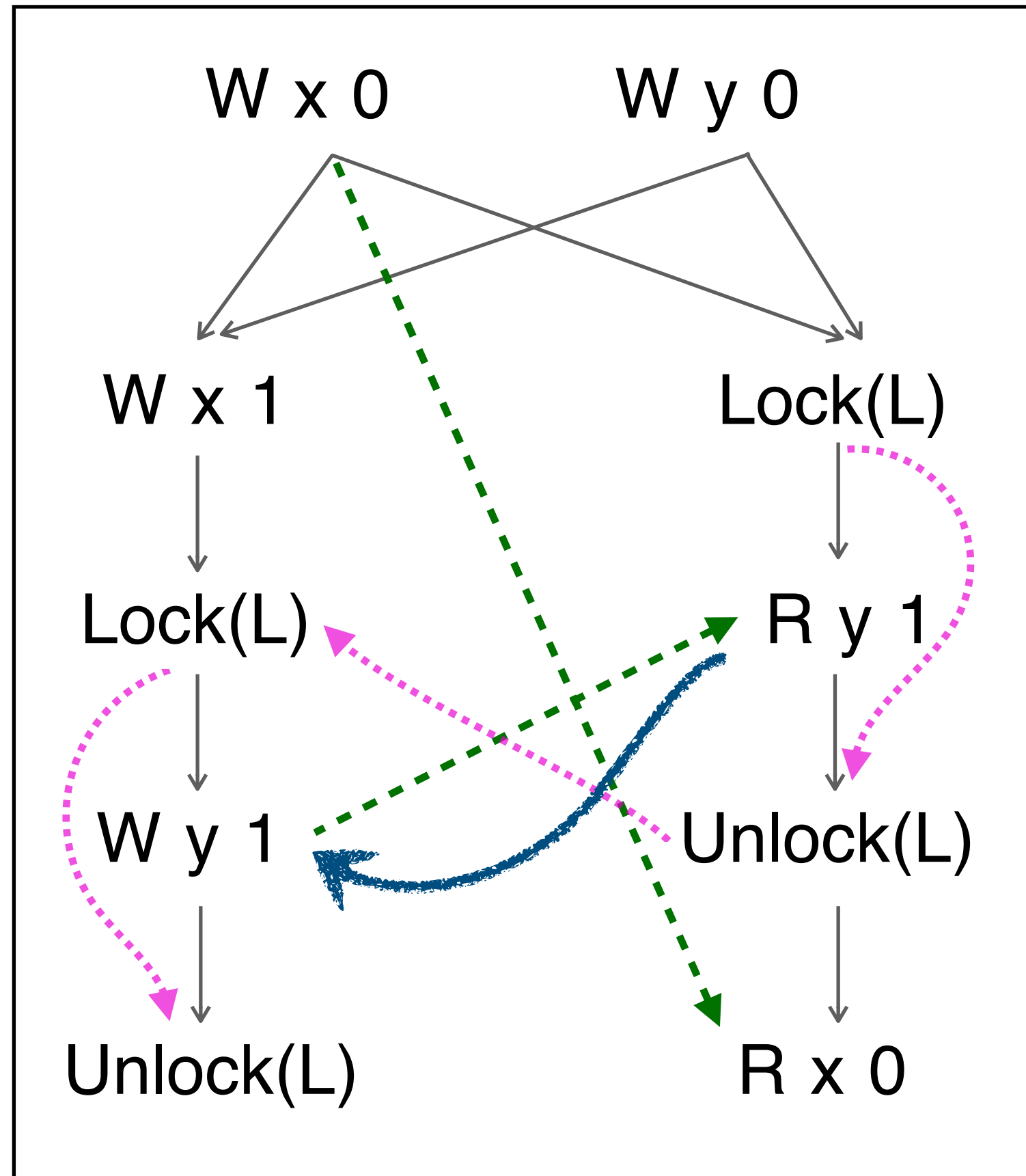
catch-fire

# Example

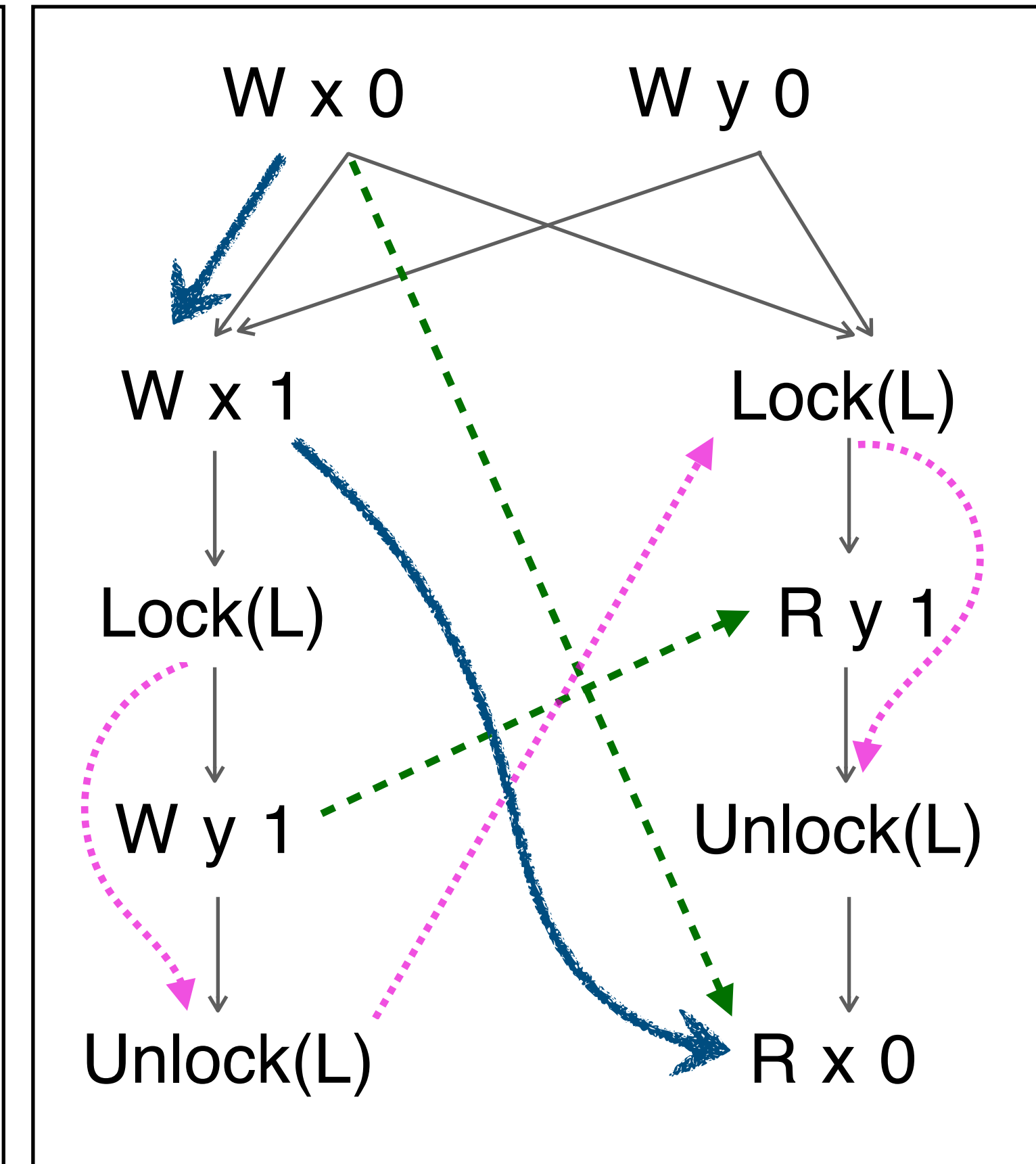
```
X := 1  
lock(L)  
Y := 1  
unlock(L)
```

```
lock(L)  
a := Y // 1  
unlock(L)  
if (a=1) then  
  b := X // 0
```

behavior  $a=1$  &  $b=0$  disallowed



inconsistent



inconsistent

+ no consistent graph of this program is racy

# Atomic accesses



relaxed  release/acquire  sc

- All atomics guarantee **coherence**
  - accesses **to each location** are in a total order (that extends *hb*) where each read reads from the last write
- Release/Acquire enforce **synchronization**
  - via another case in the definition of *hb*
- SC accesses ensure a **global** total order among them



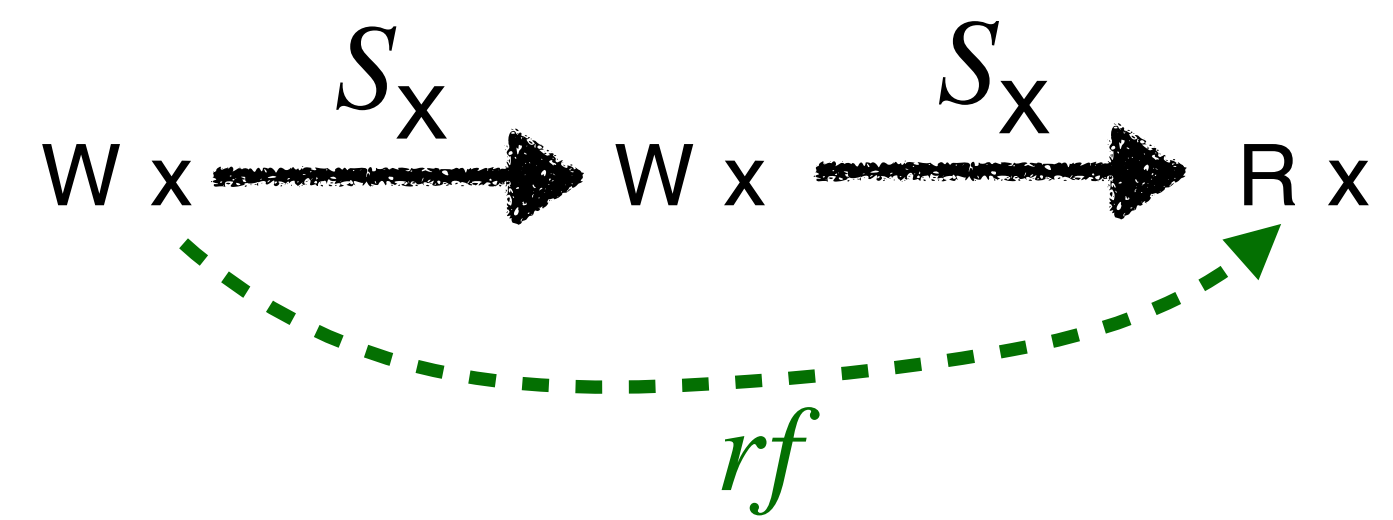
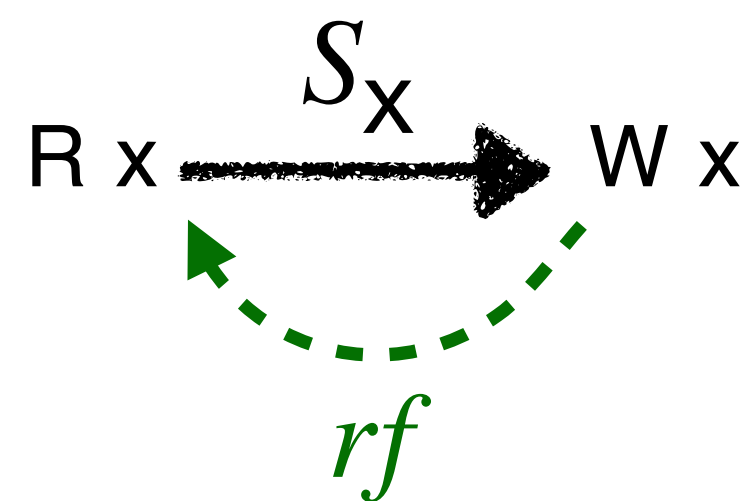
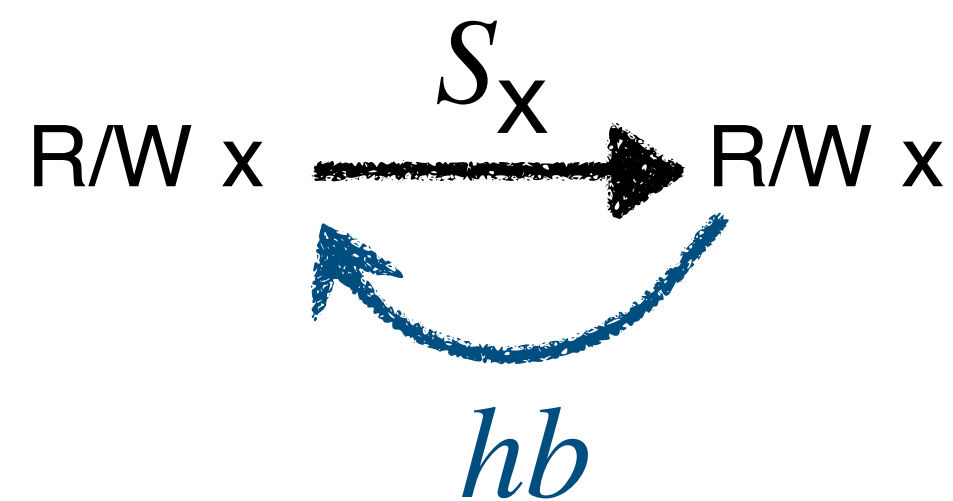
# Coherence guarantees for atomics

aka SC-per-location

For every location  $X$ , there exists a relation  $\mathcal{S}_X$  such that:

- $\mathcal{S}_X$  is a total order on all accesses to  $X$
- $\mathcal{S}_X$  contains *hb* when restricted to accesses to  $X$
- *rf* relates every read  $r$  from  $X$  to the  $\mathcal{S}_X$ -maximal write that is  $\mathcal{S}_X$ -before  $r$

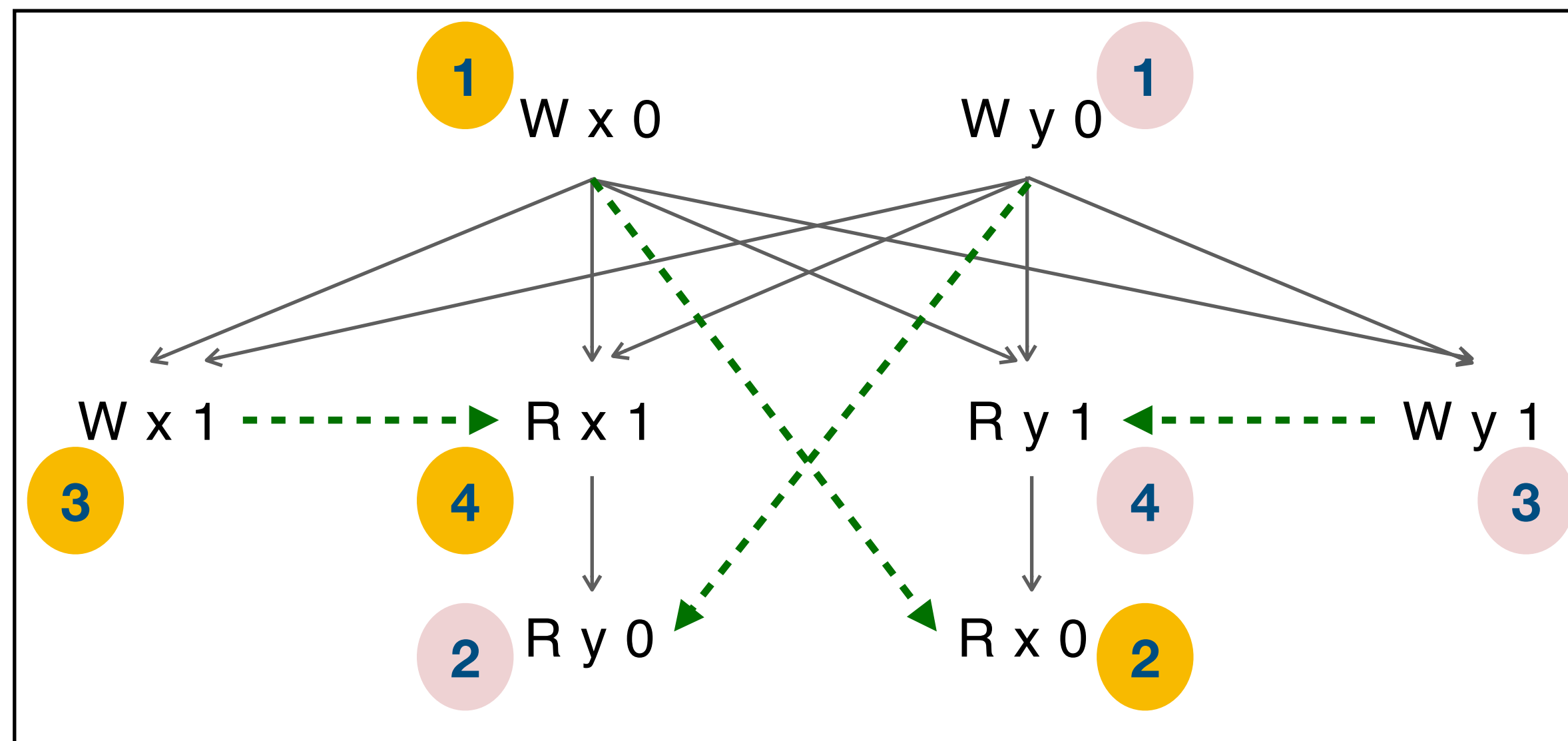
disallowed  
patterns:





# Example: IRIW (independent-reads-independent-writes)

<code>X := 1 rlx</code>	<code>a := X rlx // 1</code> <code>b := Y rlx // 0</code>	<code>c := Y rlx // 1</code> <code>d := X rlx // 0</code>	<code>Y := 1 rlx</code>
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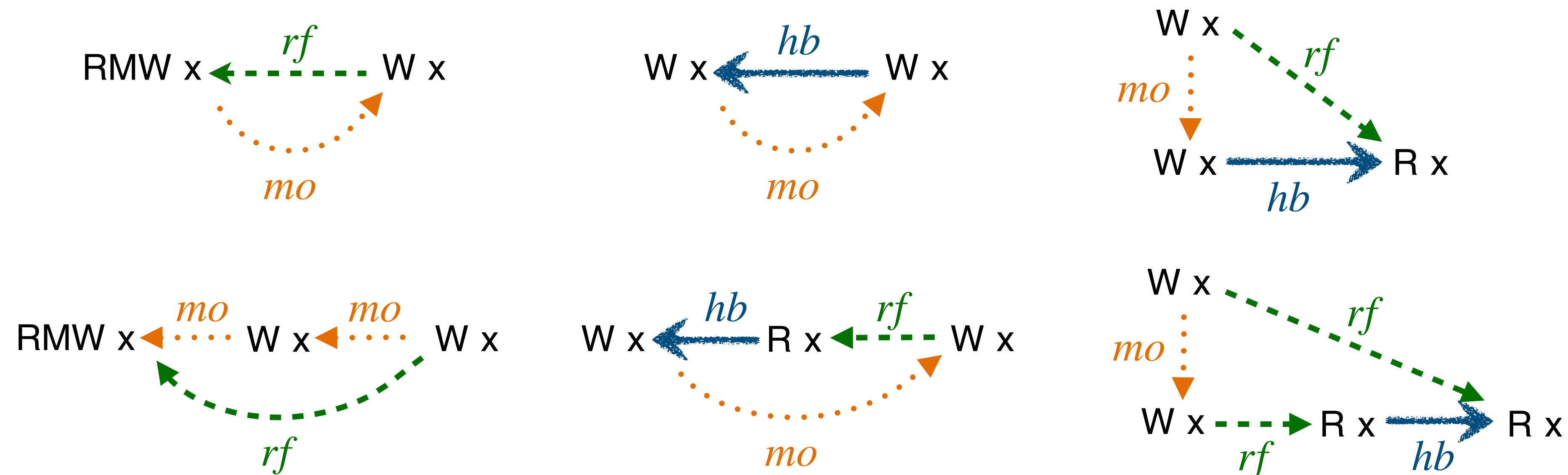


→ program-order *po*  
 - - - → reads-from *rf*

coherence allows this behavior

# Alternative formulation of coherence

- include *modification order* (aka *coherence order*) in execution graphs:  $G = \langle E, po, rf, lo, mo, \dots \rangle$ 
  - $mo = \cup_x mo_x$  where each  $mo_x$  is a total order on all **writes** to X
- forbid the following six patterns:



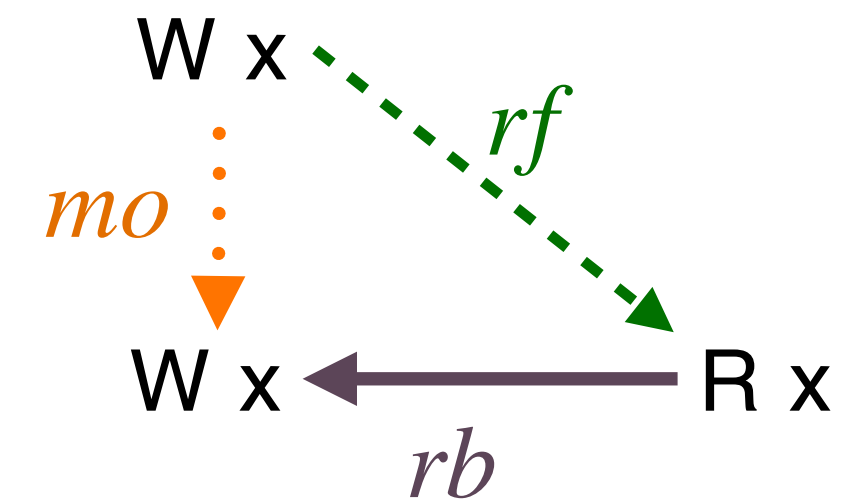
- This is equivalent to the previous formulation with a total order on all accesses to X

# A more concise formulation

- Reads-before (aka from-read) relation:

$$rb = (rf^{-1} ; mo) \setminus id$$

relation  
composition



- Coherence:

$$\text{acyclic}(hb \mid_{\text{same-location}} \cup rf \cup mo \cup rb)$$

- Compare to a standard declarative formulation of SC:  $\text{acyclic}(po \cup rf \cup mo \cup rb)$

# Another concise formulation

- Extended coherence relation:

$$eco = (rf \cup mo \cup rb)^+ = rf \cup mo \cup rb \cup (mo ; rf) \cup (rb ; rf)$$

- Coherence:

$eco ; hb^?$  is irreflexive

- Compare to another standard declarative formulation of SC:  $acyclic(eco \cup hb)$
- There is also an alternative equivalent definition that avoids  $mo$  altogether:

L, Vafeiadis: **Owicki-Gries Reasoning for Weak Memory Models**. ICALP 2015. <http://plv.mpi-sws.org/ogra/full-paper.pdf>  
(appendix B)

# Synchronization via atomic accesses



relaxed  $\square$  release/acquire  $\square$  sc



- Release/Acquire (and SC) accesses form “synchronization edges”:

$$sw = rf \cap (W^{\exists rel} \times R^{\exists acq})$$

$$hb = (po \cup lo \cup sw \cup \dots)^+$$

- The full definition of  $sw$  is more involved, allowing more synchronization patterns:
  - using relaxed accesses + release/acquire fences
  - using “release sequences” (definition was changed in C++20)

# Examples: MP (message passing)

1

```
int Y = 0
int X = 0

Y := 42
X := 1

a := X
if (a=1) then
  b := Y
```

3

```
int Y = 0
atomic<int> X = 0

Y := 42
X := 1 rel

a := X acq
if (a=1) then
  b := Y
```

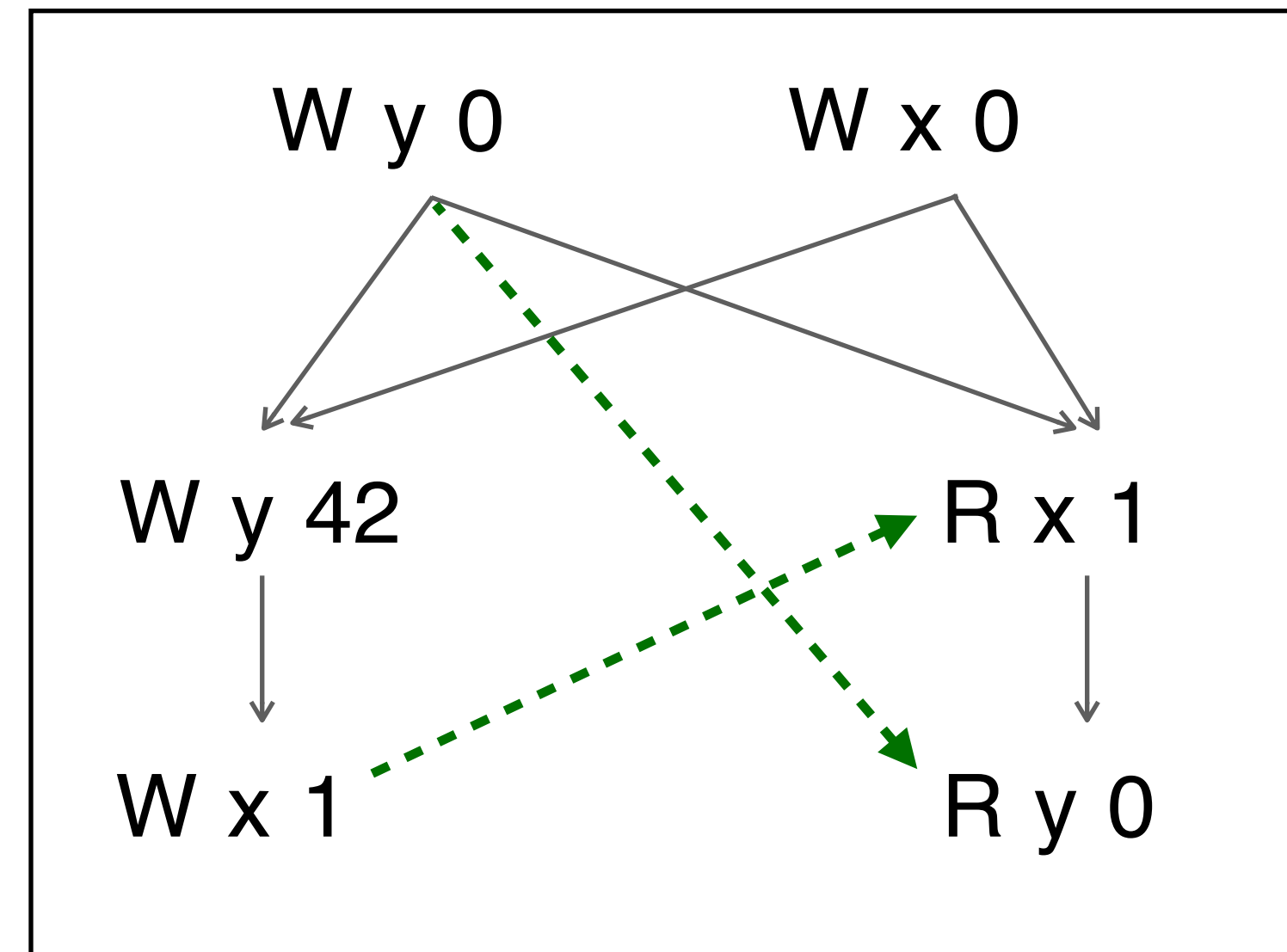
2

```
int Y = 0
atomic<int> X = 0

Y := 42
X := 1 rlx

a := X rlx
if (a=1) then
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```

behavior  $a=1$  &  $b=0$  allowed?





# Examples: MP (message passing)

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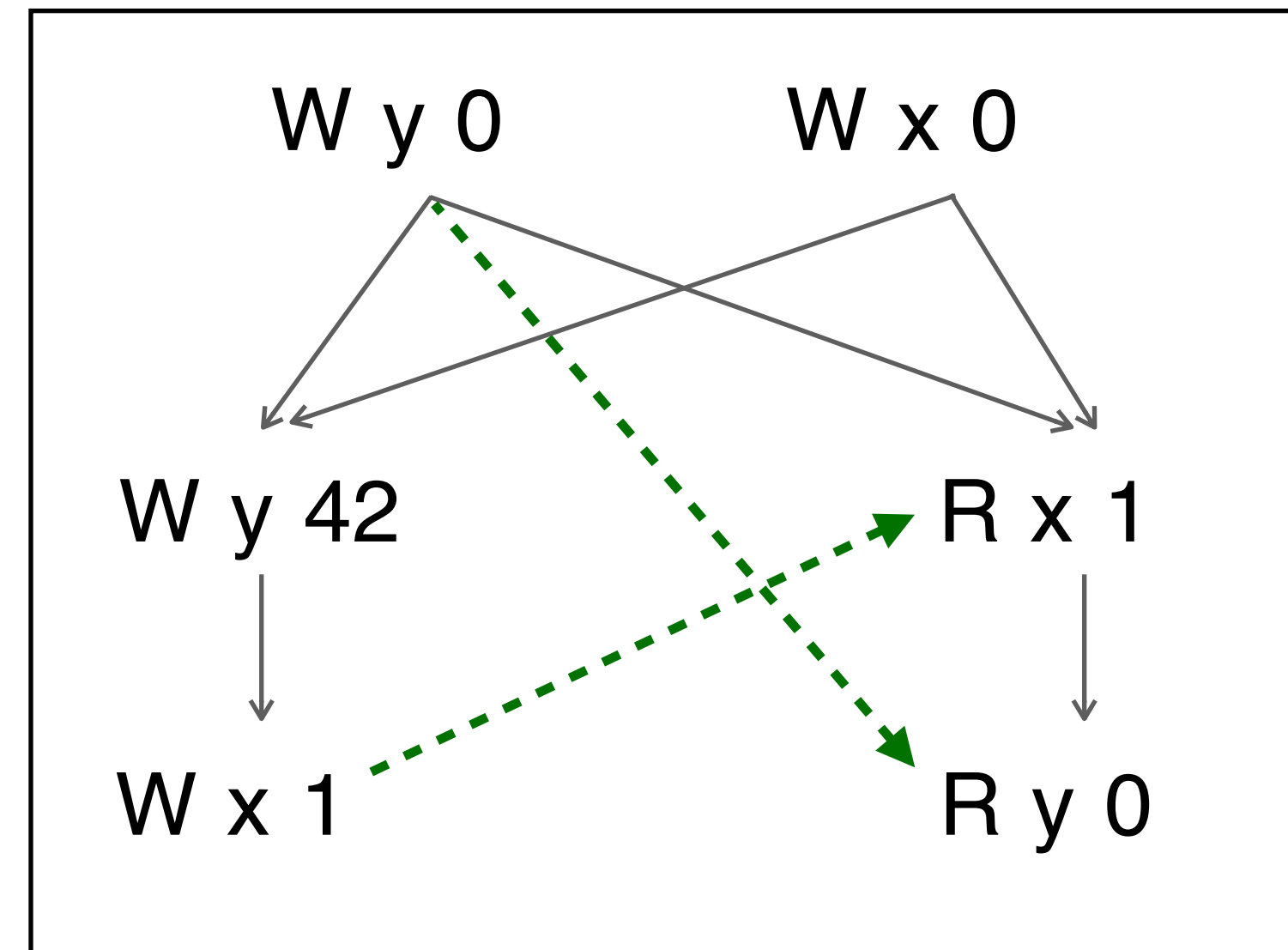
2

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atomic<int> X = 0

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a := X rlx
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behavior  $a=1$  &  $b=0$  allowed?



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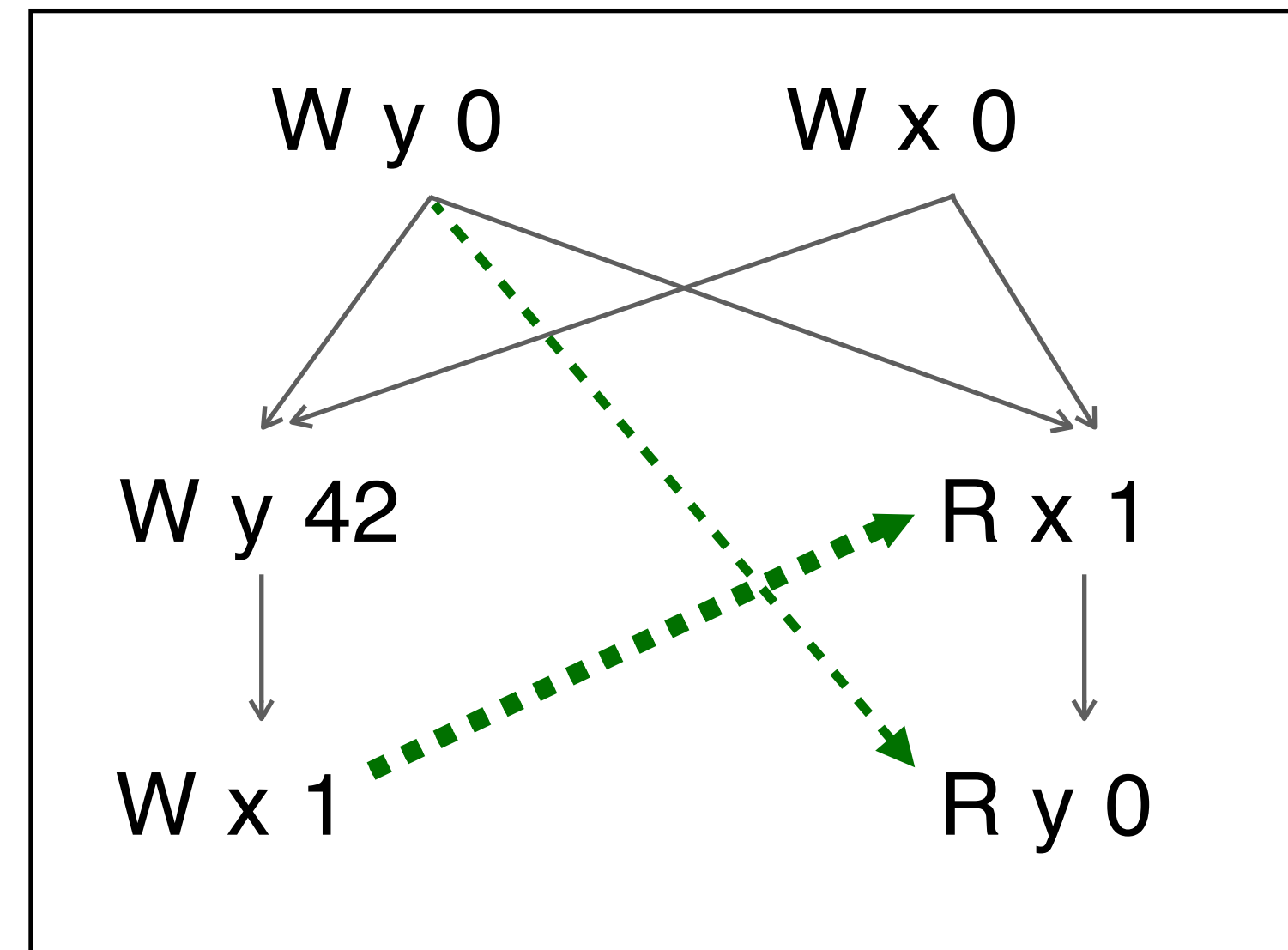
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if (a=1) then
  b := Y
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X := 1 rel

a := X acq
if (a=1) then
  b := Y
```

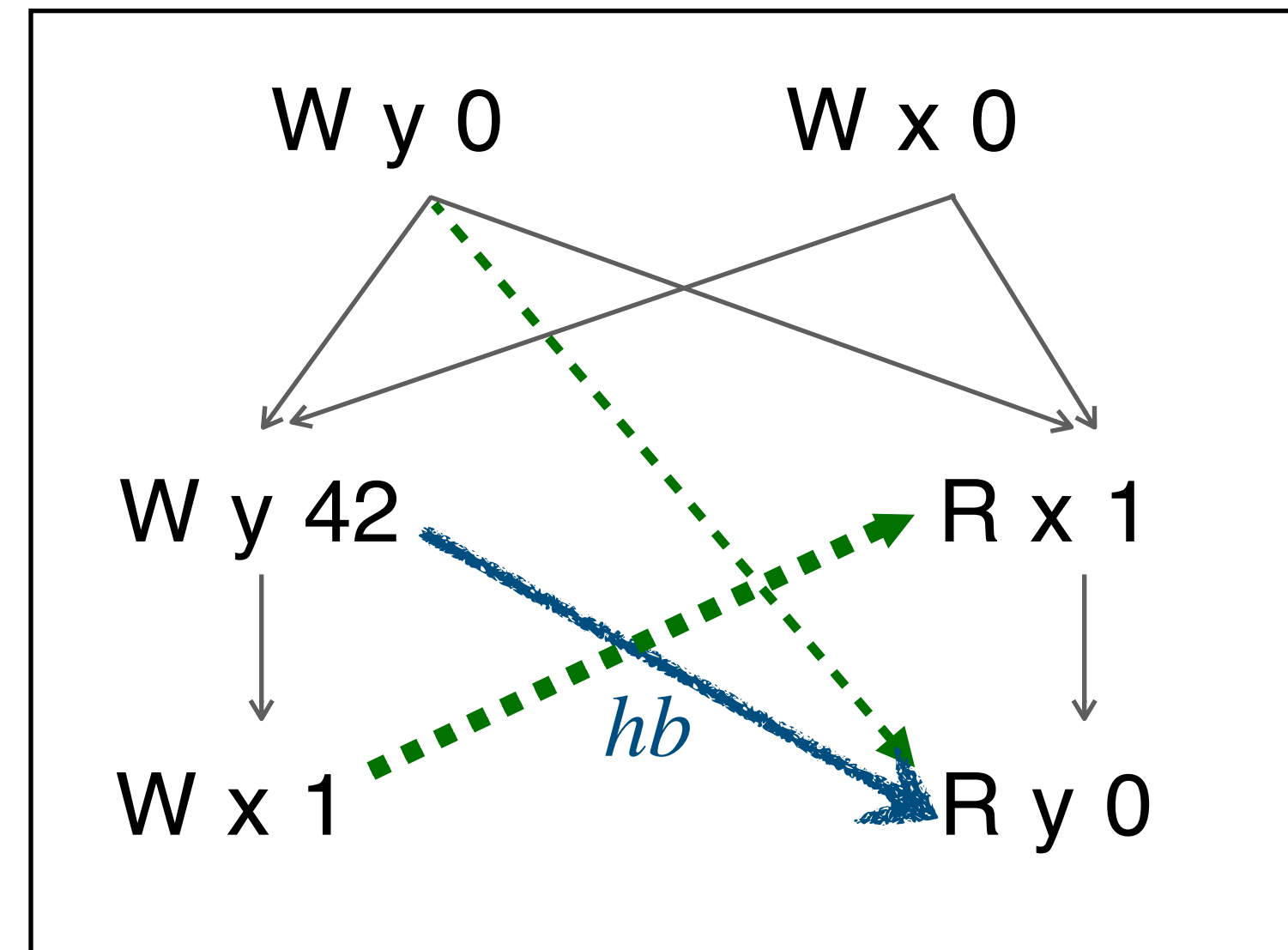
2

```
int Y = 0
atomic<int> X = 0

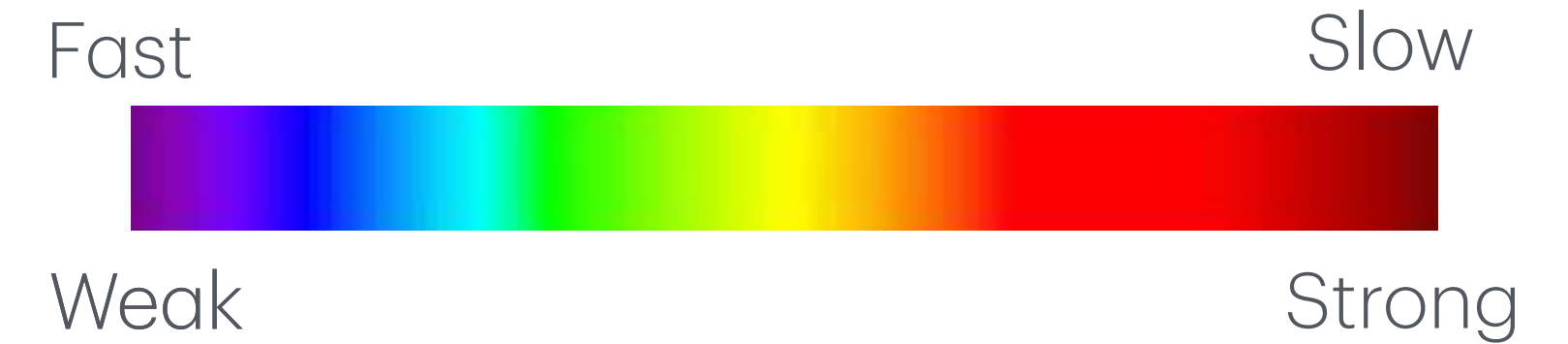
Y := 42
X := 1 rlx

a := X rlx
if (a=1) then
  b := Y
```

behavior  $a=1$  &  $b=0$  allowed?



# SC accesses



relaxed  $\square$  release/acquire  $\square$  sc



- SC accesses can be used to provide **sequentially consistent semantics** when needed
- Roughly, there should exist **a total order *sc* on all SC accesses** in which every read reads from the last write

- The precise semantics is much more complicated

**p<sub>sc</sub>** is acyclic.

$$sb|_{\neq 1oc} \triangleq sb \setminus sb|_{1oc}$$

$$scb \triangleq sb \cup sb|_{\neq 1oc}; hb; sb|_{\neq 1oc} \cup hb|_{1oc} \cup mo \cup rb$$

$$psc_{base} \triangleq ([E^{sc}] \cup [F^{sc}]; hb^?); scb; ([E^{sc}] \cup hb^?; [F^{sc}])$$

$$psc_F \triangleq [F^{sc}]; (hb \cup hb; eco; hb); [F^{sc}]$$

$$psc \triangleq psc_{base} \cup psc_F$$

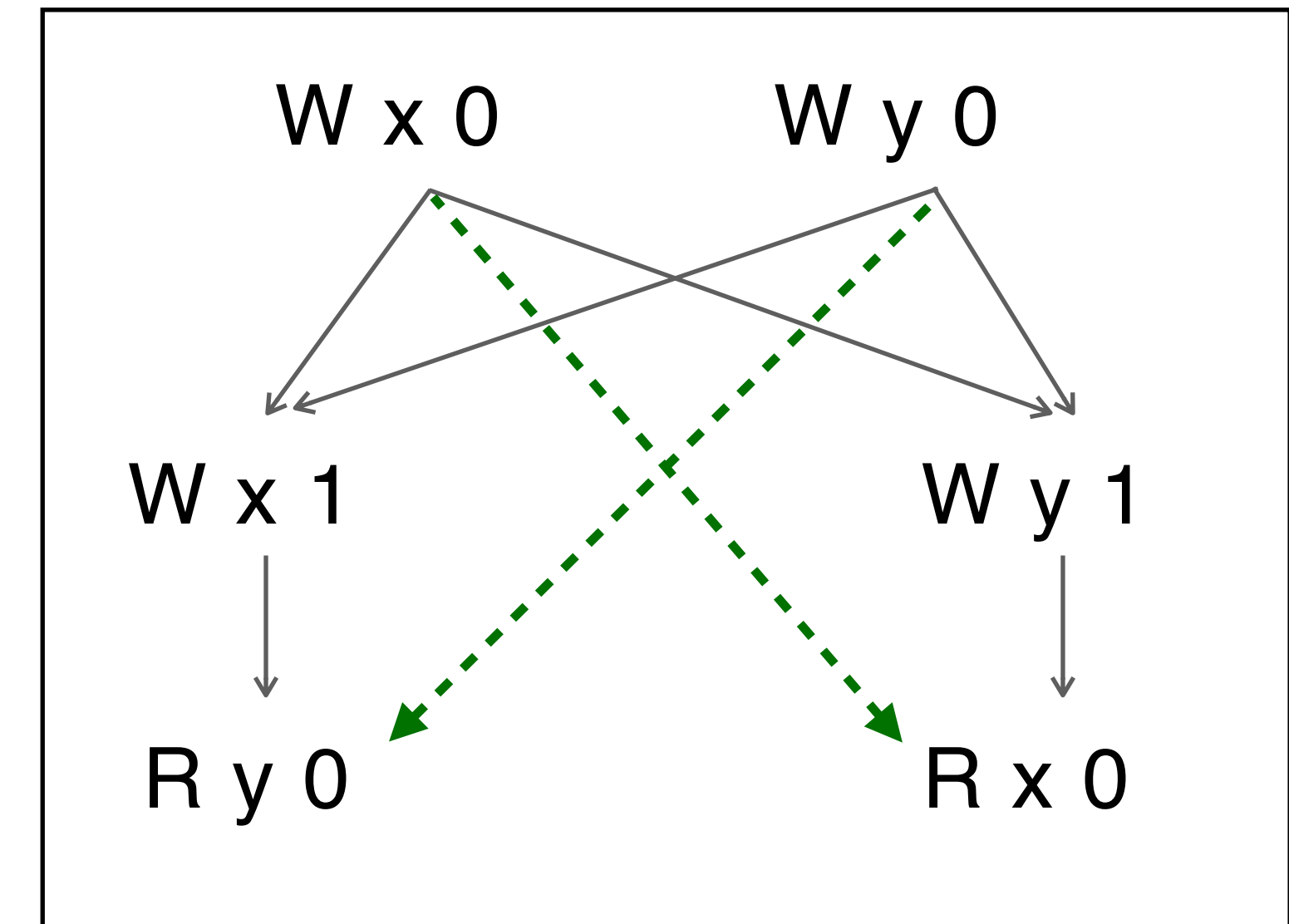
- It has been a rich source of bugs in the model, and it is currently under another revision...

# Example: SB (store buffer)

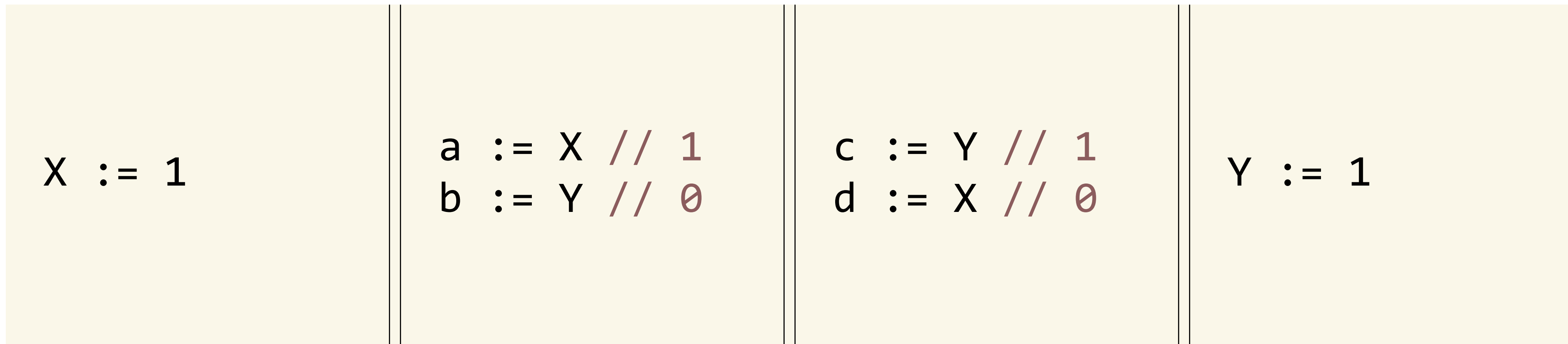
```
X := 1 sc  
a := Y sc // 0
```

```
Y := 1 sc  
b := X sc // 0
```

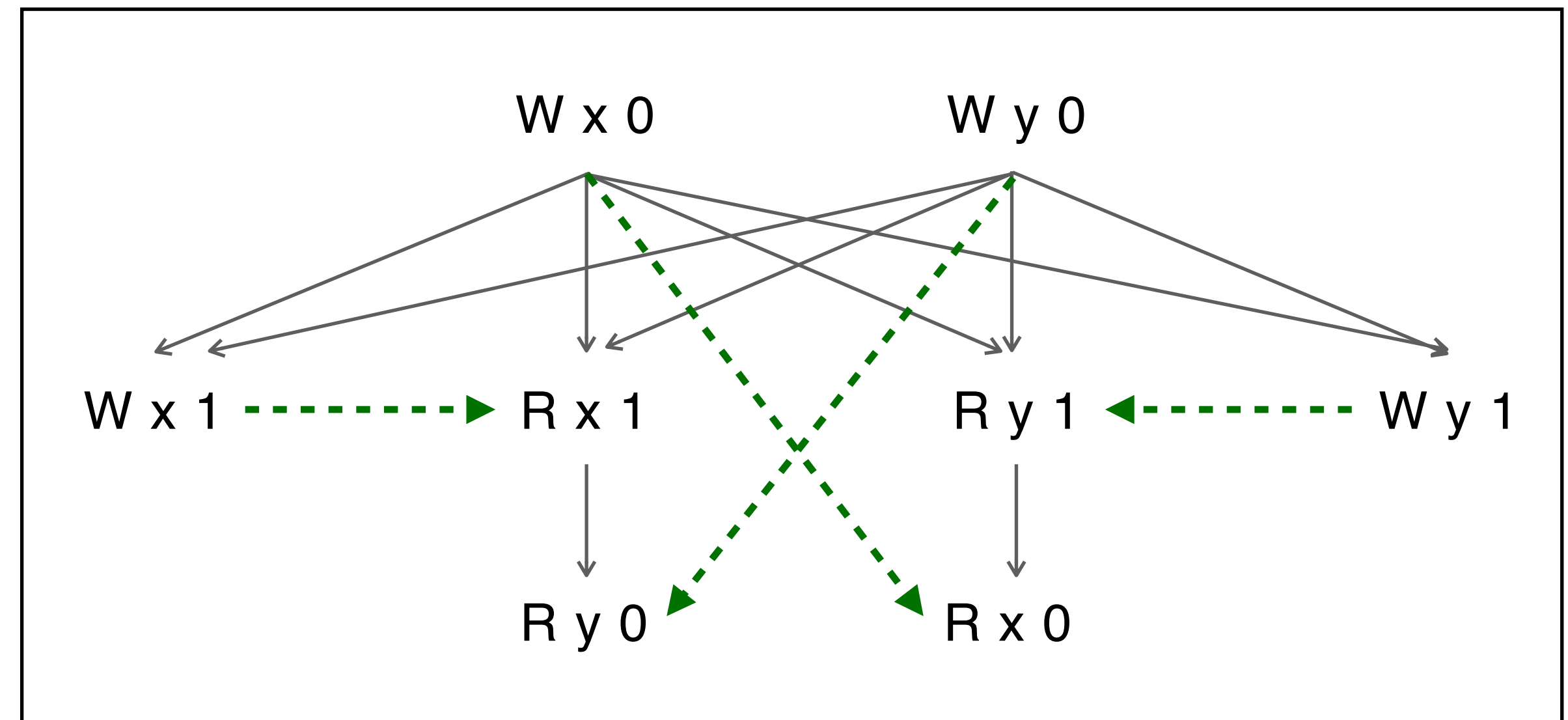
- Allowed with **release/acquire** atomic accesses
- Disallowed when all 4 accesses are **sc**



# Example: IRIW (independent-reads-independent-writes)



- Allowed with **release/acquire** atomic accesses
- Disallowed when all 6 accesses are **sc**



# Fixing SC accesses in C11

```
X := 1 sc
```

```
a := X acq // 1  
b := Y sc // 0
```

```
c := Y acq // 1  
d := X sc // 0
```

```
Y := 1 sc
```

- In the original C11 model this behavior was **disallowed** (the order **sc** had to agree with **hb**)
- But it is **allowed on POWER** multicores after compilation mapping!
- The C/C++11 was weakened in order to solve this problem

L, Vafeiadis, Kang, Hur, Dreyer: **Repairing Sequential Consistency in C/C++11**. PLDI 2017. <https://doi.org/10.1145/3140587.3062352>

# SC fences

`atomic_thread_fence(memory_order_seq_cst)`

- SC-fences provide another way to **enforce SC semantics** when needed
- Consistency essentially requires that there exists a total order  $sc_F$  on all SC fences in the graph that is a part of  $hb$ :

$$hb = (po \cup lo \cup sw \cup sc_F)^+$$



# SC fences

- Weak behaviors can be forbidden by placing SC-fences:

```
X := 1 rlx  
SC-fence  
a := Y rlx // 0
```

```
Y := 1 rlx  
SC-fence  
b := X rlx // 0
```

```
X := 1 rlx
```

```
a := X rlx // 1  
SC-fence  
b := Y rlx // 0
```

```
c := Y rlx // 1  
SC-fence  
d := X rlx // 0
```

```
Y := 1 rlx
```

- SC-fences are often preferred by expert developers, making SC accesses rather useless...
- SC-fences can be encoded as release/acquire RMWs (e.g., `FADD(F, 0)`) to a distinguished, otherwise unused location

# Recap: The C11 memory model

- **Catch-fire**: races on non-atomics  $\implies$  undefined behavior
- Relaxed atomics for racy (but non-synchronizing) accesses
- Atomics ensure coherence
- Locks and release/acquire atomics for synchronization
- SC atomics / fences for ensuring a global total order



# The out-of-thin-air problem & RC11

# The out-of-thin-air problem

- The model presented so far is *too weak*.
- Values might appear **“out-of-thin-air”**!
- For the same reason, the DRF guarantee is *broken* (we will discuss later).

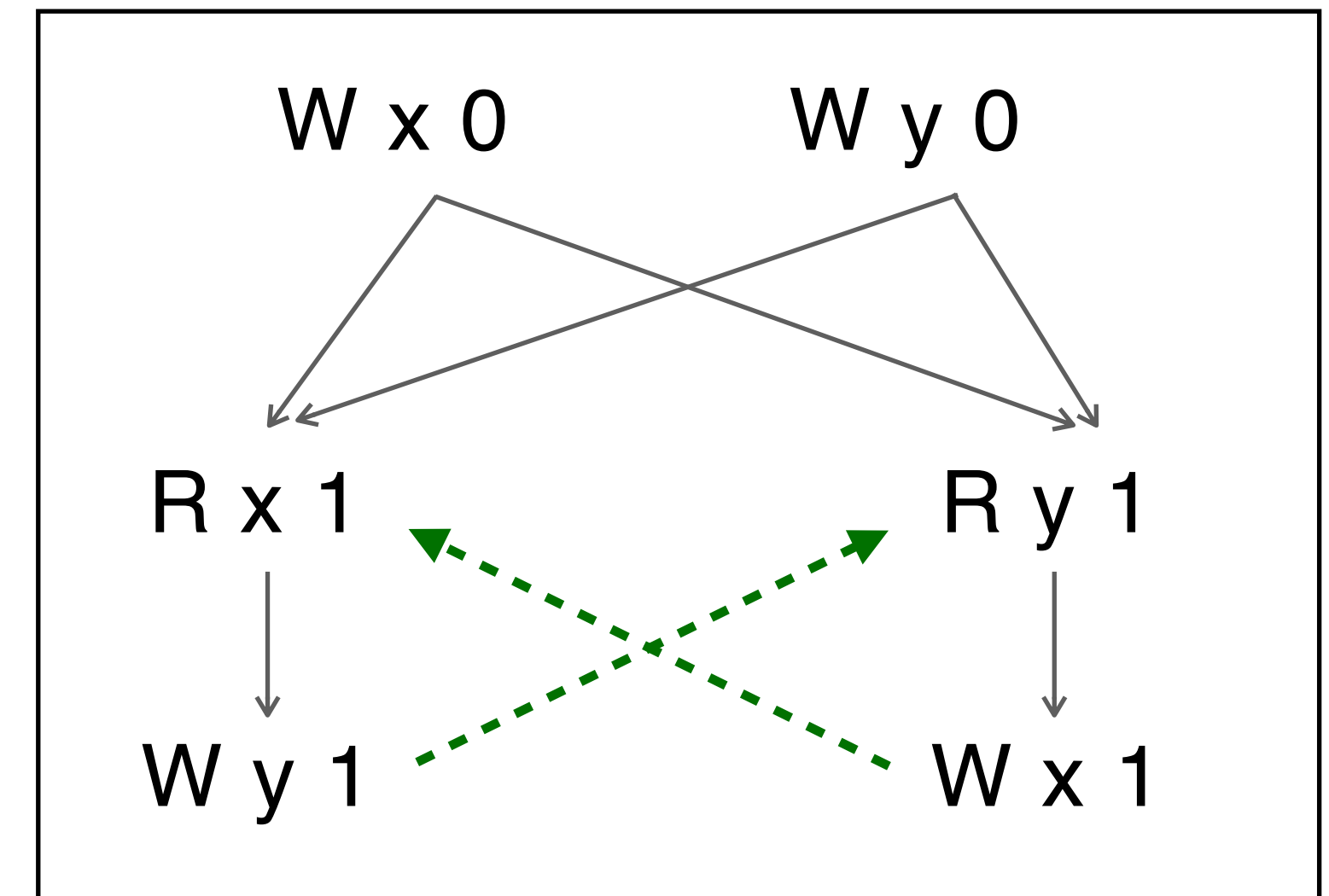
```
std::atomic<int> x = 0;
std::atomic<int> y = 0;

// Thread 1
int r1 = y.load(std::memory_order_relaxed); // A
x.store(r1, std::memory_order_relaxed); // B
// Thread 2
int r2 = x.load(std::memory_order_relaxed); // C
y.store(r2, std::memory_order_relaxed); // D
```

# Example: LB (load buffer)

```
a := X rlx // 1  
Y := 1 rlx
```

```
b := Y rlx // 1  
X := 1 rlx
```

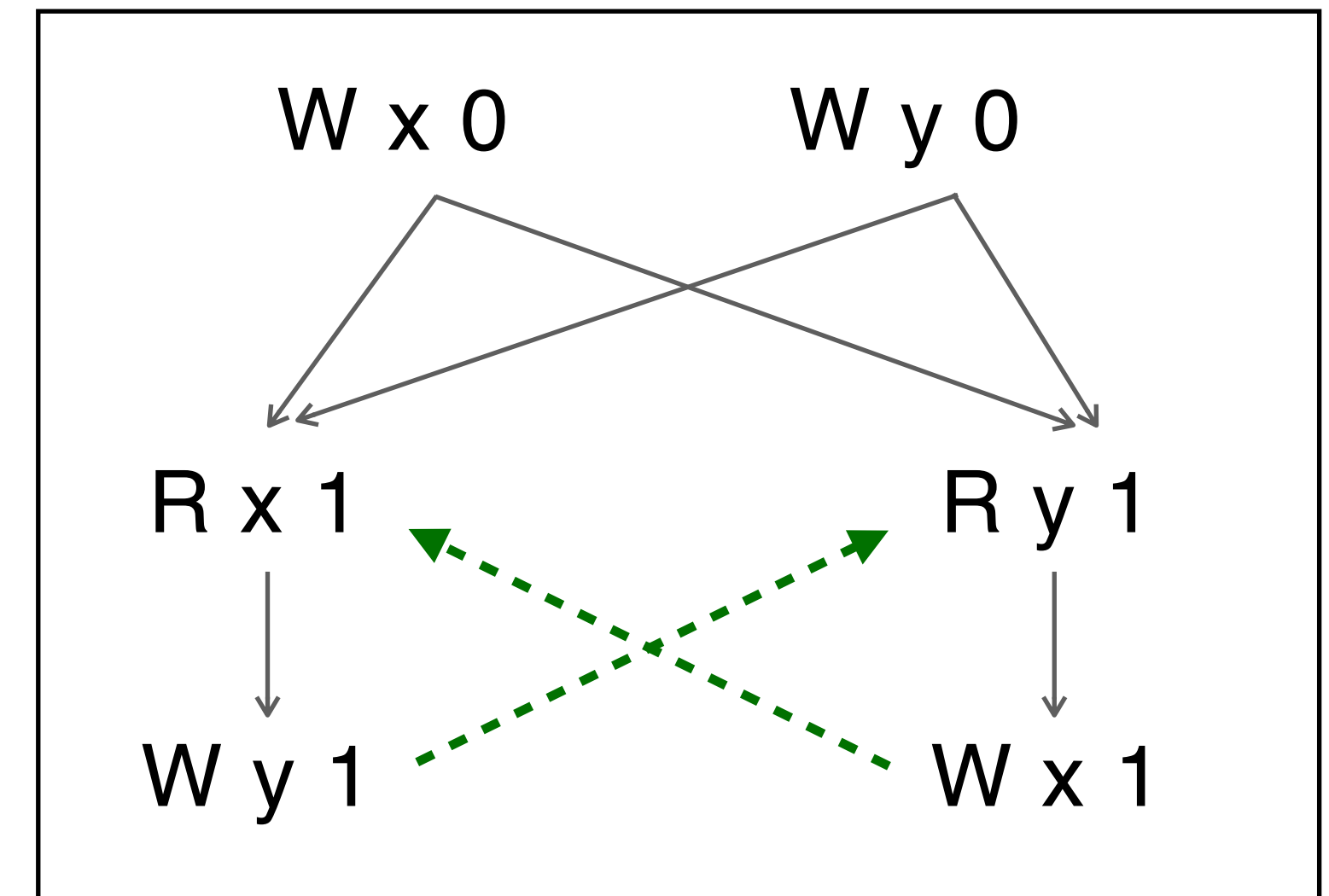


- C11 *allows* this behavior, for a good reason:
  - We want to compile relaxed accesses to *plain machine accesses*
  - *Hardware models (POWER / ARM) allow it*

# Example: LB (load buffer)

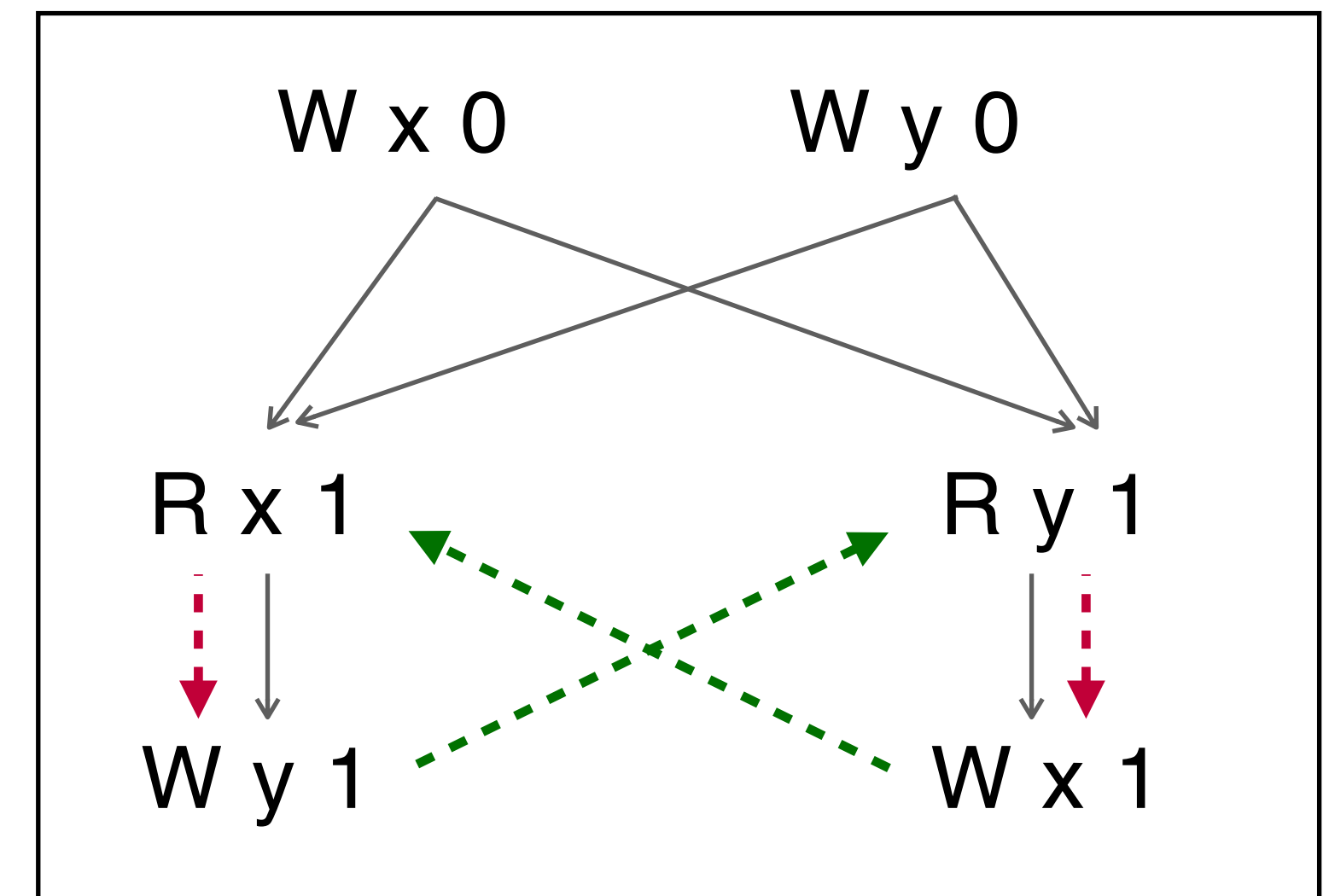
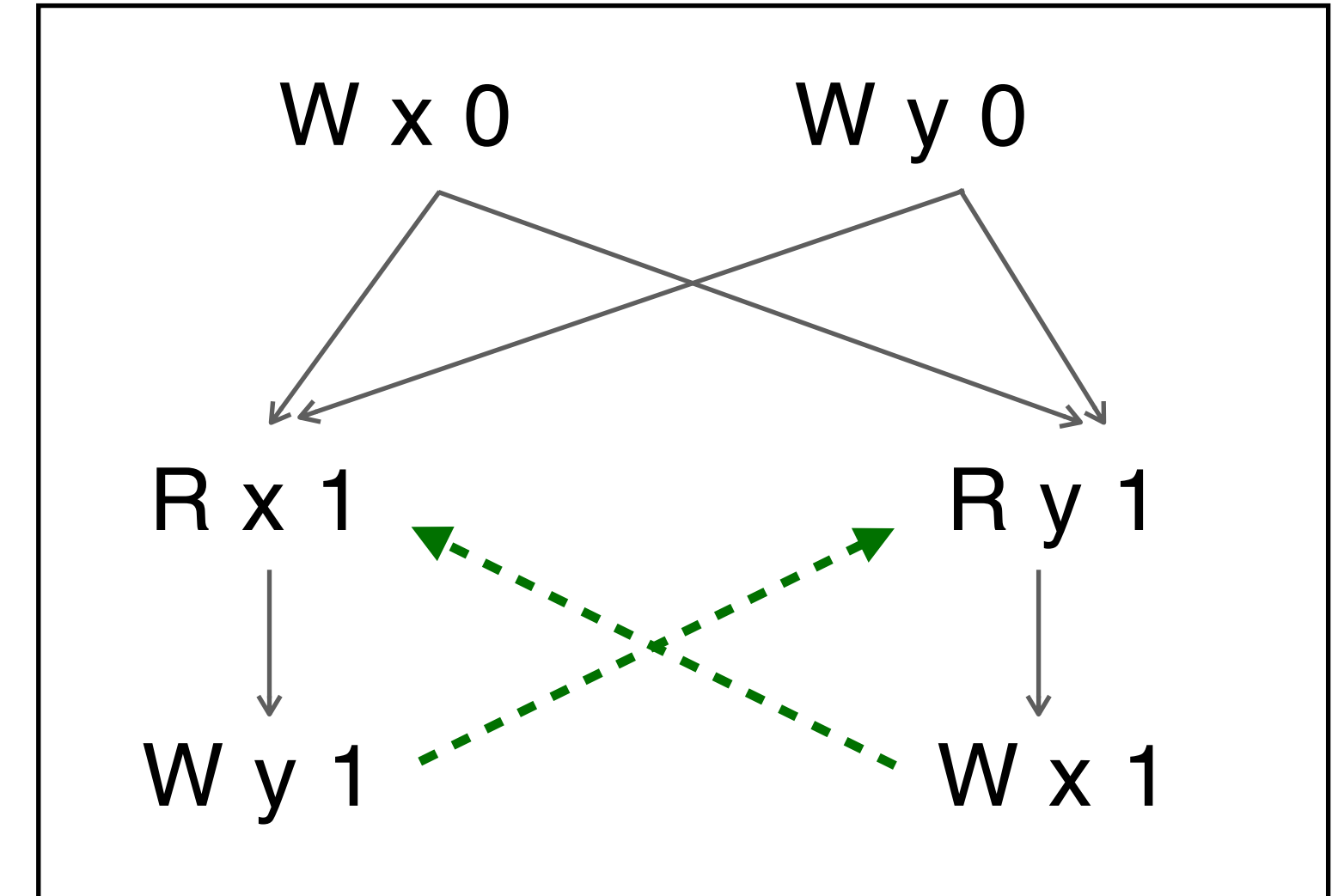
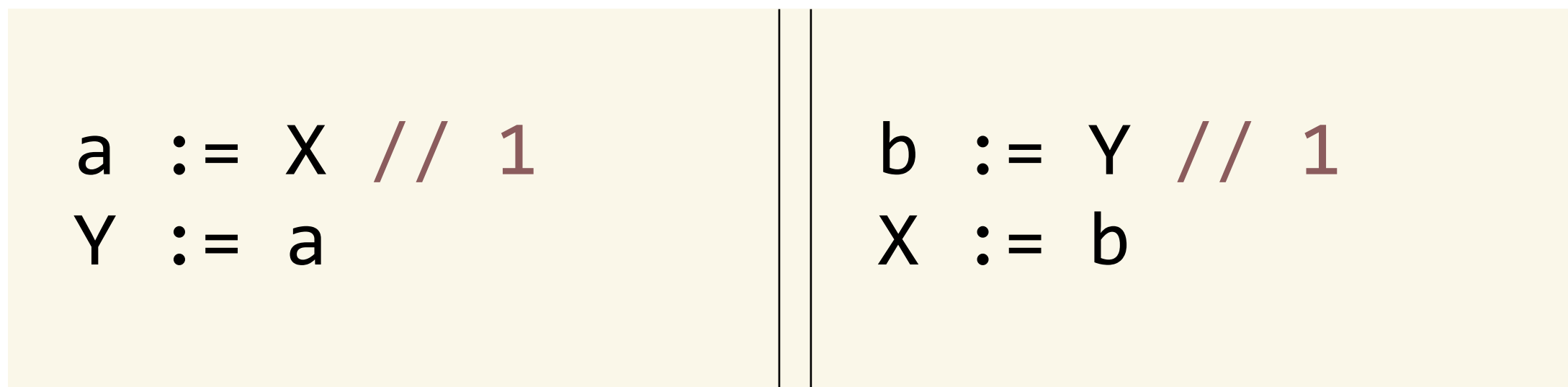
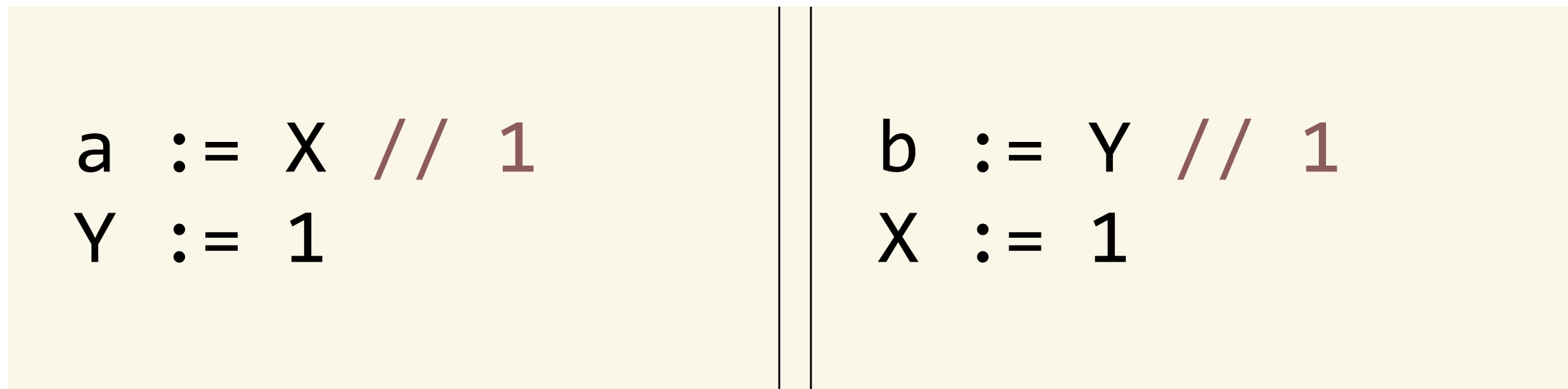
```
a := X rlx // 1  
Y := a rlx
```

```
b := Y rlx // 1  
X := b rlx
```



- But, it means that **it also allows the above behavior**
- The two behaviors are represented by the same execution graph!
- The value 1 appears ***“out-of-thin-air”***

# The hardware solution



-----> dependency *dep*

- Hardware models forbid: (*dep*  $\cup$  *rf*) cycles

# The hardware solution

- Hardware execution graph maintain *dependency relation* among events
- This is **not** a viable option for a PL since compilers may remove syntactic dependencies
- Devising a good “semantic” notion of dependency is an open challenge

```
a := X // 1  
Y := 1 + a - a
```

```
b := Y // 1  
X := 1 + b - b
```

  
compiler  
optimization

```
a := X // 1  
Y := 1
```

```
b := Y // 1  
X := 1
```



# The out-of-thin-air problem

- The C++14 standard states:

*“Implementations should ensure that no “out-of-thin-air” values are computed that circularly depend on their own computation.”*

- But doesn't give a sufficiently formal definition...

*“Disturbingly, 40+ years after the first relaxed-memory hardware was introduced (the IBM 370/158MP), the field still does not have a credible proposal for the concurrency semantics of any general-purpose high-level language that includes high performance shared-memory concurrency primitives. This is a major open problem for programming language semantics.”*

# RC11: a conservative approach

- Disallow  $(po \cup rf)$  cycles altogether.
  - Implementation cost: **forbid RW-reordering** for relaxed accesses
  - Importantly, reordering of **non-atomic accesses** is still sound!
  - Different strategies and their performance implications were investigated:  
Ou, Demsky: **Towards understanding the costs of avoiding out-of-thin-air results**. OOPSLA 2018. <https://doi.org/10.1145/3276506>
- The obtained model is called **RC11** (“repaired C11”).

Boehm, Demsky: **Outlawing ghosts: avoiding out-of-thin-air results**. MSPC 2014. <https://doi.org/10.1145/2618128.2618134>

L, Vafeiadis, Kang, Hur, Dreyer: **Repairing Sequential Consistency in C/C++11**. PLDI 2017. <https://doi.org/10.1145/3140587.3062352>

# Alternative proposals

- Solving the out-of-thin-air problem without changing the compilation schemes requires a **major revision** of the standard
- We **cannot have a per-execution definition**: validity of one execution depends on what happens in other executions
- Some prominent proposals:
  - Chakraborty, Vafeiadis. **Grounding thin-air reads with event structures**. POPL 2019. <https://doi.org/10.1145/3290383>
  - Jeffrey, Riely, Batty, Cooksey, Kaysin, Podkopaev. **The leaky semicolon: compositional semantic dependencies for relaxed-memory concurrency**. POPL 2022. <https://doi.org/10.1145/3498716>
  - Kang, Hur, L, Vafeiadis, Dreyer. **A promising semantics for relaxed-memory concurrency**. POPL 2017. <https://doi.org/10.1145/3009837.3009850>

# RC11

- In the rest of this presentation, we mostly assume RC11:  $(po \cup rf)$  is acyclic
- This model has been extensively studied in recent years:
  - acyclicity of  $(po \cup rf)$  allows adaptations of existing techniques
  - we think about the system executing the program **“in-order”** on top of a non-standard memory system

# Operationalizing RC11

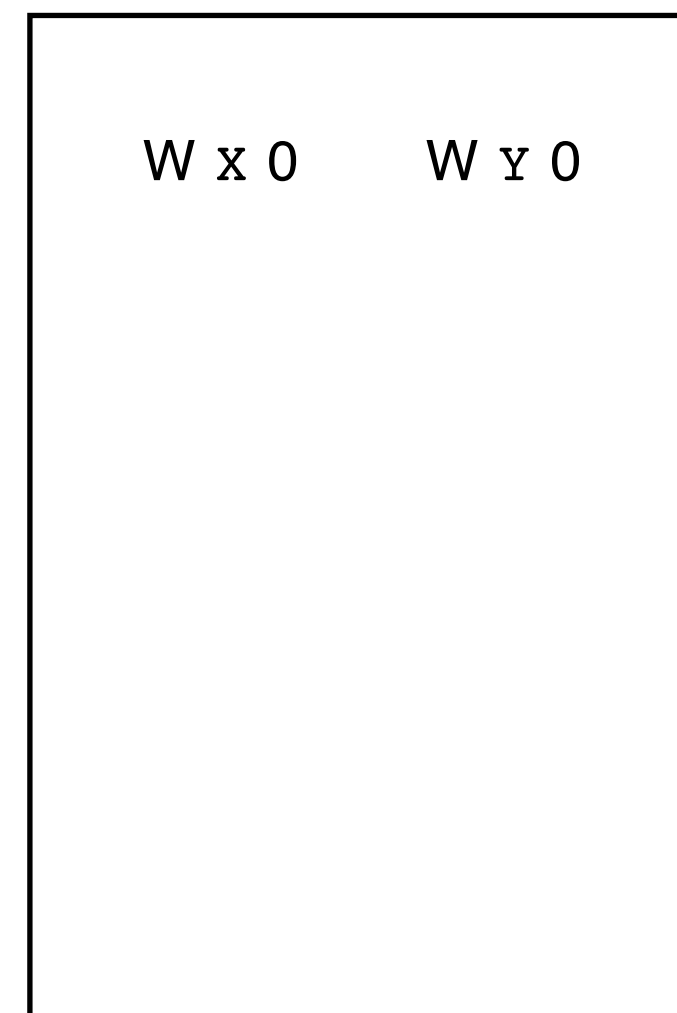
- State is the **execution graph** produced so far
- **Non-deterministic** choice where to read from (and where to place writes in the modification order)
- **Consistency** is checked at every step
- This memory system is synchronized with an “in-order” **program semantics**

```
X := 1 rlx  
a := Y rlx // 0
```

```
Y := 1 rlx  
b := X rlx // 0
```

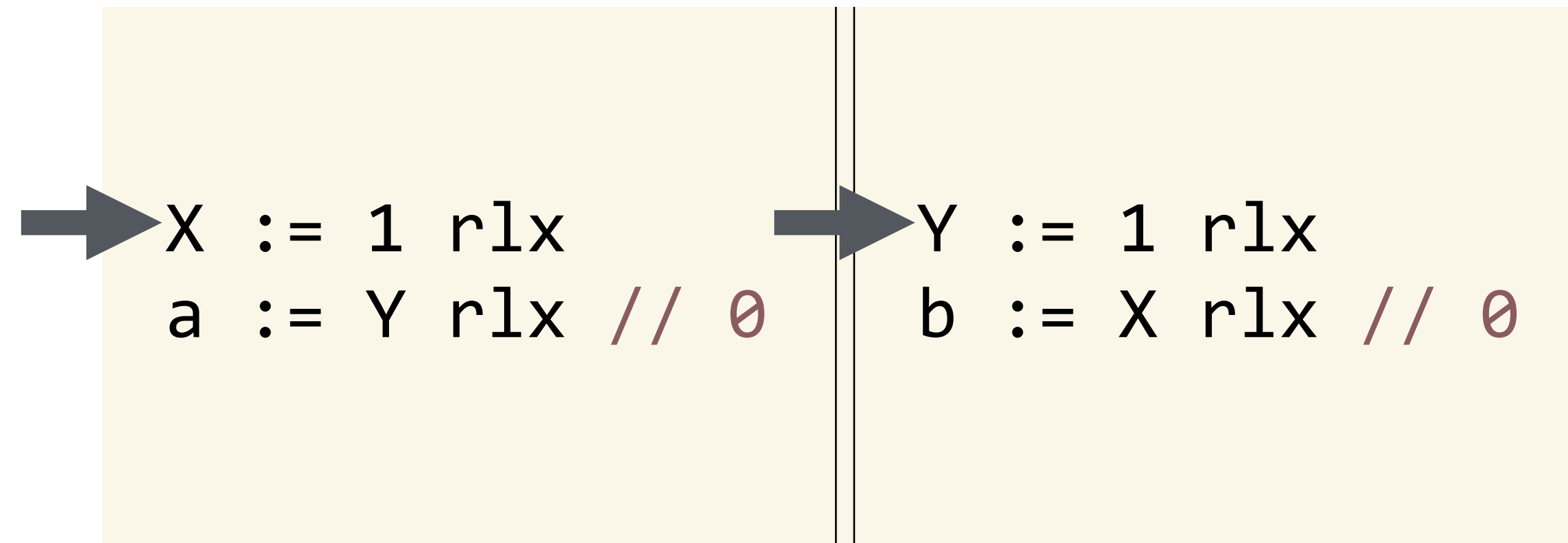
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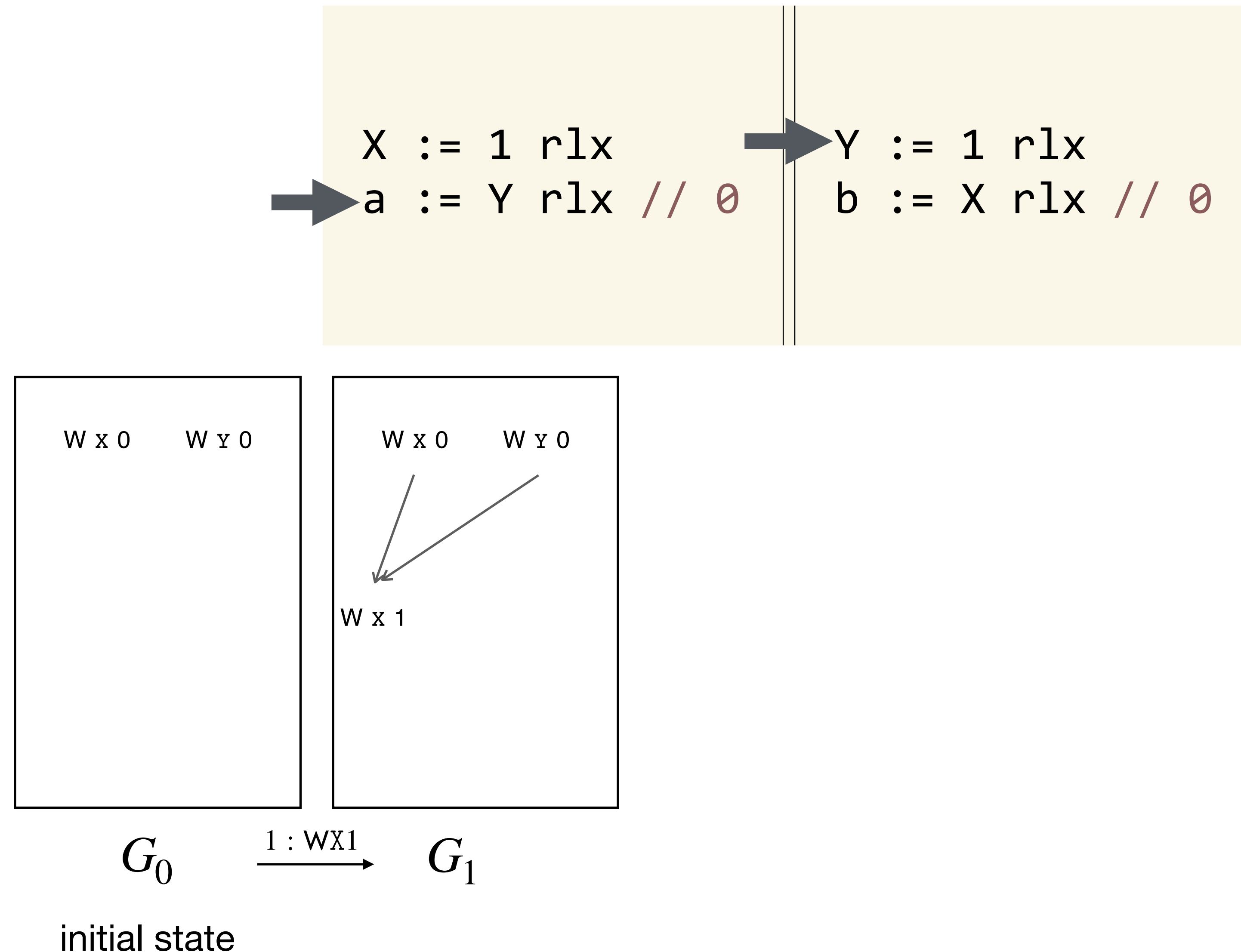
$G_0$

initial state



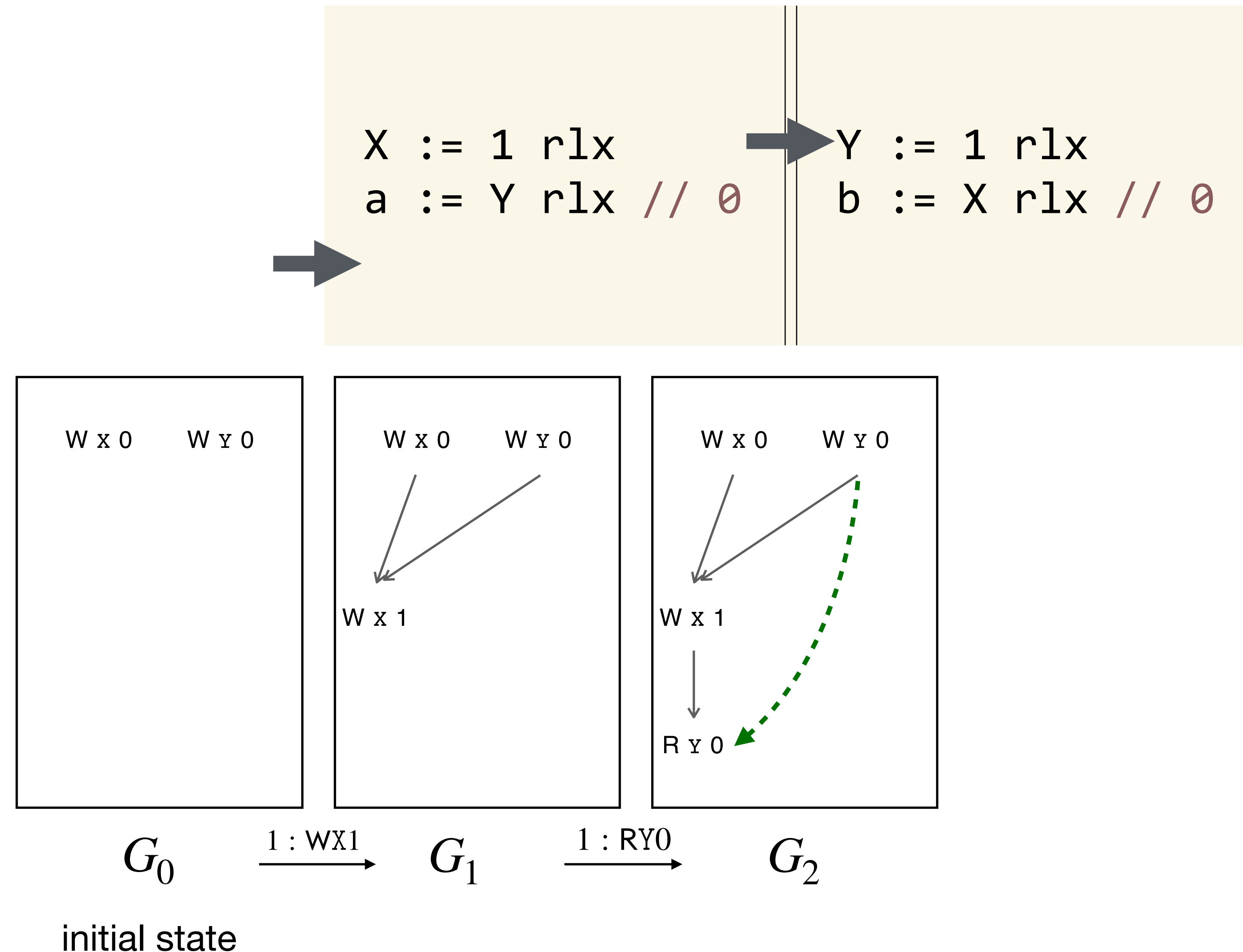
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# Operationalizing RC11

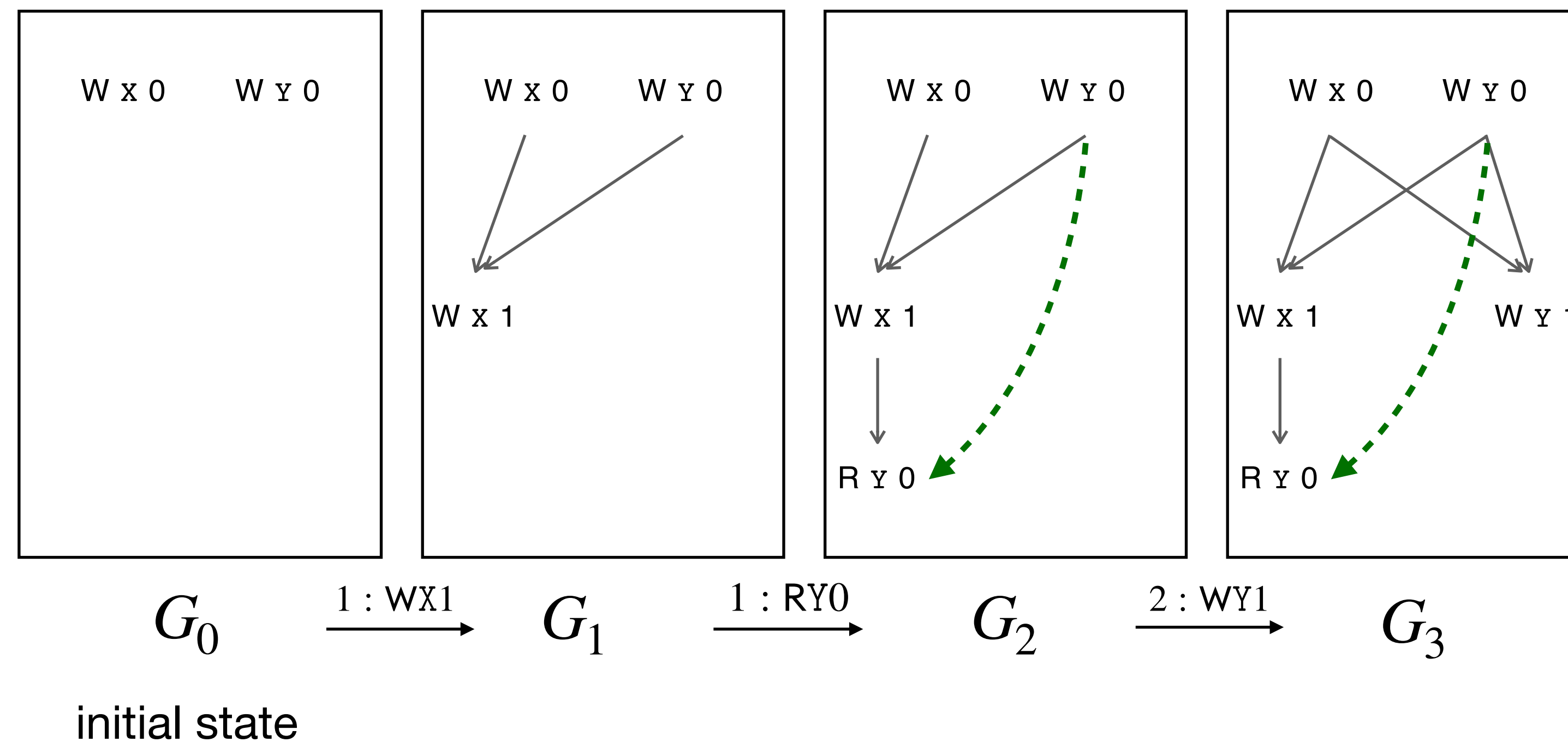
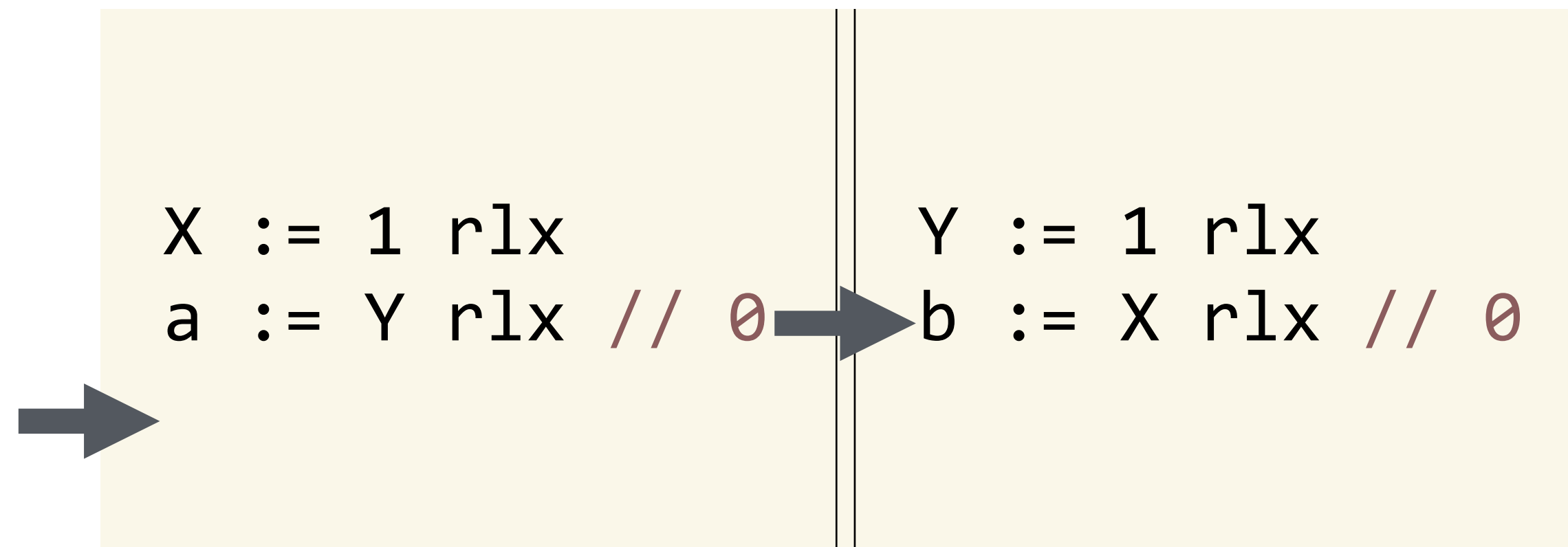
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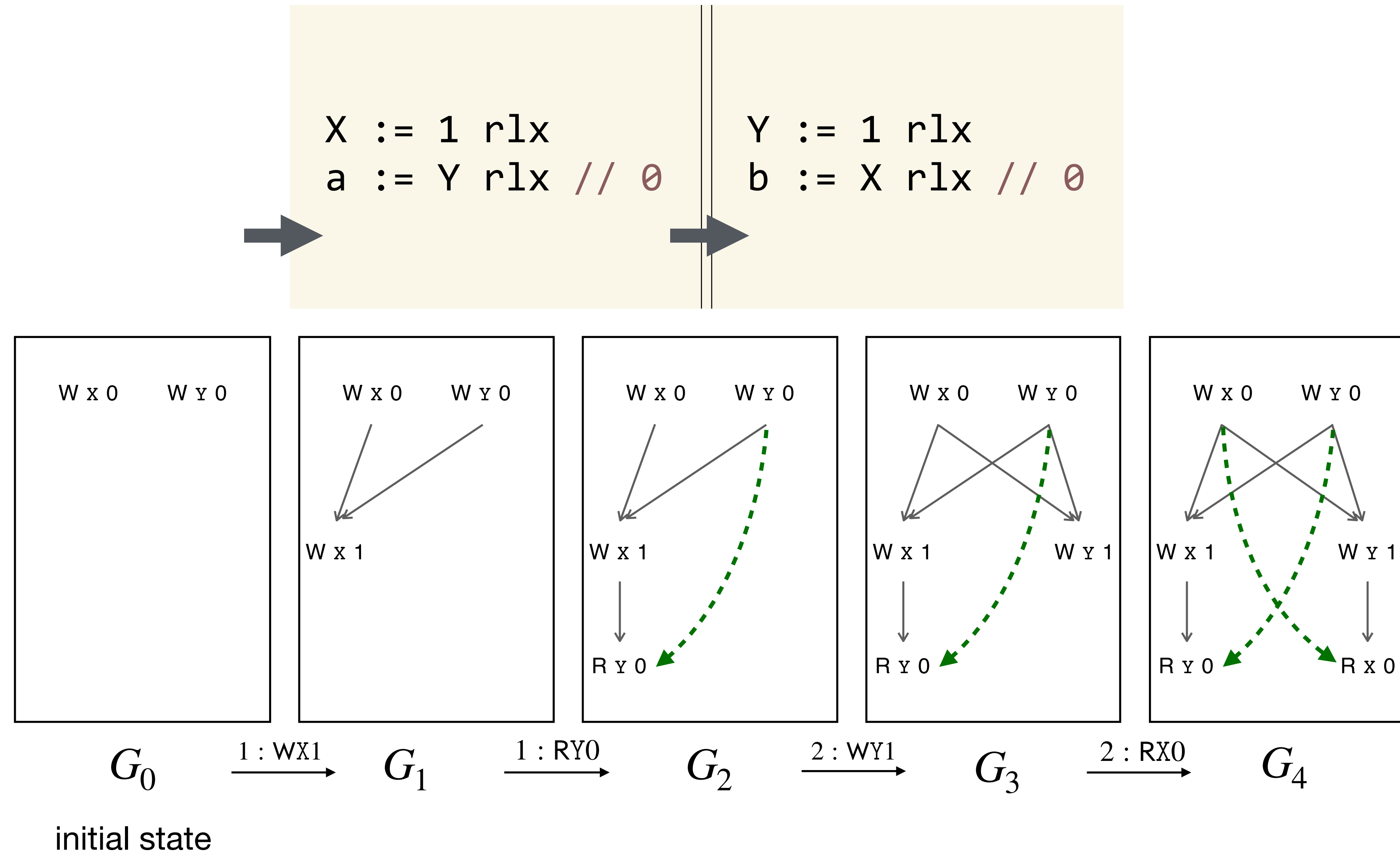
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# Operationalizing RC11

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- **Consistency** is checked at every step
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# Operationalizing RC11

- Observation:
  - we don't need the full execution graph in the state
  - we only need the part that can affect consistency of later accesses
- This leads to more **compact presentations** (somewhat similar to distributed implementations)
- Note: The state space remains **infinite** (for program with loops)
  - important implications for algorithmic verification
- Next, we demonstrate this idea for the **RA fragment**



relaxed  $\square$  release/acquire  $\square$  sc



# The RA memory model

- An well-studied fragment of C11 is RA (intricate but not overwhelmingly detailed)
- Ensures **causal consistency** & **coherence**
- Supports “**flag-based synchronization**”

```
Y := 42
X := 1
```

```
a := X
if (a=1) then
  b := Y // 0
```

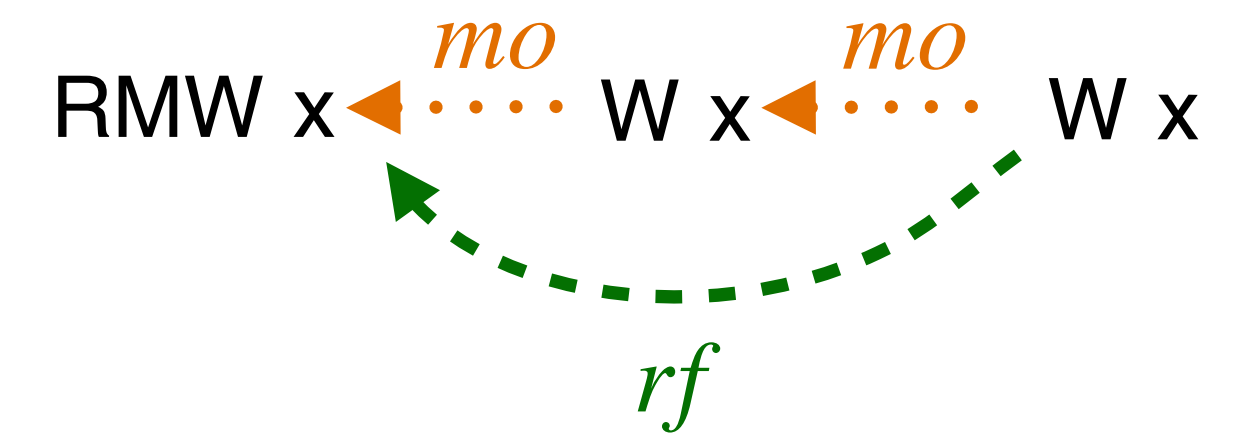
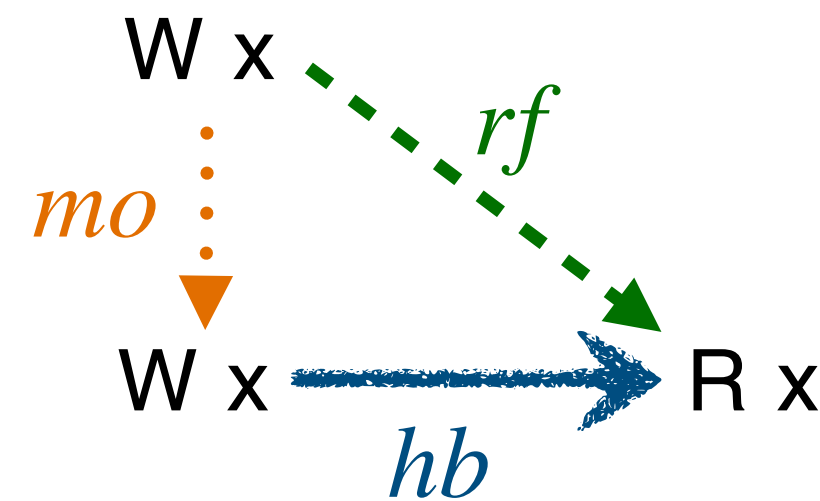
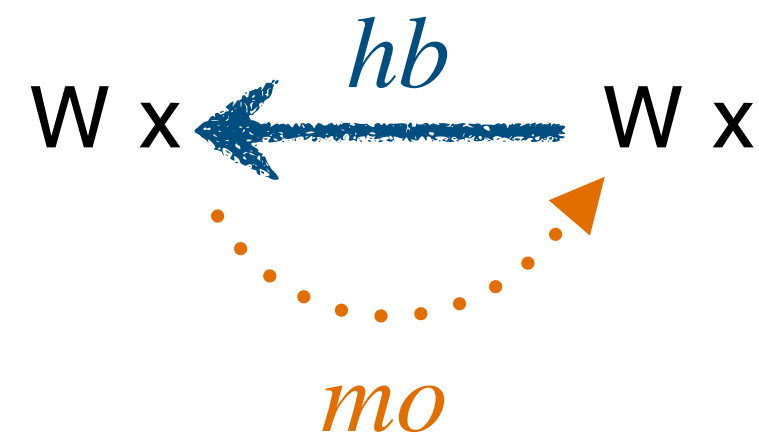
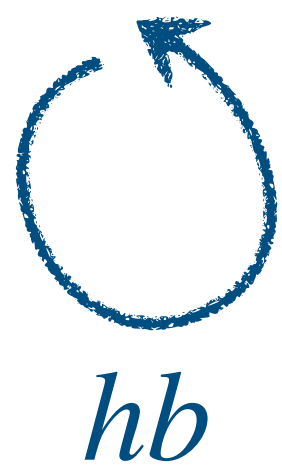
- Allows **WR-reordering**
- Threads can disagree about the order of writes: **non-multi-copy-atomic**
- **Locks** can be implemented using RMWs
- **SC-fences** can be encoded as RMWs to a distinguished otherwise unused location

# Declarative RA

- When restricting RC11 to only release/acquire accesses:

- $hb = (po \cup rf)^+$

- Four disallowed patterns:

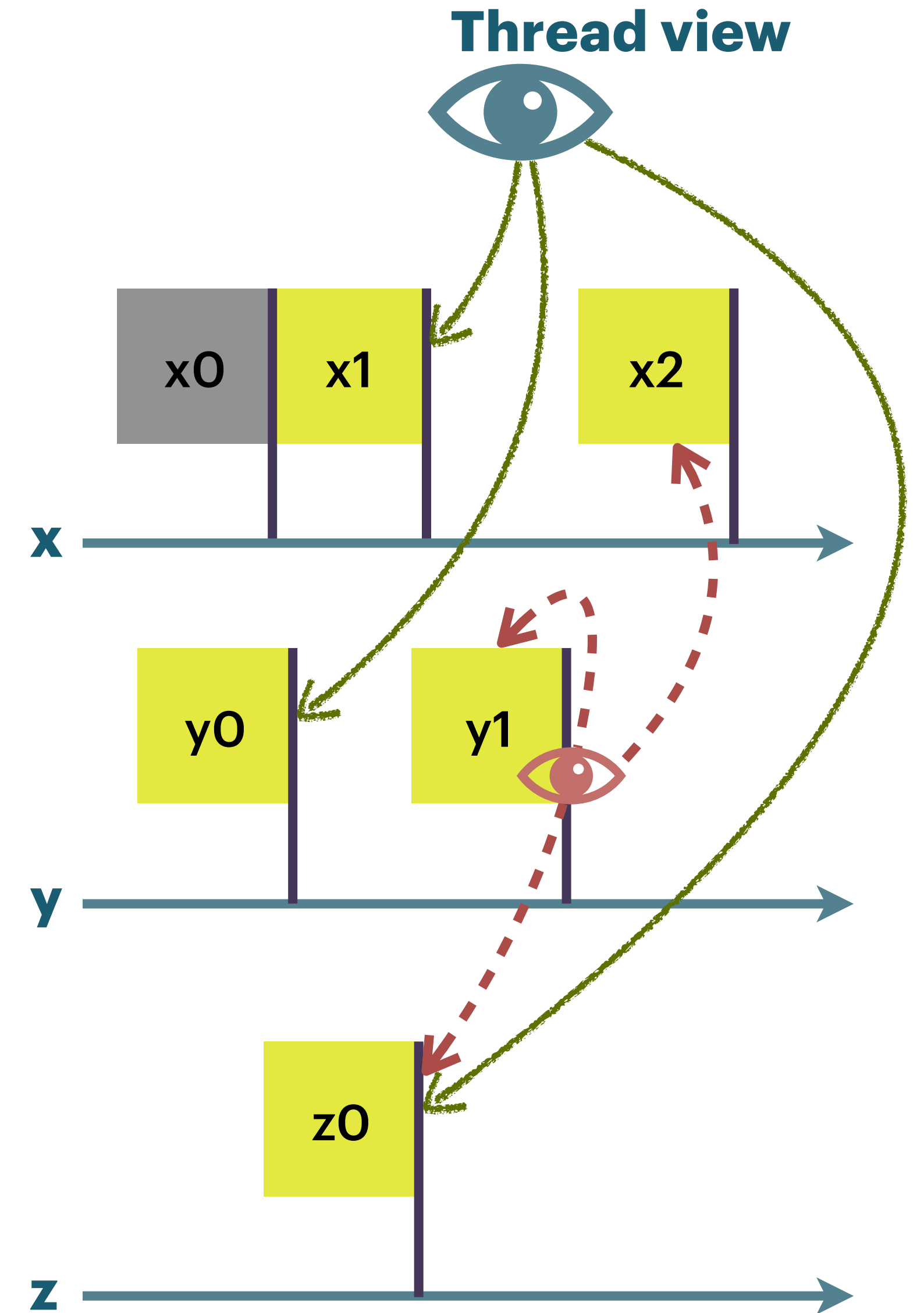


- Concise formulation:  $acyclic(hb \upharpoonright_{same\text{-}location} \cup mo \cup rb)$

# Operational formulation

## A view-based semantics

- Memory: Timeline per location (represents *mo*)
- Populated with immutable *messages* holding values
- Each *view* points to msgs on each timeline
- *Threads have views* — cannot read from “the past”
- *Msgs have views* for enforcing causal propagation
- Simulates the *graph-based* operational semantics



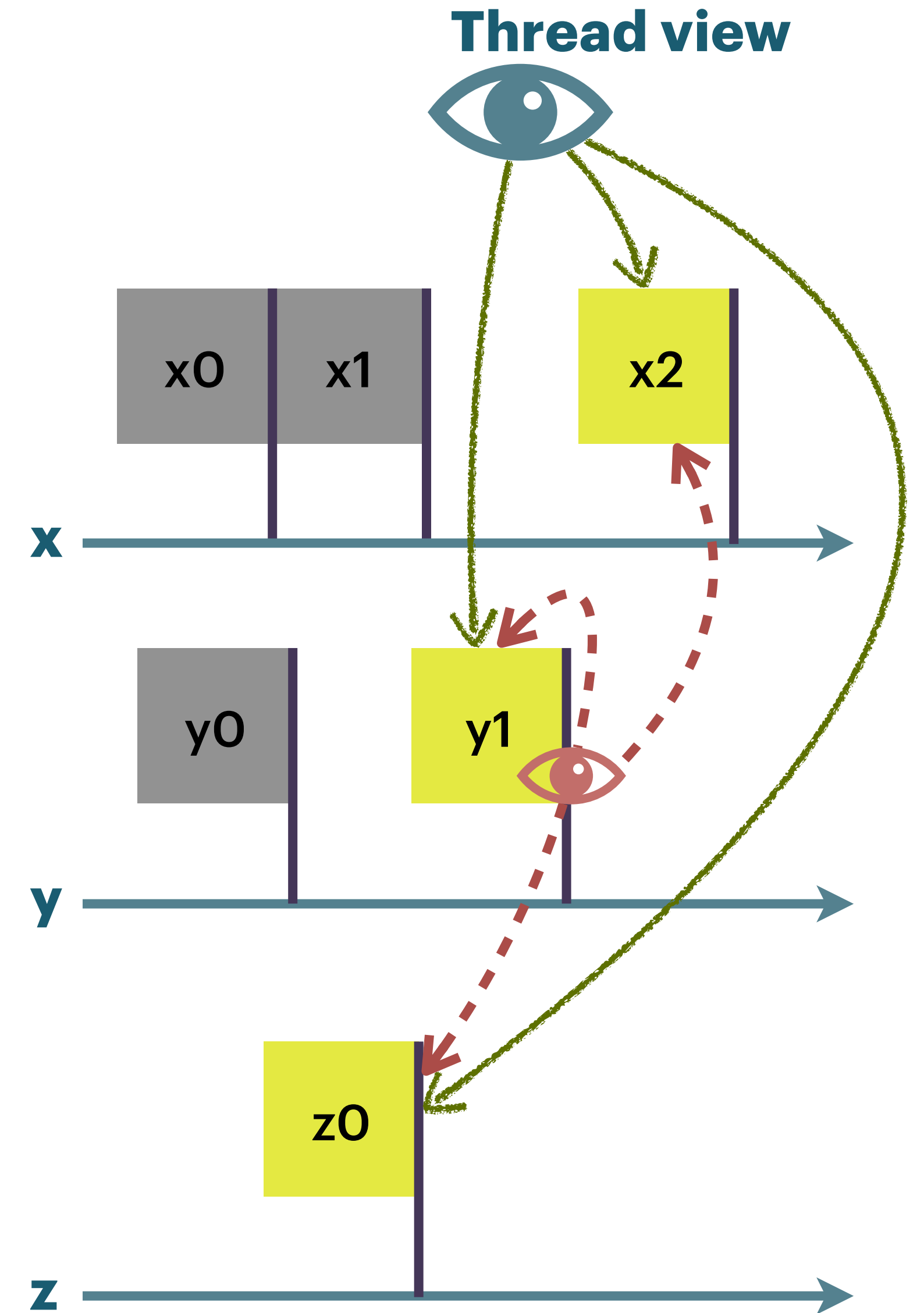
Kang, Hur, L, Vafeiadis, Dreyer: **A promising semantics for relaxed-memory concurrency**. POPL 2017. <https://doi.org/10.1145/3009837.3009850>

Dvir, Kammar, L: **A Denotational Approach to Release/Acquire Concurrency**. ESOP 2024. [https://doi.org/10.1007/978-3-031-57267-8\\_5](https://doi.org/10.1007/978-3-031-57267-8_5) 75

# Operational formulation

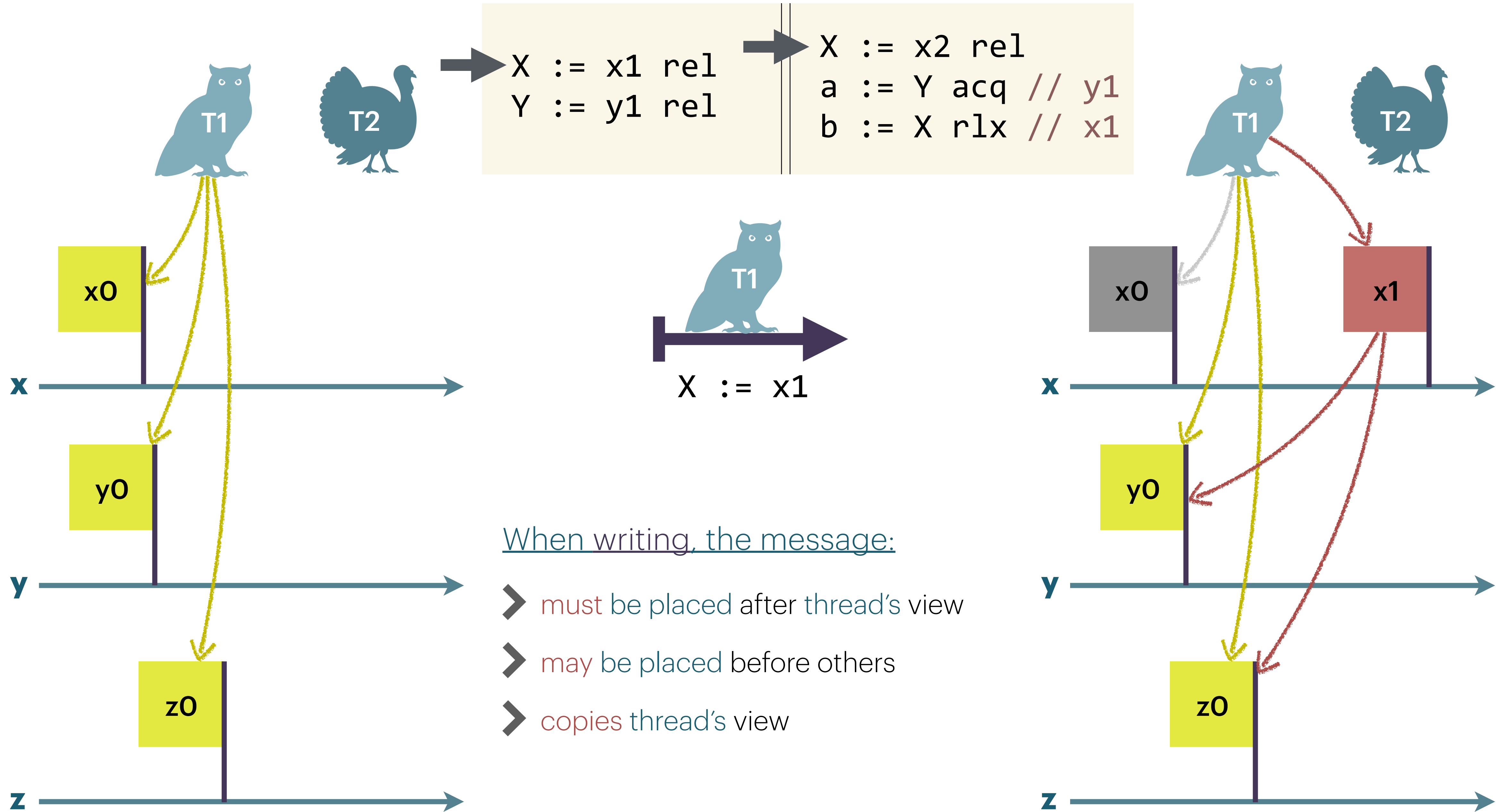
## A view-based semantics

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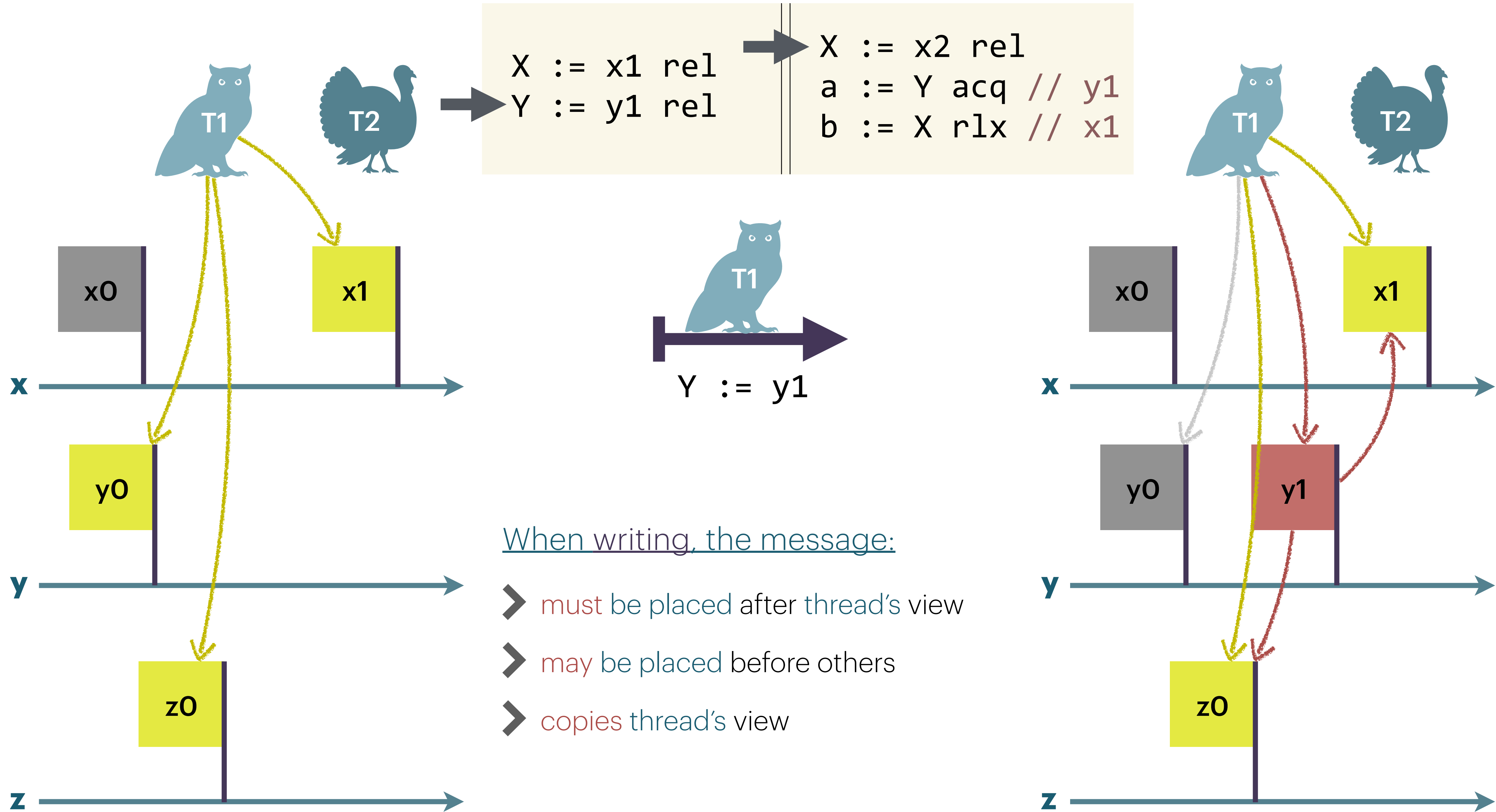


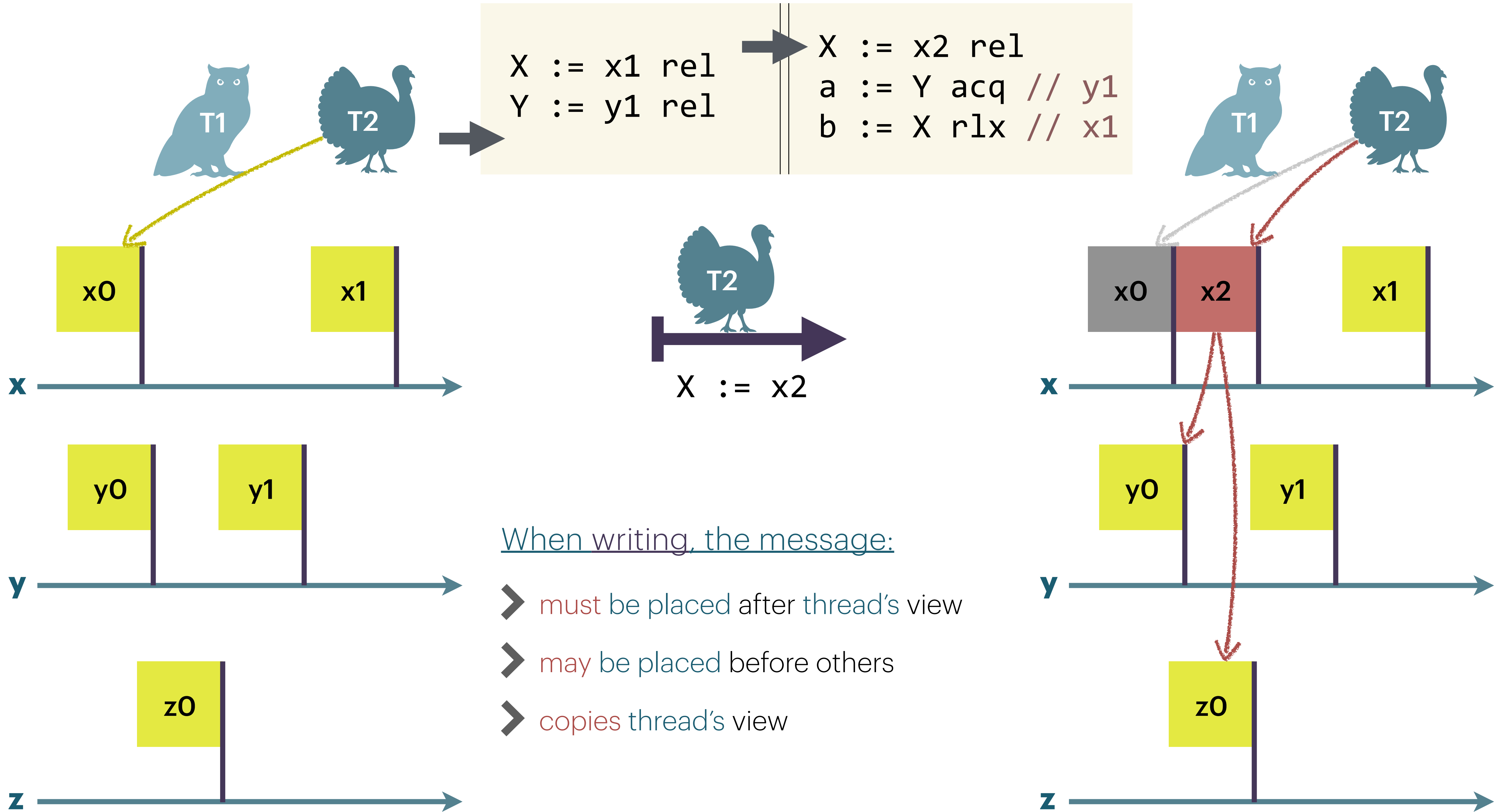
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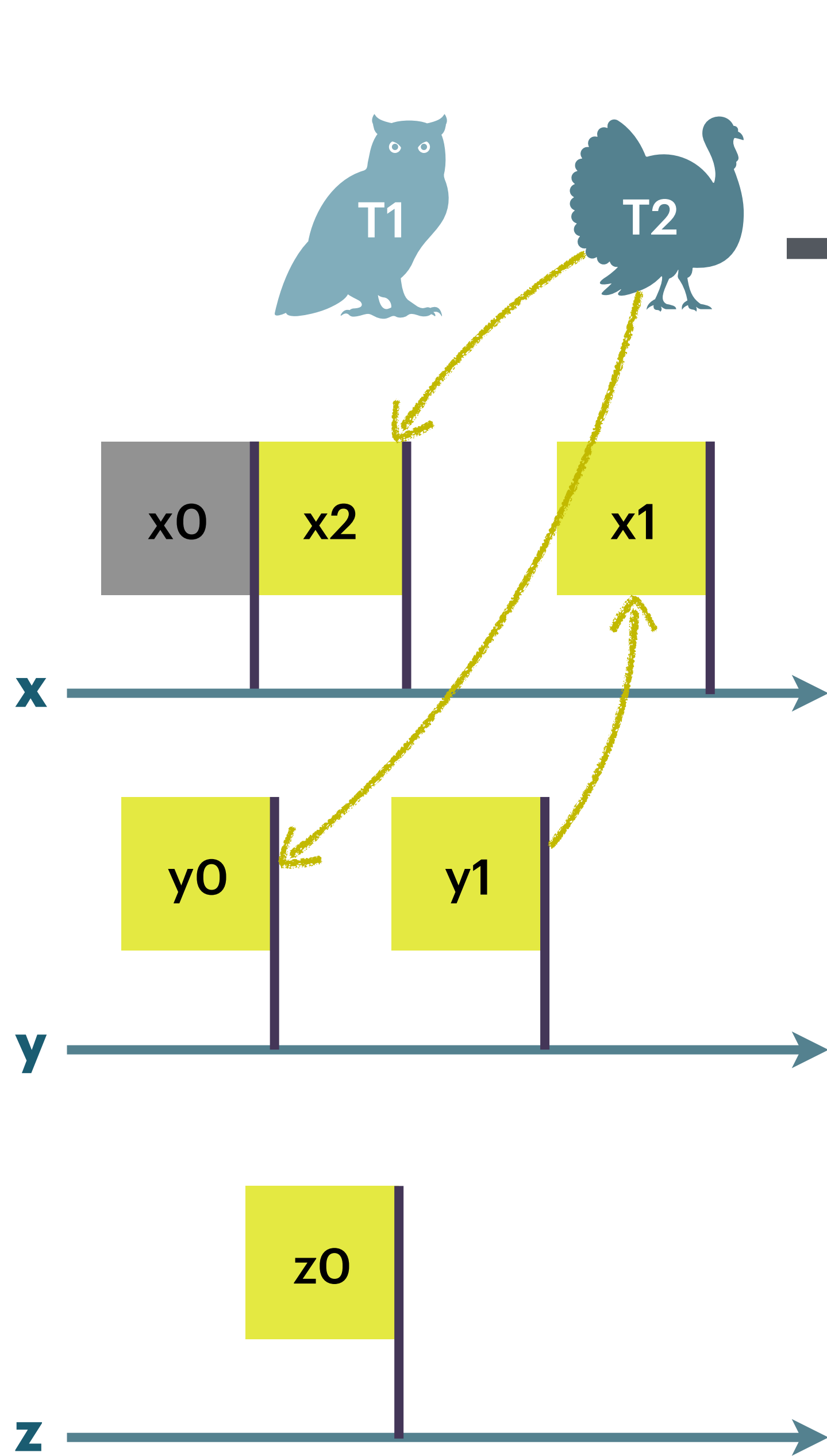
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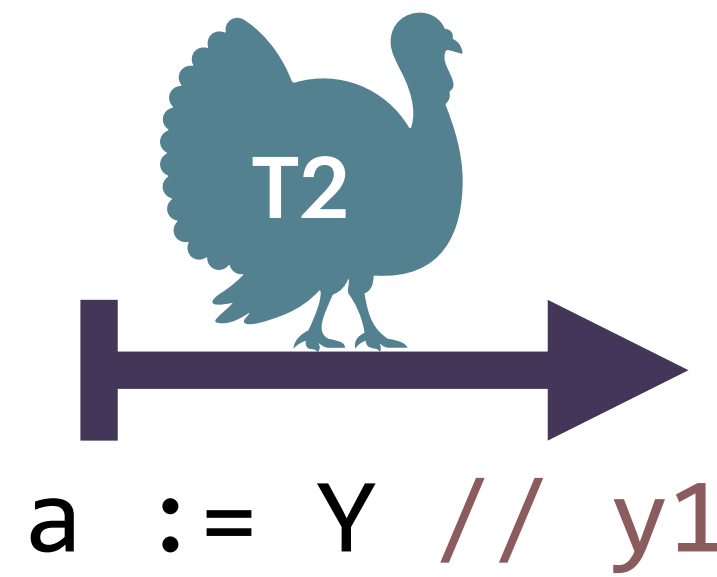




```

X := x1 rel
Y := y1 rel
-----
X := x2 rel
a := Y acq // y1
b := X rlx // x1

```

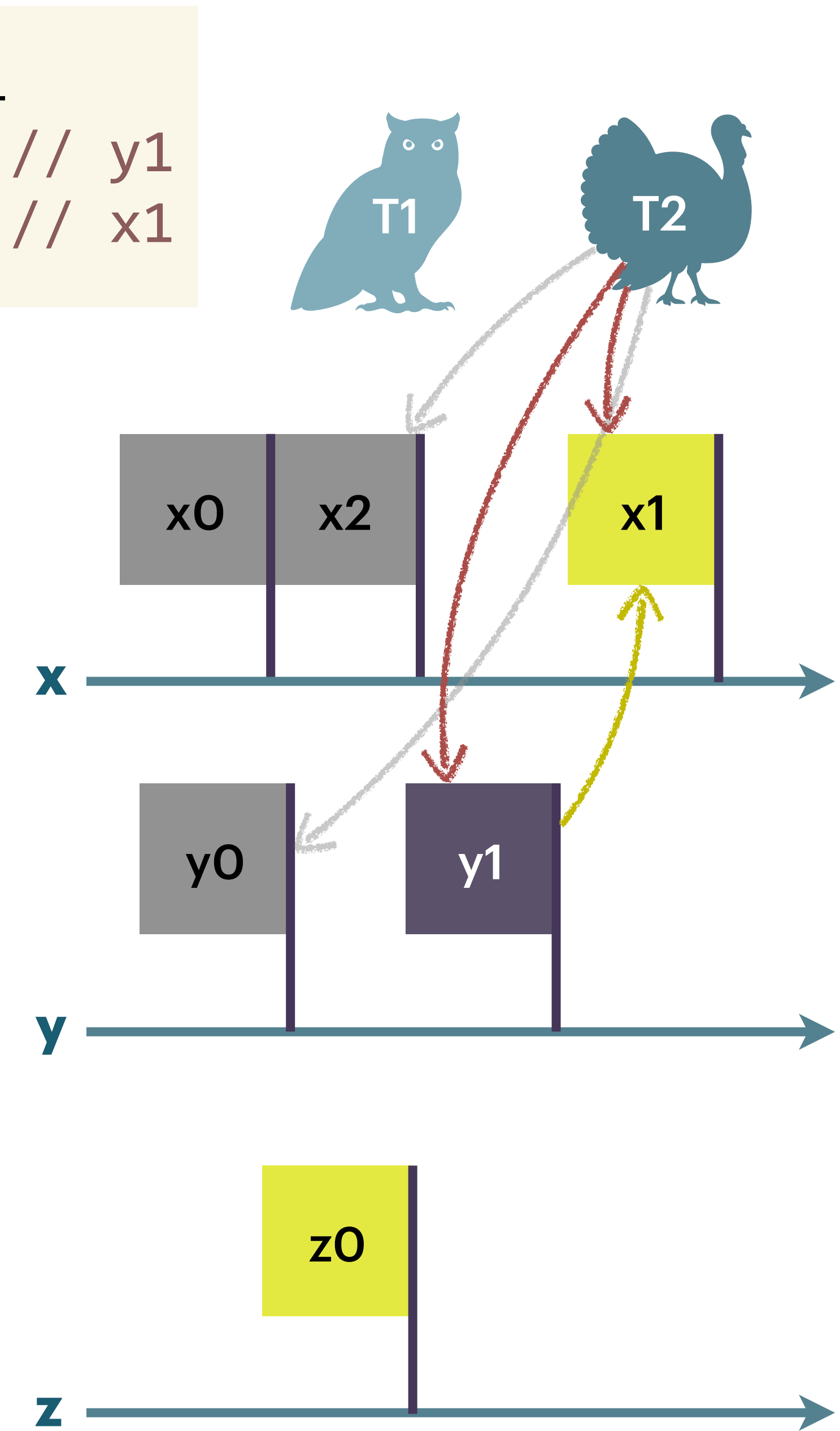


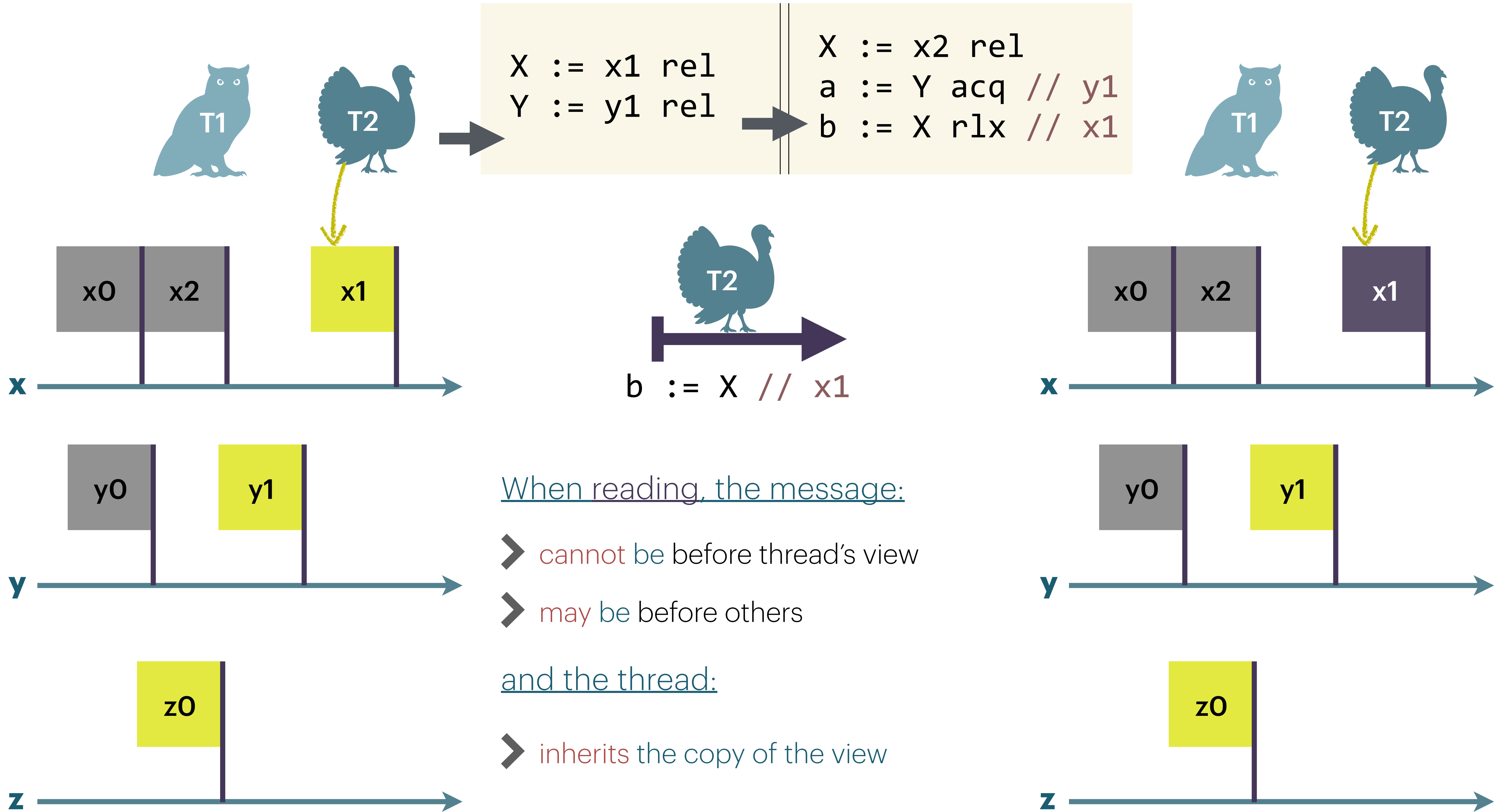
When reading, the message:

- cannot be before thread's view
- may be before others

and the thread:

- inherits the copy of the view



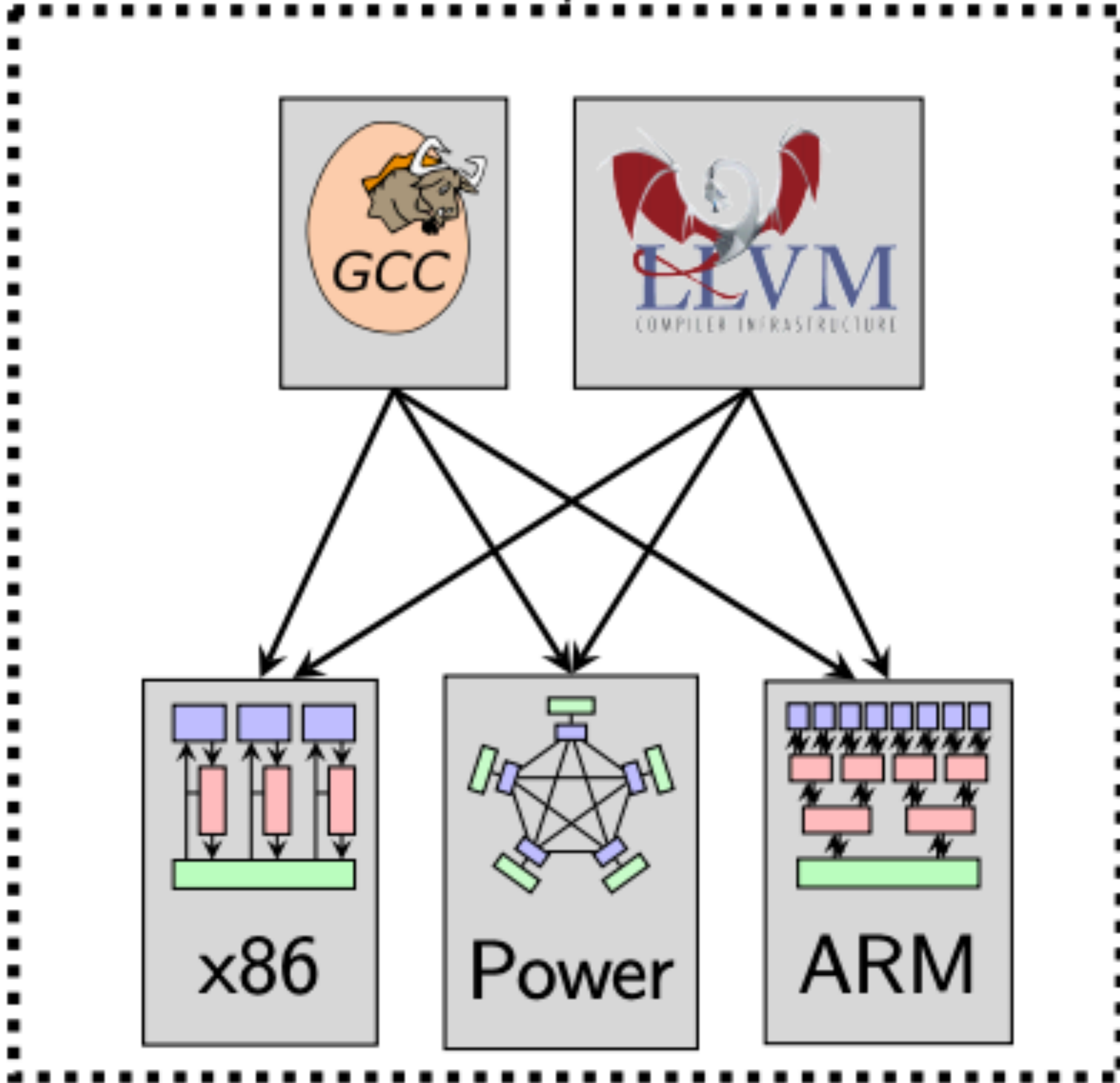


# Implementability of (R)C11



programmers

The (R)C11 memory model



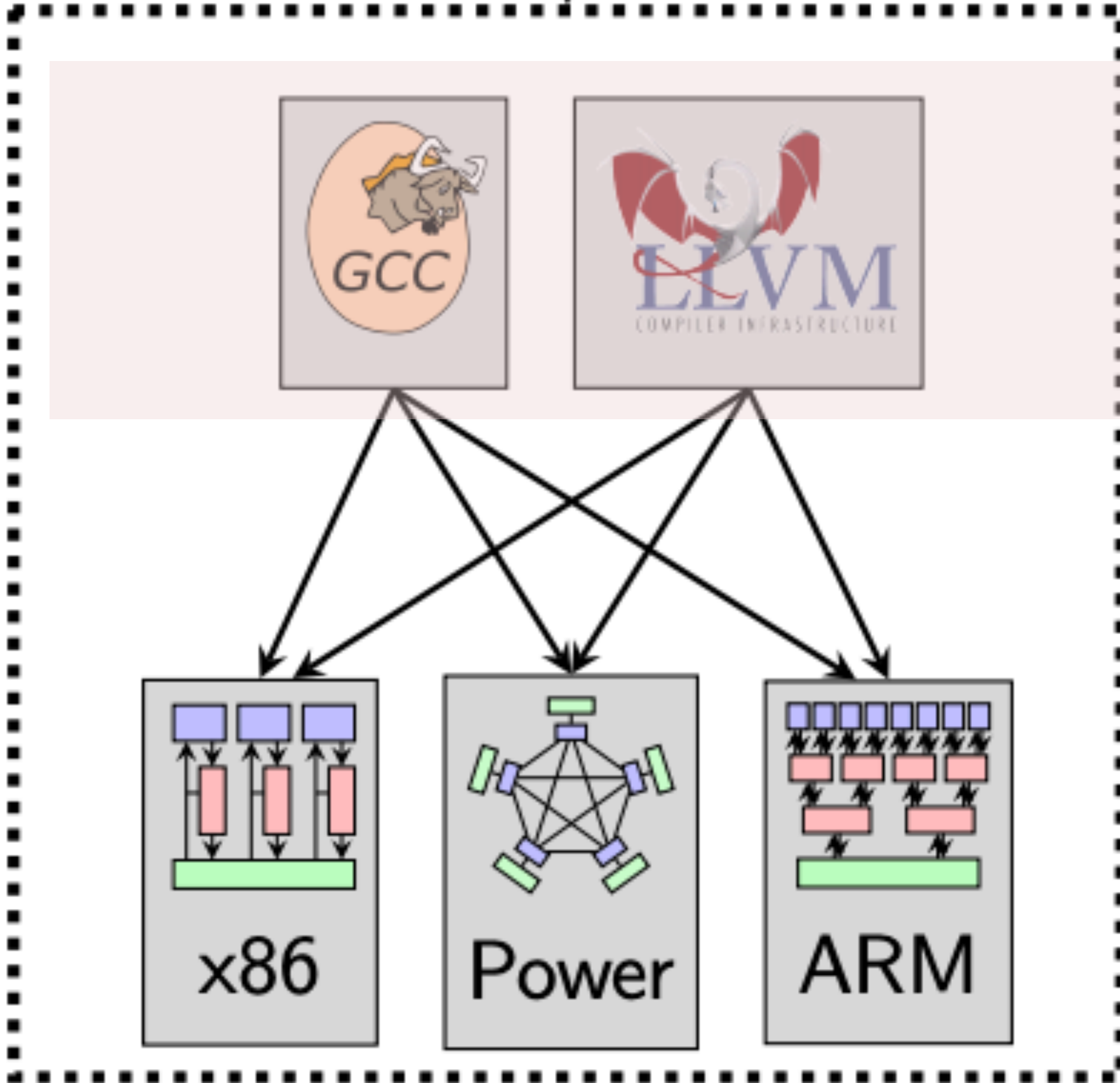
compilers

multicore architectures



programmers

The (R)C11 memory model



compilers

multicore architectures

# Compiler optimizations

- A formal analysis of soundness of optimizations in C11:

Vafeiadis, Balabonski, Chakraborty, Morisset, Zappa Nardelli: **Common Compiler Optimisations are Invalid in the C11 Memory Model and what we can do about it**. POPL 2015. <https://doi.org/10.1145/2676726.2676995>

- In RC11:

L, Vafeiadis, Kang, Hur, Dreyer: **Repairing Sequential Consistency in C/C++11**. PLDI 2017. <https://doi.org/10.1145/3140587.3062352>

- A lot of focus on optimizations on **atomics** (eliminations and reorderings)
- Current compilers mostly optimize **non-atomics**



# Transformation soundness

- Standard notion of **soundness** of local program transformations:

$C_{src} \rightsquigarrow C_{tgt}$  if  $\mathbf{Behaviors}(P[C_{tgt}]) \subseteq \mathbf{Behaviors}(P[C_{src}])$  for every program context  $P$

- For **catch-fire** semantics (as (R)C11), it implies:
  - if  $C_{src}$  is not racy, then  $C_{tgt}$  is not racy (the compiler must not introduce races)
  - if  $C_{src}$  is racy, then  $C_{src} \rightsquigarrow C_{tgt}$  is sound for every  $C_{tgt}$  (the compiler may exploit races)

# Allowed eliminations in RC11

$$\begin{array}{l} R^o; R^o \rightsquigarrow R^o \\ W^{sc}; R^{sc} \rightsquigarrow W^{sc} \end{array}$$

$$\begin{array}{l} W^o; W^o \rightsquigarrow W^o \\ W^o; R^{acq} \rightsquigarrow W^o \end{array}$$

together with **access strengthening** RC11 allows, e.g.:

$$WW \rightsquigarrow W: X := 1 \text{ rel} ; X := 2 \text{ rlx} \rightsquigarrow X := 2 \text{ rel}$$

$$RR \rightsquigarrow R: a := X \text{ acq} ; b := X \text{ rlx} \rightsquigarrow a := X \text{ acq} ; b := a$$

$$\begin{array}{l} WR \rightsquigarrow W: X := 1 \text{ rlx} ; a := X \text{ acq} \rightsquigarrow X := 1 \text{ rlx} ; a := 1 \\ X := 1 \text{ rlx} ; a := X \text{ sc} \rightsquigarrow X := 1 \text{ sc} ; a := 1 \end{array}$$

# Allowed read-write reordering in RC11

$X ; Y \rightsquigarrow Y ; X$

X \ Y	$R_y^{o_2}$	$W_y^{o_2}$
$R_x^{o_1}$	$o_1 \sqsubseteq \text{rlx}$	$o_1, o_2 \sqsubseteq \text{rlx} \wedge (o_1 = \text{na} \vee o_2 = \text{na})$
$W_x^{o_1}$	$o_1 \neq \text{sc} \vee o_2 \neq \text{sc}$	$o_2 \sqsubseteq \text{rlx}$

thanks to  
"catch-fire"

e.g.,

RR:  $a := X ; b := Y \text{ acq} \rightsquigarrow b := Y \text{ acq} ; a := X$

WW:  $X := 1 \text{ rel} ; Y := 2 \rightsquigarrow Y := 2 ; X := 1 \text{ rel}$

WR:  $X := 1 \text{ rel} ; b := Y \text{ acq} \rightsquigarrow b := Y \text{ acq} ; X := 1 \text{ rel}$

RW:  $a := X ; Y := 1 \rightsquigarrow Y := 1 ; a := X$

"roach motel"

# Optimizing non-atomics

Thanks to catch-fire, non-atomics can be generally optimized as sequential code

```
lock(L)
```

```
a := X
```

```
Y := 1
```

```
unlock(L)
```

```
lock(L)
```

```
b := Y
```

```
X := b
```

```
unlock(L)
```

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<code>lock(L)</code>	<code>lock(L)</code>
① <code>a := X</code>	<code>b := Y</code> ③
② <code>Y := 1</code>	<code>X := b</code> ④
<code>unlock(L)</code>	<code>unlock(L)</code>

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well-locked ①, ② → ③, ④ or ③, ④ → ①, ②

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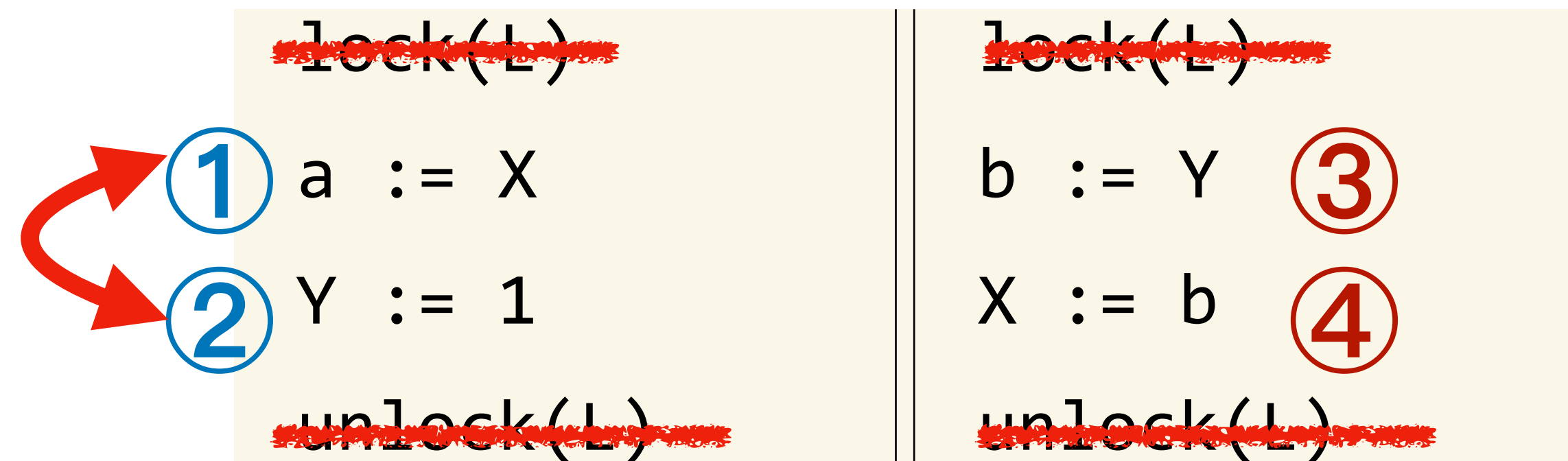
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not well-locked e.g., ② → ③ → ④ → ①



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not well-locked e.g., ② → ③ → ④ → ①

Switching ① and ② makes a difference, but the program is wrong (racy)!

<del>lock(L)</del>	<del>lock(L)</del>
① a := X	b := Y ③
② Y := 1	X := b ④
<del>unlock(L)</del>	<del>unlock(L)</del>

# Load introduction is unsound

- But, irrelevant load introduction is **unsound** in catch-fire semantics!
- This is something compilers actually perform :(

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GCC, LLVM



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A load from `x` introduced!



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n = 0 →  
spawn a thread writing to x

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```
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    sum += x;
```

no data race

n = 0 →  
spawn a thread writing to x

GCC, LLVM



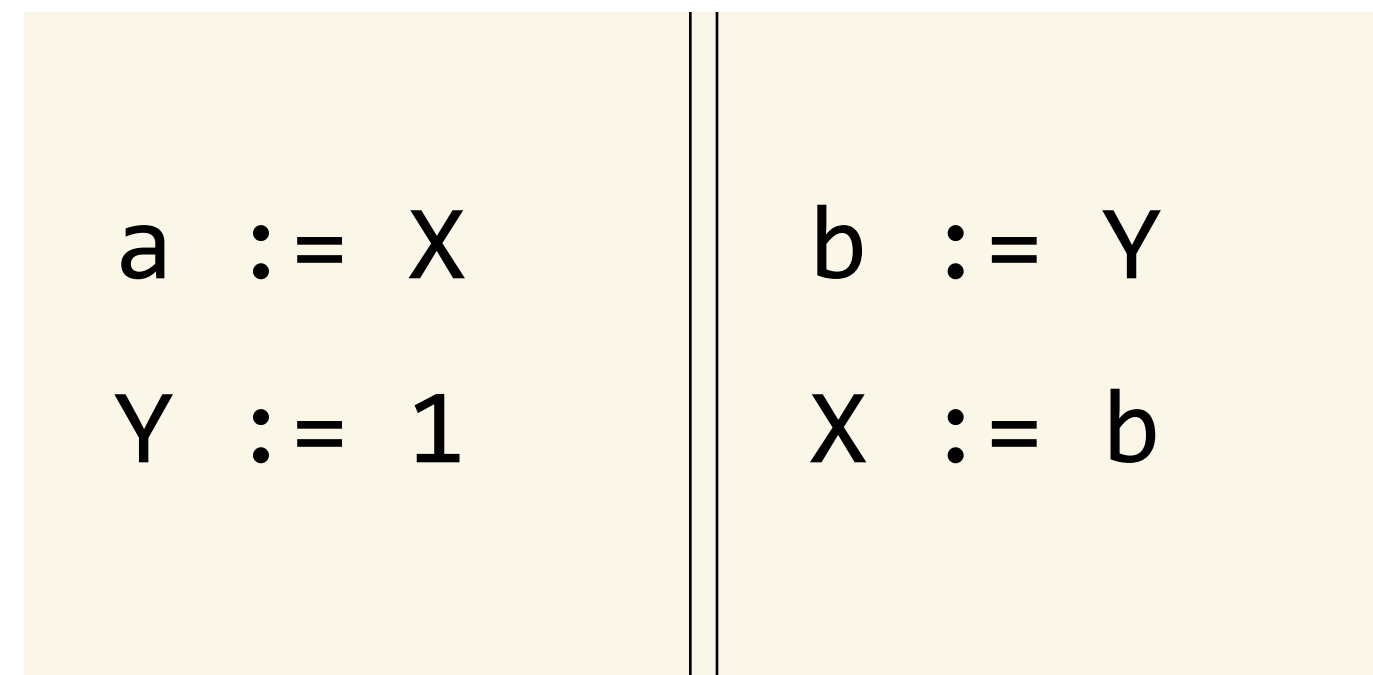
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```

A load from x introduced!

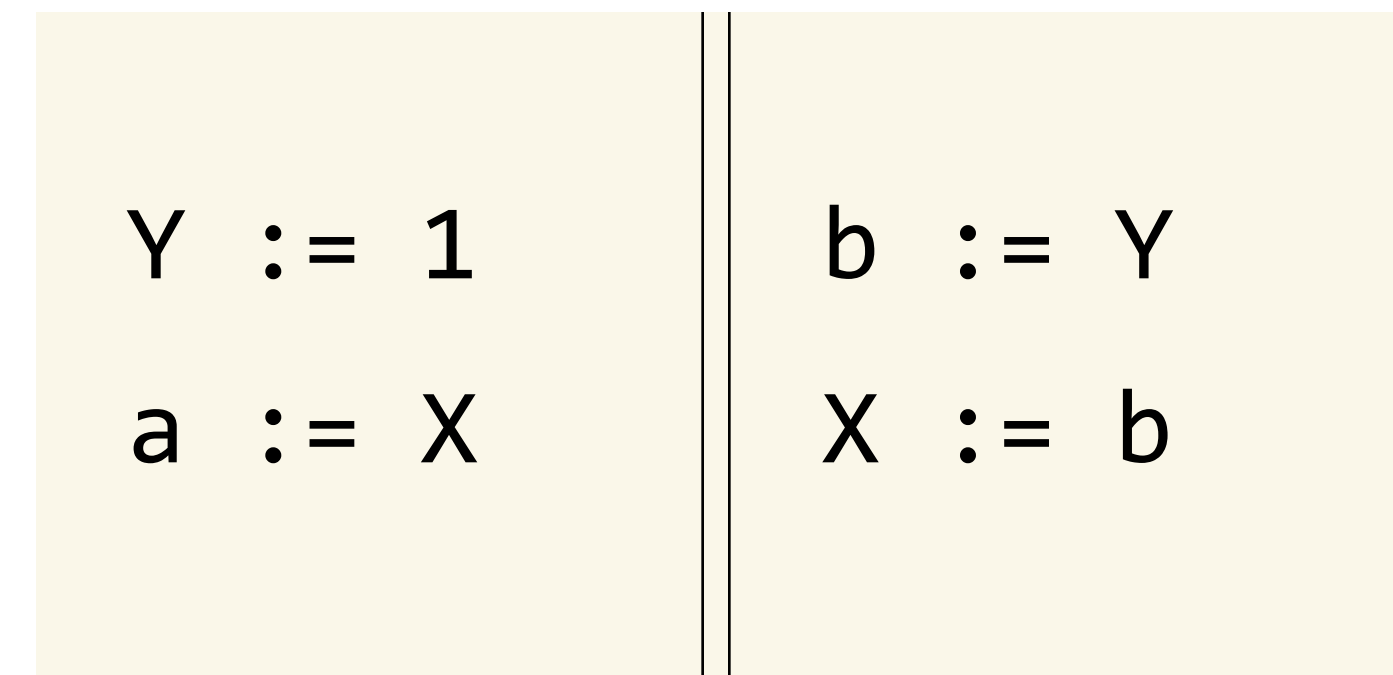
data race

# Undefined value as a solution?

- Execute “in-order” and read “undefined value” for every race



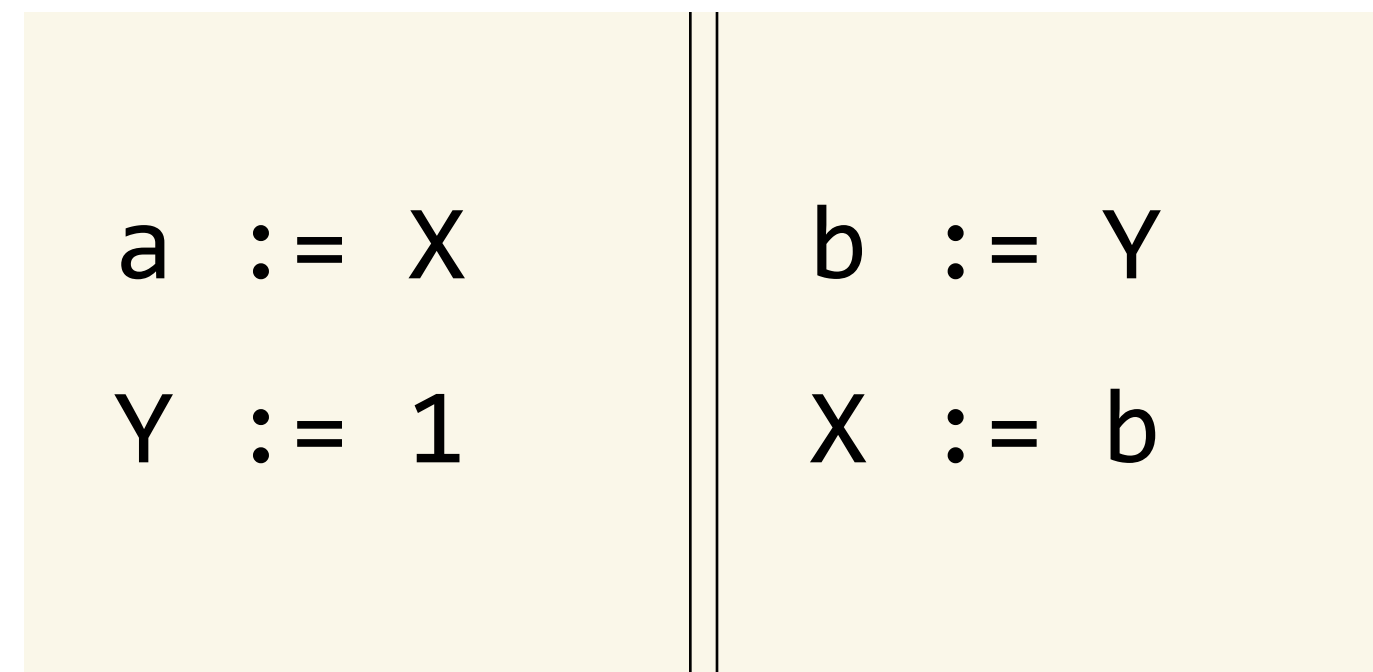
`a = b = 1` disallowed



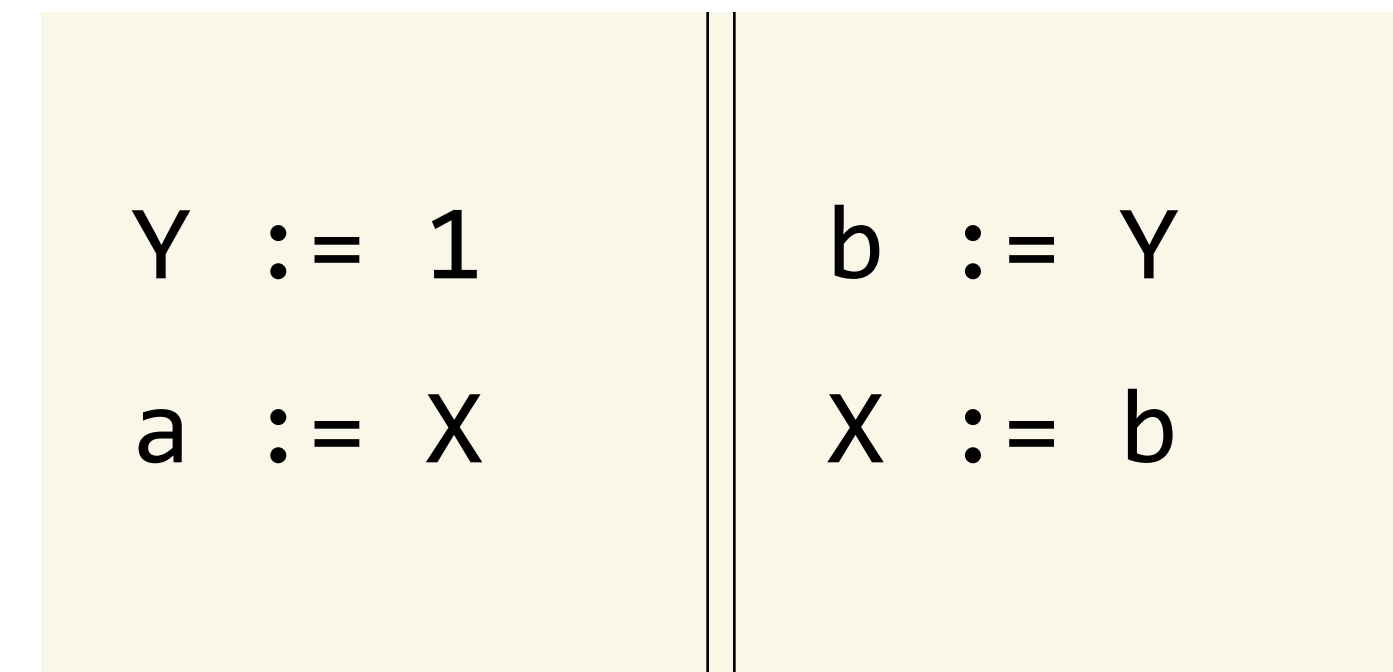
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`a = b = 1` disallowed



`a = b = 1` allowed

RW reordering is **unsound** in this model

# Our Solution: An intermediate memory model

Lee, Cho, Margalit, Hur, L: **Putting Weak Memory in Order via a Promising Intermediate Representation**. PLDI 2023. <https://doi.org/10.1145/3591297>

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source (C/C++) model:  
**in-order**

IU

compiler IR model:  
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programming & reasoning

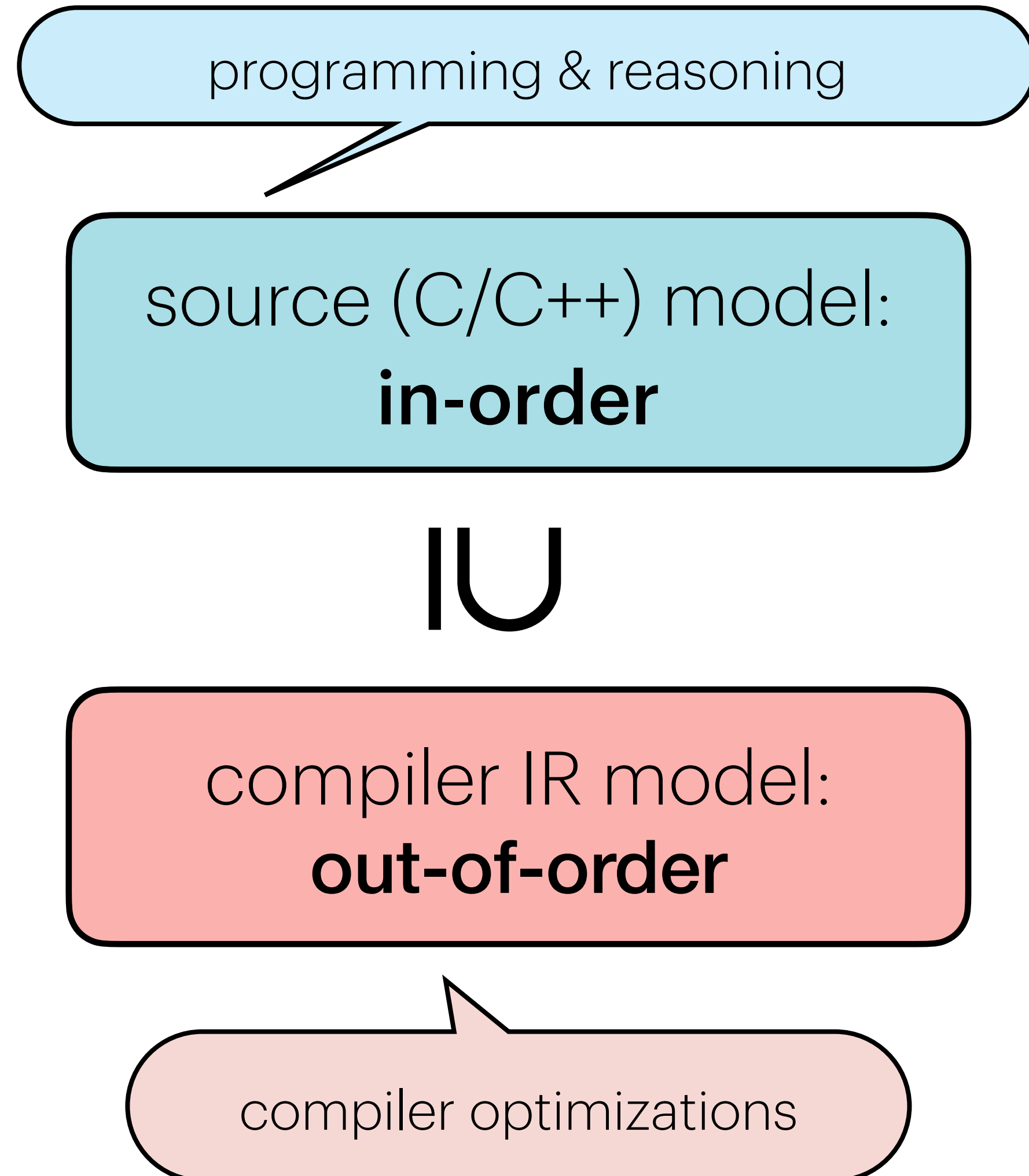
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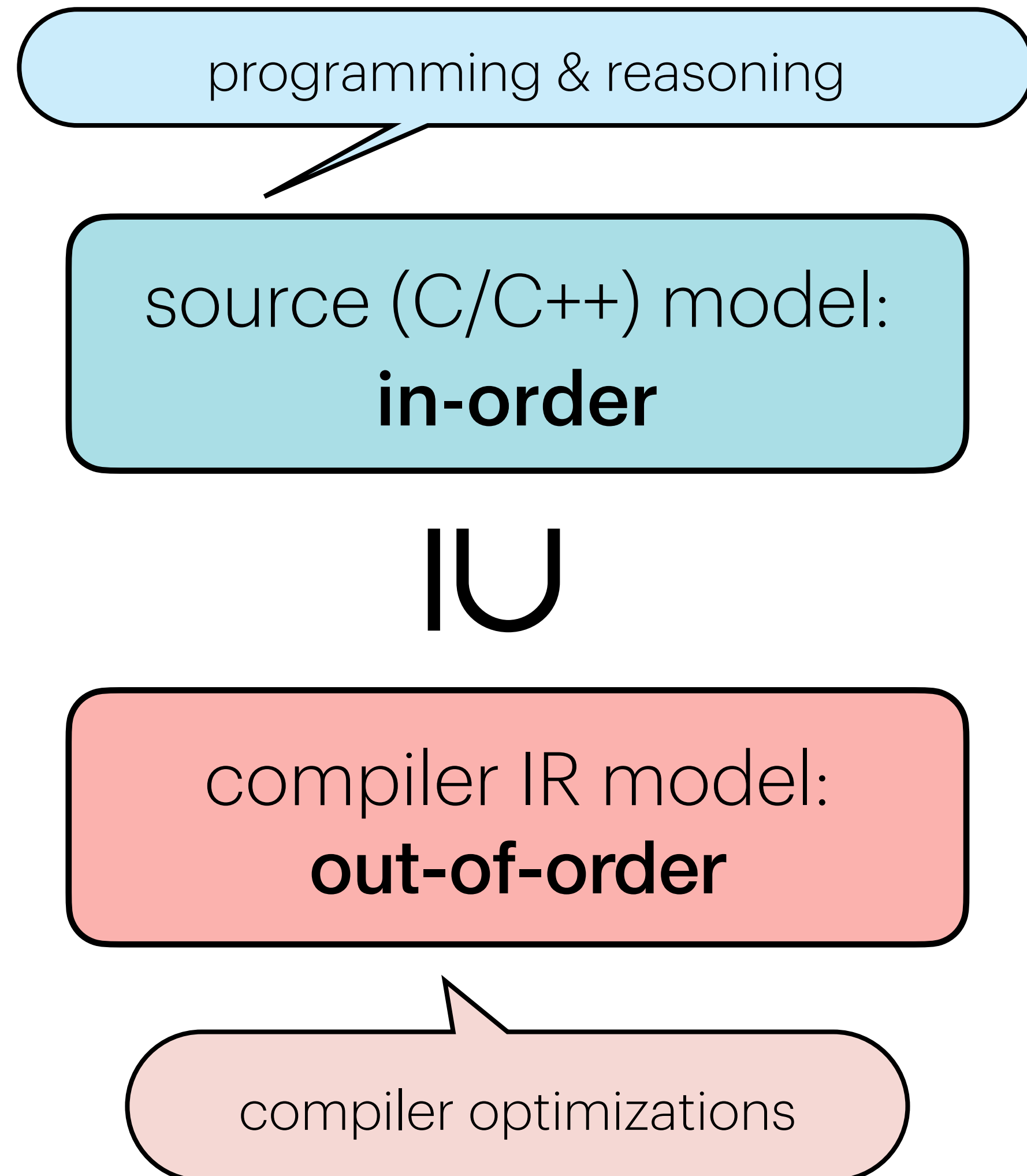
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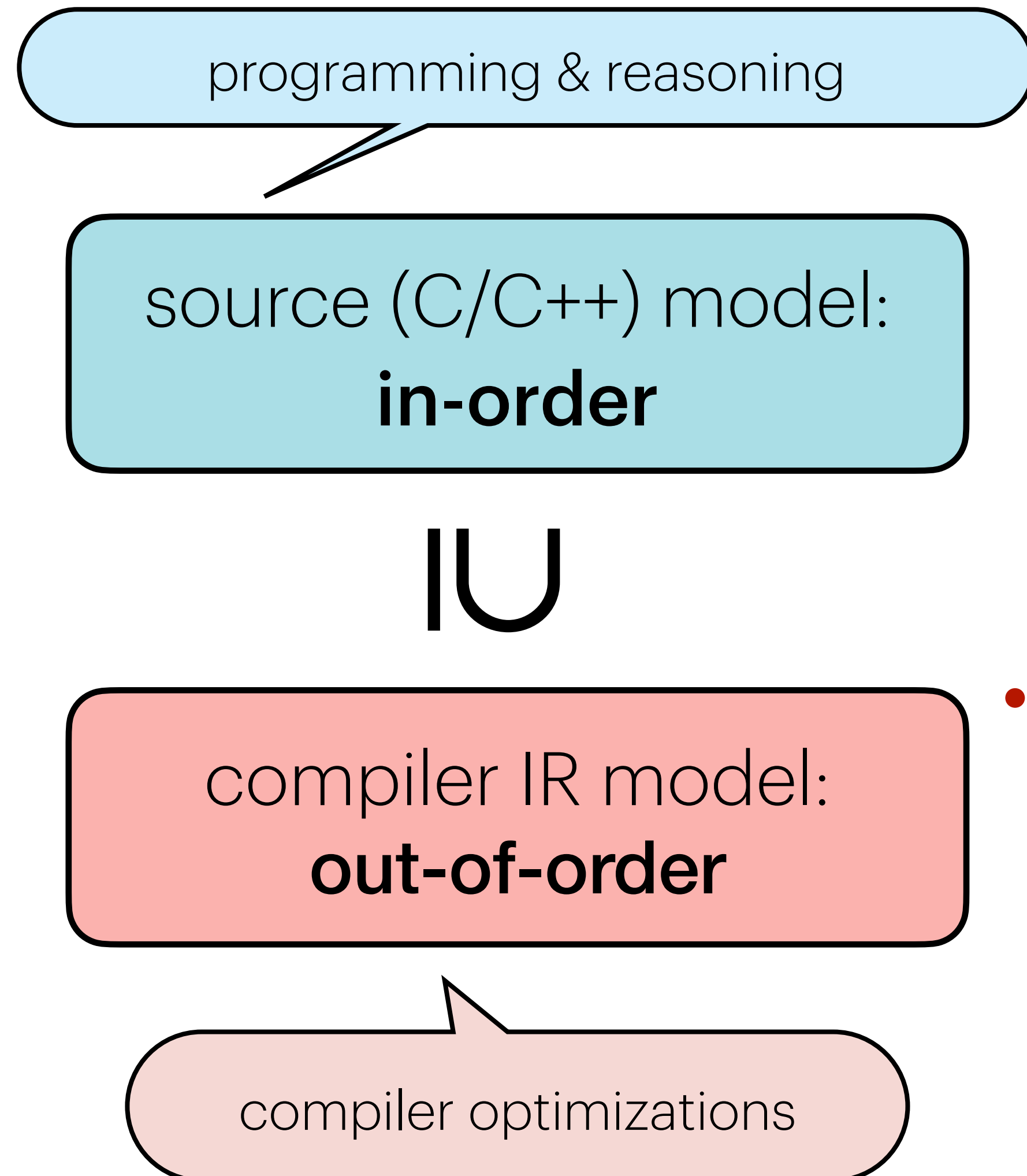
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- In-order source model
  - Undefined behavior on WW & WR races
  - Based on RC11

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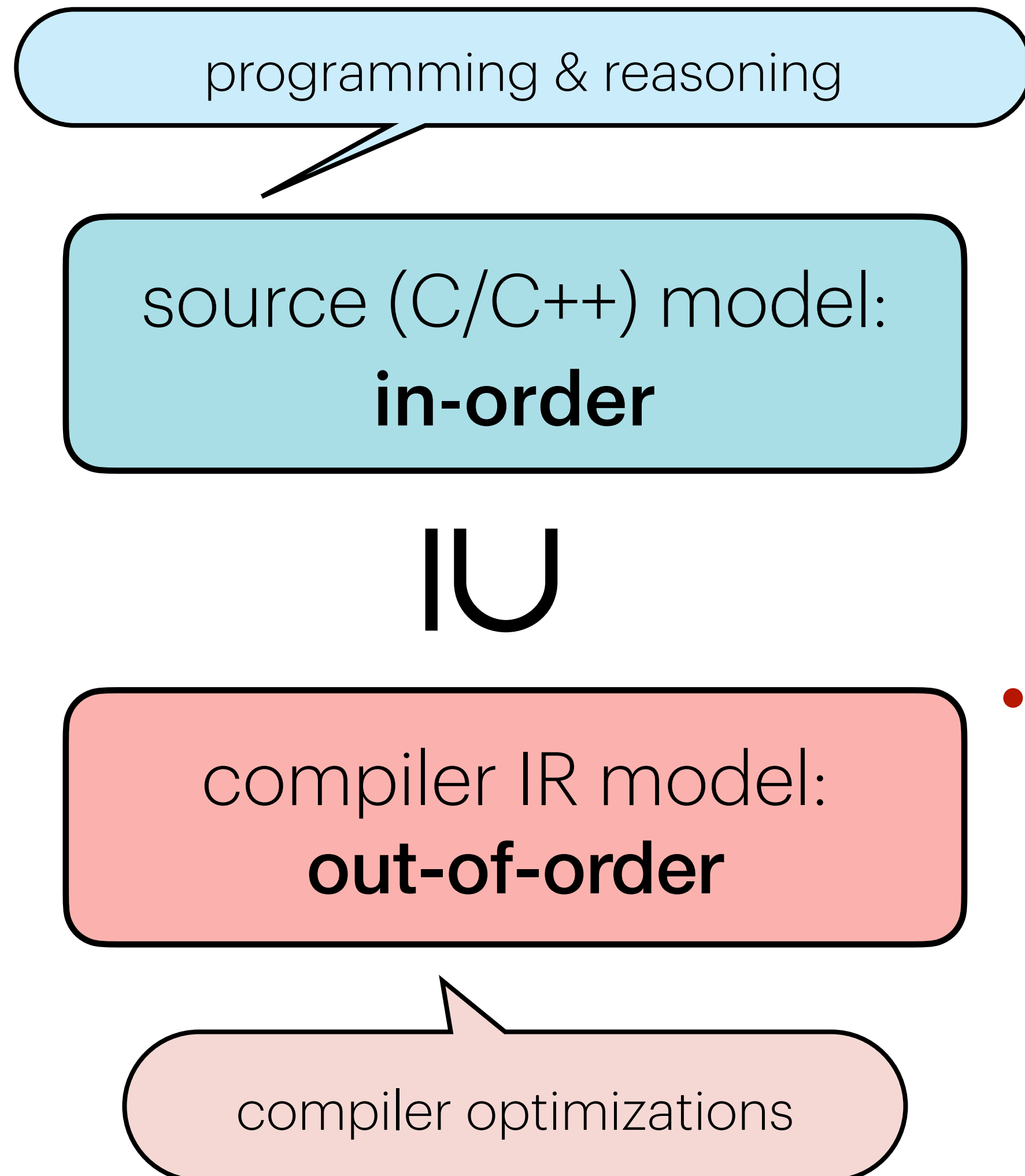
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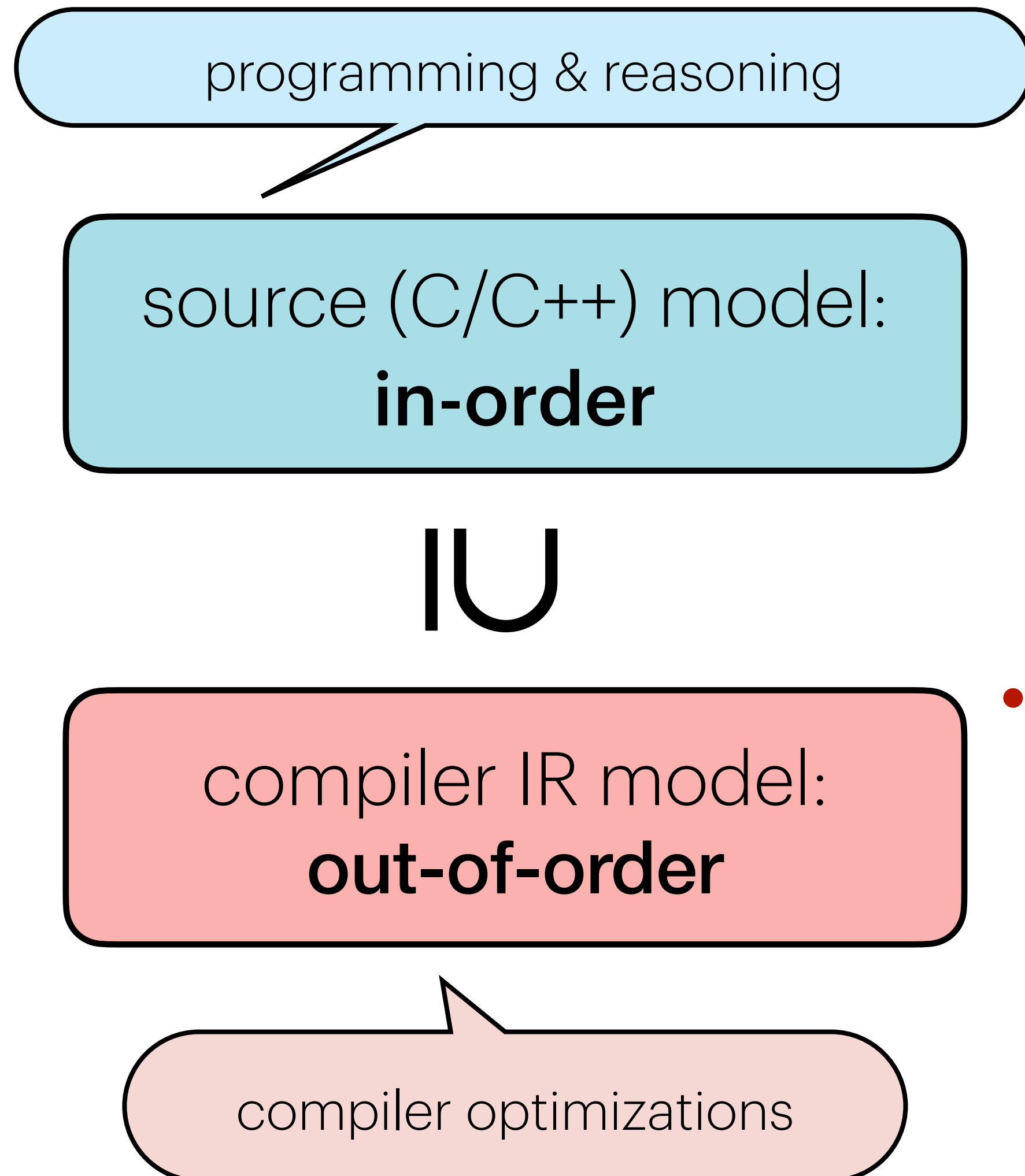
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RW reordering
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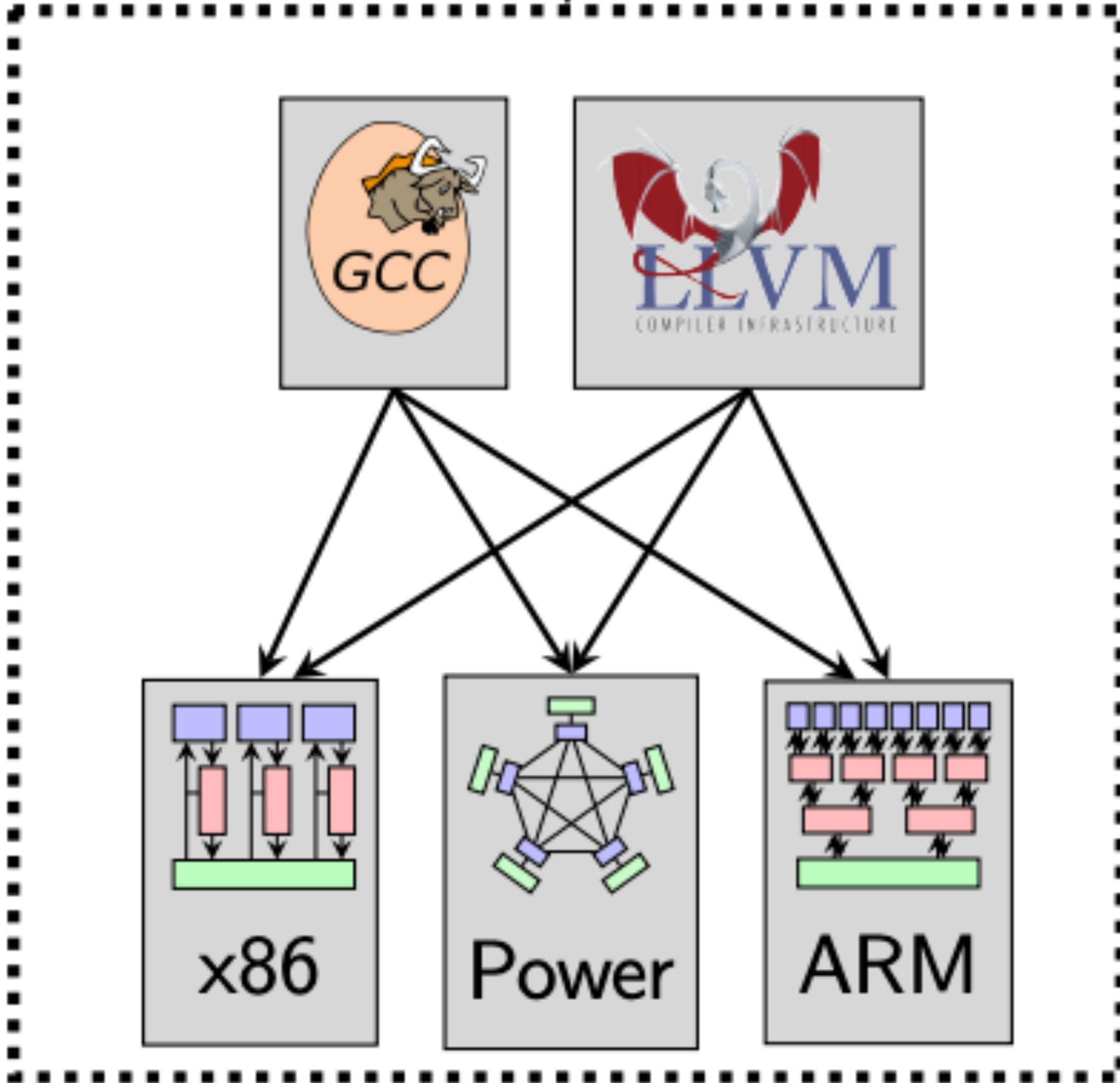
RW reordering

load  
introduction



programmers

The (R)C11 memory model



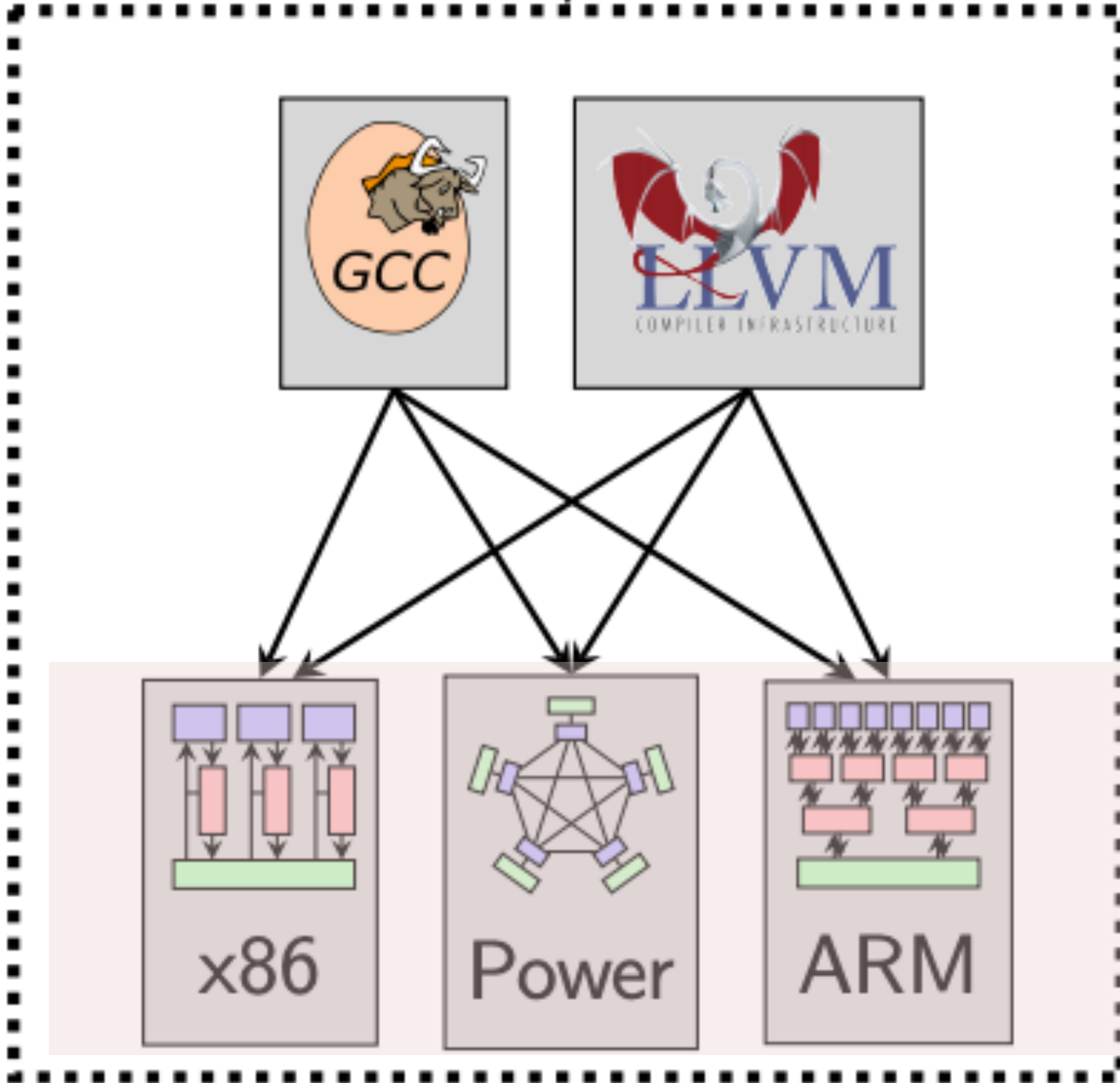
compilers

multicore architectures



programmers

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# Implementability on multicore hardware

<https://www.cl.cam.ac.uk/~pes20/cpp/cpp0xmappings.html>

$(R)$	$\triangleq$ MOV (from memory)	$(W^{\square rel})$	$\triangleq$ MOV (to memory)
$(W^{sc})$	$\triangleq$ MOV; MFENCE	$(RMW)$	$\triangleq$ CMPXCHG
$(F^{\neq sc})$	$\triangleq$ No operation	$(F^{sc})$	$\triangleq$ MFENCE

**Figure 8.** Compilation to TSO.

$(R^{na})$	$\triangleq$ ld	$(W^{na})$	$\triangleq$ st
$(R^{rlx})$	$\triangleq$ ld;c	$(W^{rlx})$	$\triangleq$ st
$(R^{acq})$	$\triangleq$ ld;cmp;bc;isync	$(W^{rel})$	$\triangleq$ lwsync;st
$(F^{\neq sc})$	$\triangleq$ lwsync	$(F^{sc})$	$\triangleq$ sync
$(RMW^{rlx})$	$\triangleq$ L:lwarx;cmp;bc Le;stwcx.;bc L;Le:		
$(RMW^{acq})$	$\triangleq$ $(RMW^{rlx});isync$		
$(RMW^{rel})$	$\triangleq$ lwsync; $(RMW^{rlx})$		
$(RMW^{acqrel})$	$\triangleq$ lwsync; $(RMW^{rlx});isync$		

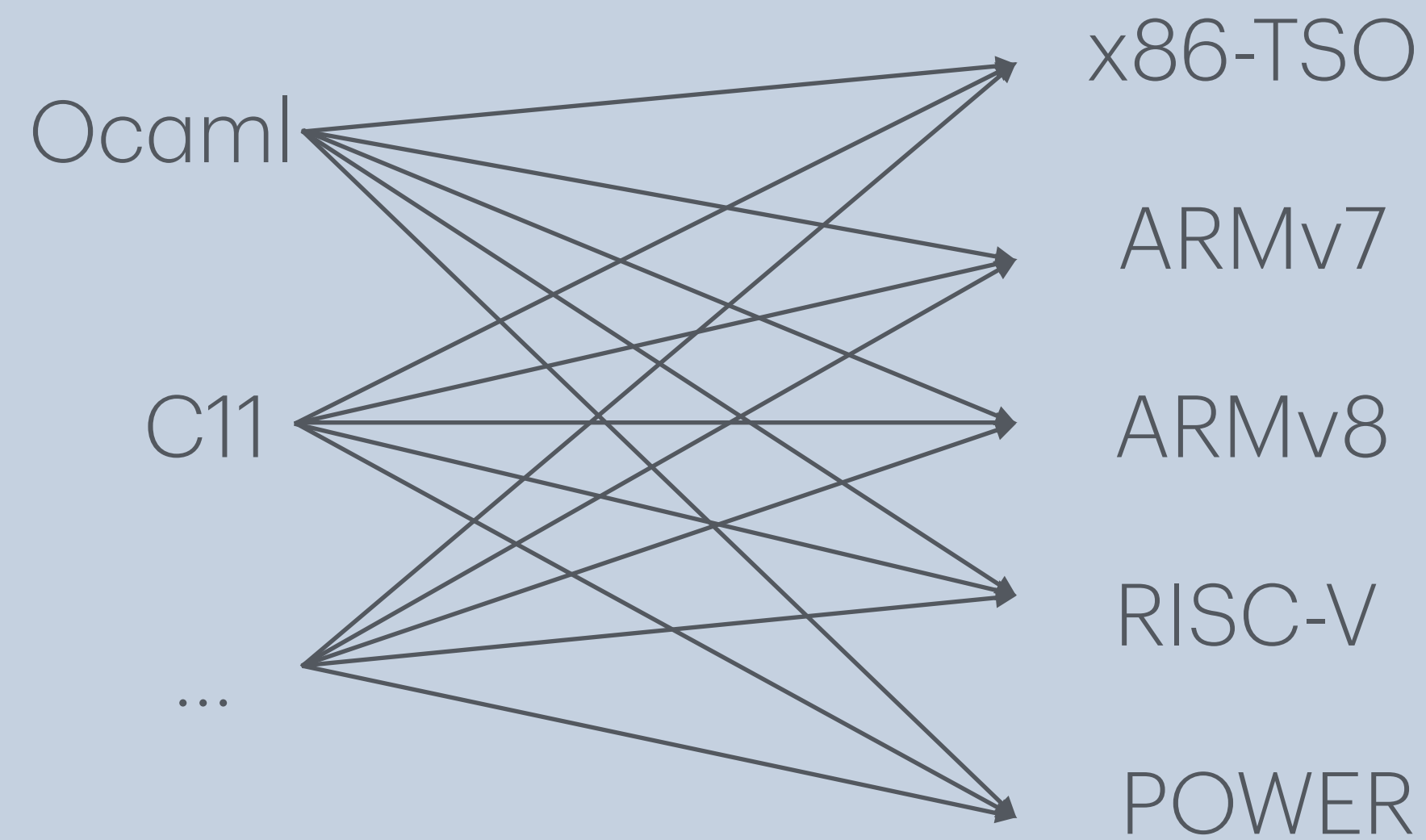
**Figure 9.** Compilation of non-SC primitives to Power.

Leading sync	Trailing sync		
$(R^{sc})$	$\triangleq$ sync; $(R^{acq})$	$(R^{sc})$	$\triangleq$ ld;sync
$(W^{sc})$	$\triangleq$ sync;st	$(W^{sc})$	$\triangleq$ $(W^{rel});sync$
$(RMW^{sc})$	$\triangleq$ sync; $(RMW^{acq})$	$(RMW^{sc})$	$\triangleq$ $(RMW^{rel});sync$

**Figure 10.** Compilations of SC accesses to Power.

# Mapping correctness

- $\llbracket C \rrbracket$  : mapping a program  $C$  to a given hardware
- A mapping is *correct* if  $\text{Behaviors}_{PL}(C) \subseteq \text{Behaviors}_{Hardware}(\llbracket C \rrbracket)$

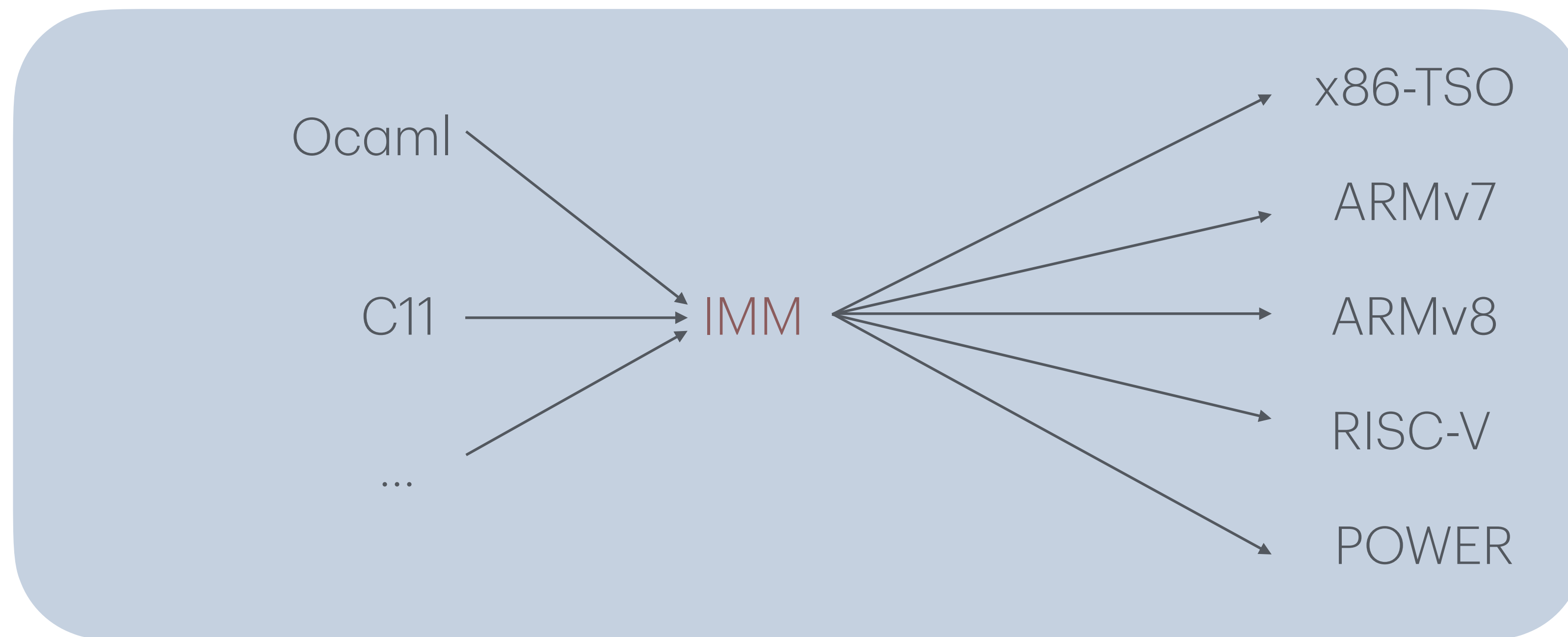




# An intermediate memory model

Podkopaev, L, Vafeiadis: **Bridging the gap between programming languages and hardware weak memory models**. POPL 2019. <https://doi.org/10.1145/3290382>

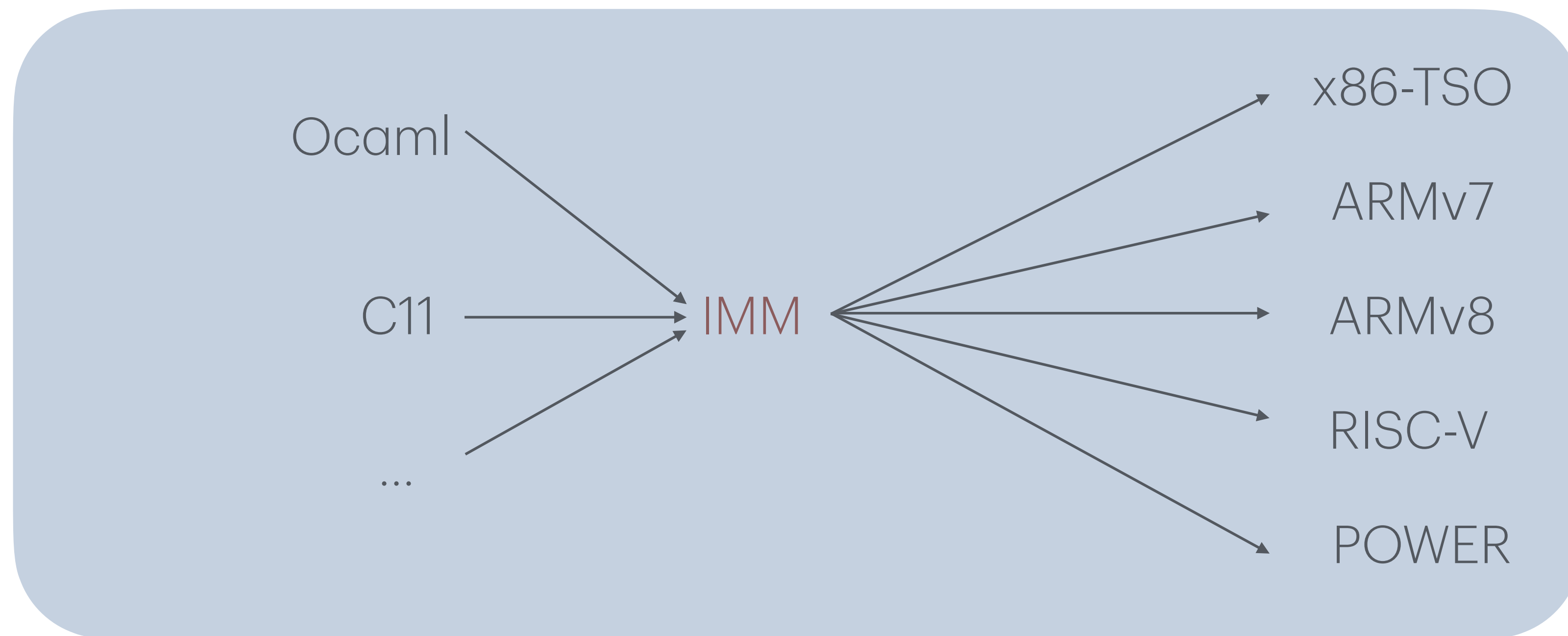
- **IMM model** as a common denominator of existing hardware weak memory models



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- **IMM model** as a common denominator of existing hardware weak memory models



# Programming guarantees

- Data-race-freedom (DRF) theorems
- Library abstraction

# Motivation for the DRF guarantee

- Weak memory models are **complex**
  - most programmers do not understand the underlying model
- We would like to provide a **defensive programming discipline** for non-experts:
  - ensures **strong and more intuitive semantics**
  - can be followed **without understanding** the full underlying weak memory model
- This was a main design goal for C11, let's make it more formal...



# The DRF guarantee

First attempt

If a program P satisfies:

- has only non-atomics and locks
- is race-free

Then:

- P has only SC behaviors

no consistent execution graph  
has conflicting events unordered by *hb*

```
X := 1
lock(L)
Y := 1
unlock(L)
```

```
lock(L)
a := Y
unlock(L)
if (a=1) then
  b := X
```

# The DRF guarantee

First attempt

If a program P satisfies:

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- Definition of races still requires to understand execution graphs, consistency, *hb*...

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  - Definition of races still requires to understand execution graphs, consistency, *hb*...
  - We also want to allow the use of **atomics** for avoiding races



# The DRF guarantee

## Second attempt

```
X := 1
lock(L)
Y := 1
unlock(L)
```

```
lock(L)
a := Y
unlock(L)
if (a=1) then
  b := X
```

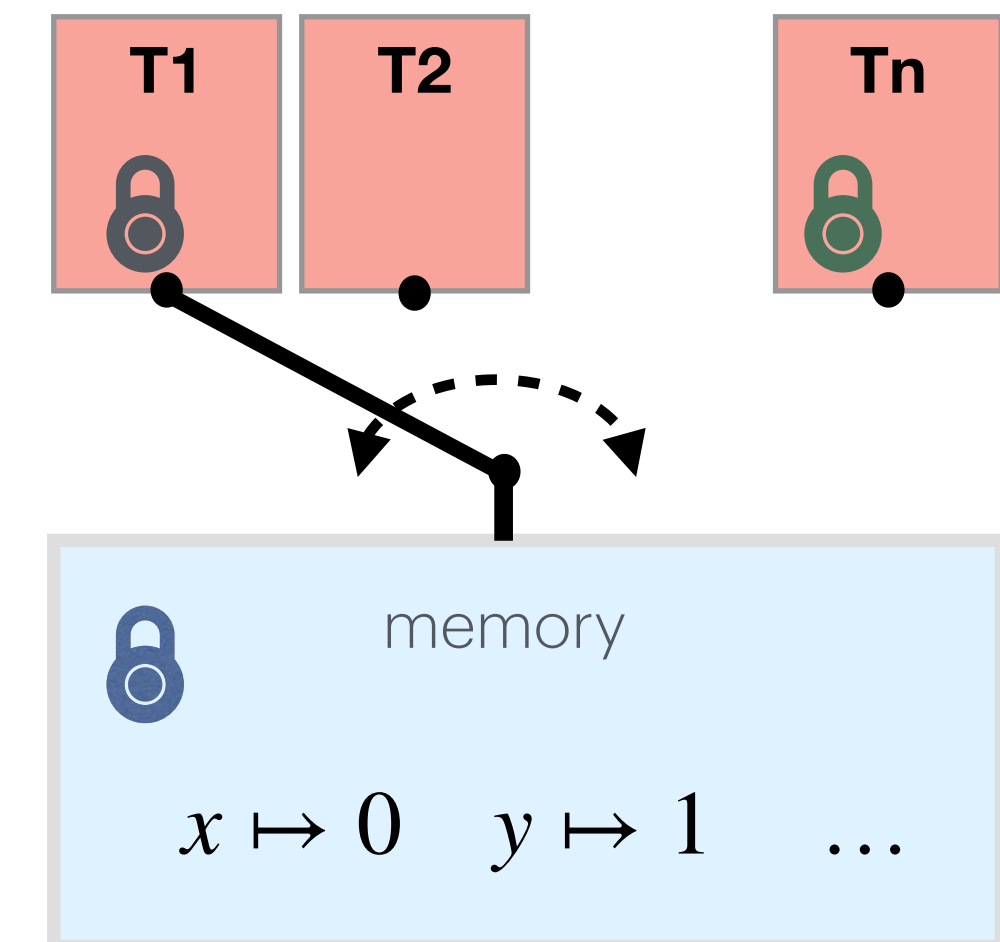
If a program P satisfies:

- has only non-atomics and locks
- is race-free under SC

Then:

- P has only SC behaviors

in every SC operational trace of P there are no consecutive conflicting accesses by different threads



- Is this good enough?

- ~~Definition of races still requires to understand execution graphs, consistency, *hb*...~~
- We also want to allow the use of **atomics** for avoiding races

# The DRF guarantee

Final formulation

```
X := 1
Y := 1 sc
```

```
a := Y sc
if (a=1) then
  b := X
```

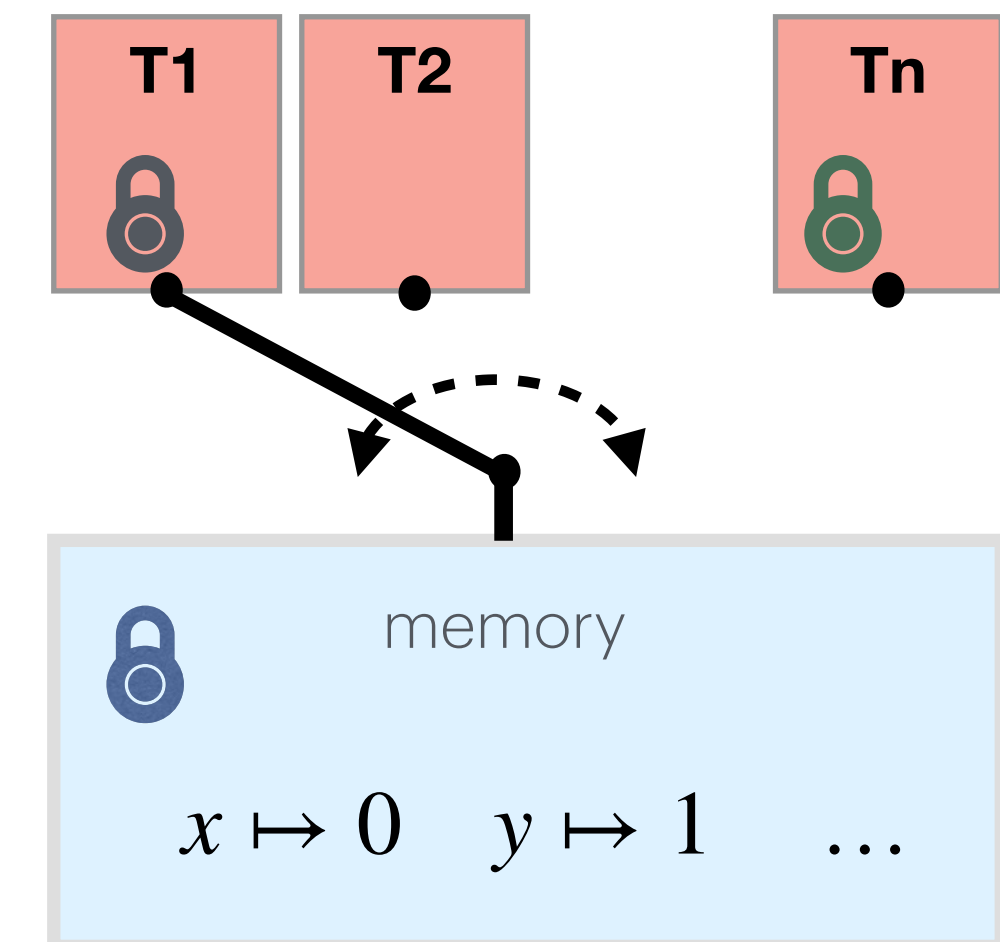
If a program P satisfies:

- ~~has only non-atomics and locks~~
- all races under SC semantics are on **sc** atomic accesses

Then:

- P has only SC behaviors

in every SC operational trace of P,  
all consecutive conflicting accesses by  
different threads **are marked as SC** in P



- Is this good enough?

- ~~Definition of races still requires to understand execution graphs, consistency, *hb*...~~
- ~~We also want to allow the use of **atomics** for avoiding races~~

# Other DRF guarantees

- The assumption of the DRF guarantee is sometimes *expensive* to satisfy
- The conclusion is also very strong
- We would like to use *another semantics* instead of SC in the role of the *strong semantics*
- Let's see how it works for the Release/Acquire model (DRF-RA)

# The DRF-RA guarantee

```
X := 1
Y := 1 rel
```

```
a := Y acq
if (a=1) then
  b := X
```

If a program P satisfies:

- all races under RA semantics are on **rel/acq** atomic accesses

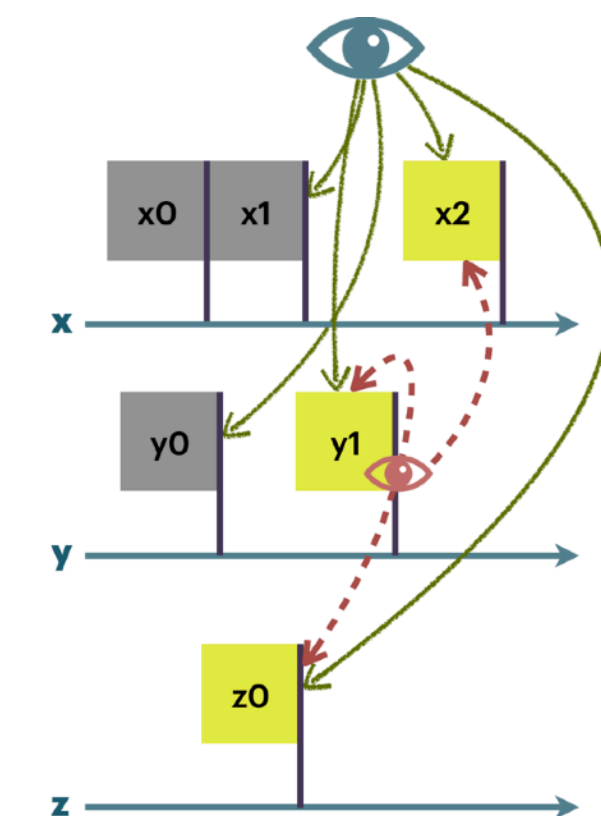
Then:

- P has only **RA** behaviors

in every RA-consistent execution graph of P,  
every pair of conflicting accesses unordered by *hb*  
are marked as **rel/acq** in P

There is also a formulation using the  
RA view-based semantics:

- A race = thread accesses X but not aware of the latest msg



# Local DRF-SC guarantee

- The assumptions above are *global*, which hinders modularity

- A local version can consider a set *Loc* of locations

- Let  $P[\mathbf{Loc} := \mathbf{sc}]$  denote the program P where all accesses to *Loc* are strengthened to **sc**

If:

- all races of  $P[\mathbf{Loc} := \mathbf{sc}]$  on locations in *Loc* under RC11 semantics are on accesses marked as **sc** in P

Then:

- every behavior of P is a behavior of  $P[\mathbf{Loc} := \mathbf{sc}]$

```
a := pop(S)
X := a
Y := 1 sc
```

```
b := pop(S)
c := Y sc
if (c=1) then
  d := X
```

$\mathbf{Loc} = \{X, Y\}$

Dolan, Sivaramakrishnan, Madhavapeddy: **Bounding data races in space and time**. PLDI 2018. <https://doi.org/10.1145/3192366.3192421>

Cho, Lee, Hur, L: **Modular data-race-freedom guarantees in the promising semantics**. PLDI 2021. <https://doi.org/10.1145/3453483.3454082>

# Local DRF-RA guarantee

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a := pop(S)
X := a
Y := 1 rel
```

```
b := pop(S)
c := Y acq
if (c=1) then
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```

$Loc = \{X, Y\}$

- Let  $P[Loc := \mathbf{ra}]$  denote the program  $P$  where all accesses to  $Loc$  are strengthened to **rel/acq**

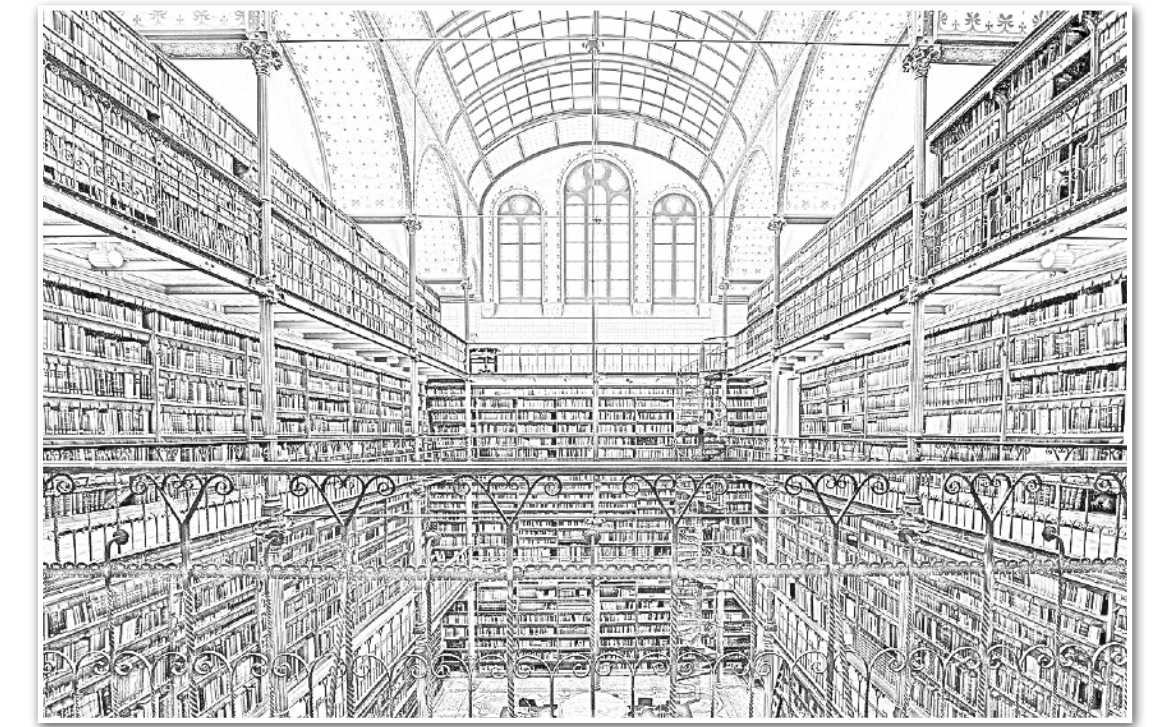
If:

- all races of  $P[Loc := \mathbf{ra}]$  on locations in  $Loc$  under RC11 semantics are on accesses marked as **rel/acq** in  $P$

Then:

- every behavior of  $P$  is a behavior of  $P[Loc := \mathbf{ra}]$

# Library abstraction



- **Experts** develop optimized concurrent objects implementations (aka *libraries*)
  - once and for all establish correctness w.r.t. their specifications
- **Clients** of these implementations reason about program behaviors assuming only the specifications
- Essential in programming, and even more critical in complicated concurrency models

This part is based on:

Singh, L: **An Operational Approach to Library Abstraction under Relaxed Memory Concurrency**. POPL 2023.

<https://doi.org/10.1145/3571246>

# Code as specification

- Specification = *reference implementation*
- Simpler (and less efficient) than the implementation
- Derive a reference implementation from a standard sequential specification *Spec* (assuming SC):

Take some sequential implementation of *Spec* and wrap each method in an *atomic block*

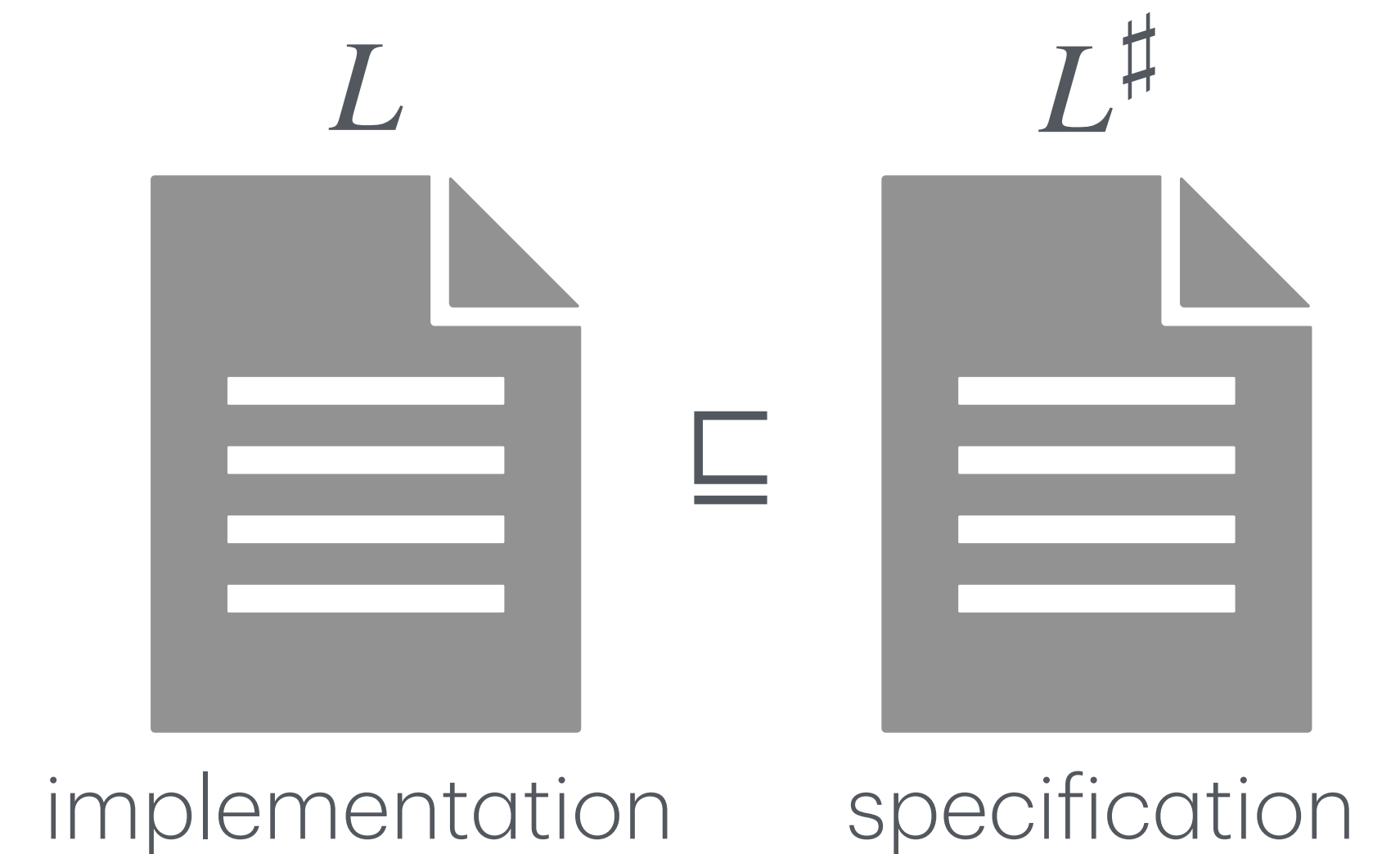
e.g., `enqueue(v) { ... } → enqueue(v) { atomic { ... } }`



specification construct



# Library correctness



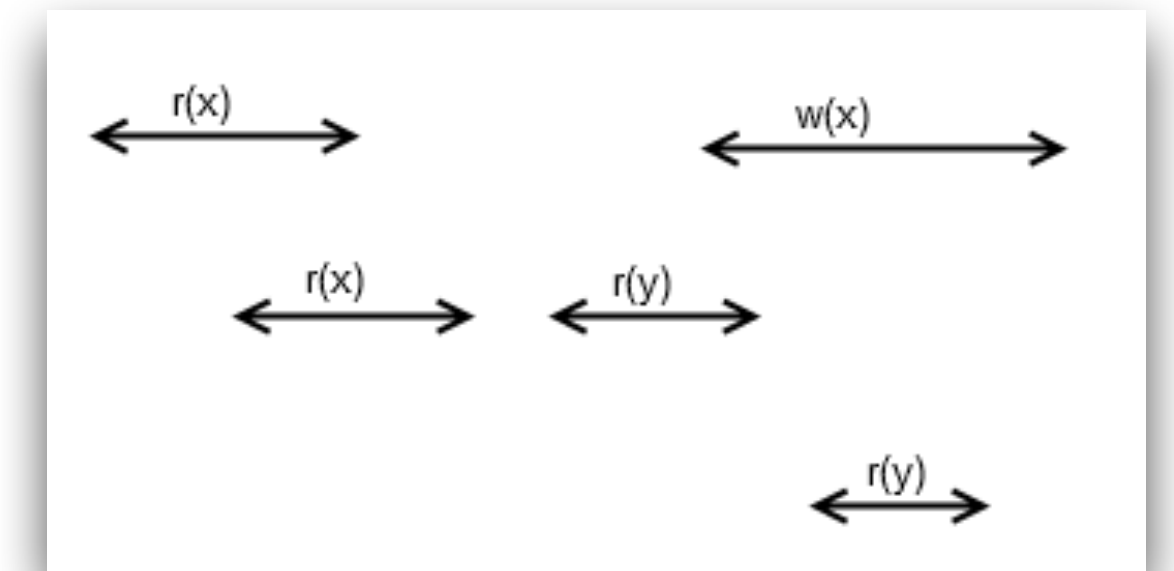
- We aim to have *contextual refinement*:

for every program  $P$ ,  $\mathbf{Behaviors}(P[L]) \subseteq \mathbf{Behaviors}(P[L^\#])$

- We assume that the client and the library use *disjoint set of locations*
- What *correctness condition* ensures contextual refinement?

# Linearizability

as a library correctness condition under SC



For concurrent data-structures, under SC, **linearizability ensures refinement**:

Filipović, O'Hearn, Rinetzky, Yang: **Abstraction for concurrent objects**. Theoretical Computer Science 2010. <https://doi.org/10.1016/j.tcs.2010.09.021>

- If  $L$  is linearizable wrt a sequential specification  $Spec$ , then for every program  $P$ ,  
 $\mathbf{Behaviors}(P[L]) \subseteq \mathbf{Behaviors}(P[L^\#(Spec)])$  under SC

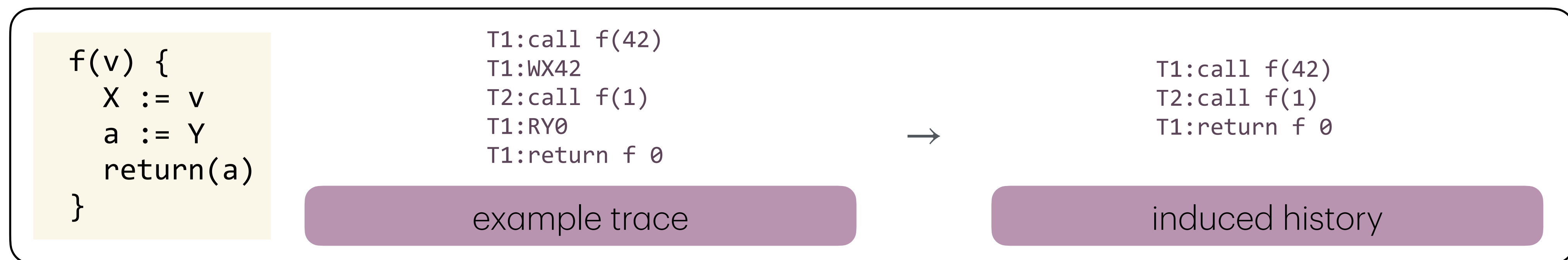
the reference implementation  
derived from  $Spec$

- The converse direction also holds

# Linearizability = history inclusion

Linearizability of  $L$  wrt  $Spec$  holds iff  $\text{Histories}(MGC[L]) \subseteq \text{Histories}(MGC[L^\#(Spec)])$ , where:

- $L^\#(Spec)$  is the reference implementation derived from the sequential specification  $Spec$
- $MGC$  denotes the **most general client**:  
concurrently and repeatedly call the methods of the library with arbitrary arguments
- **History** is a restriction of an operational trace to call/return



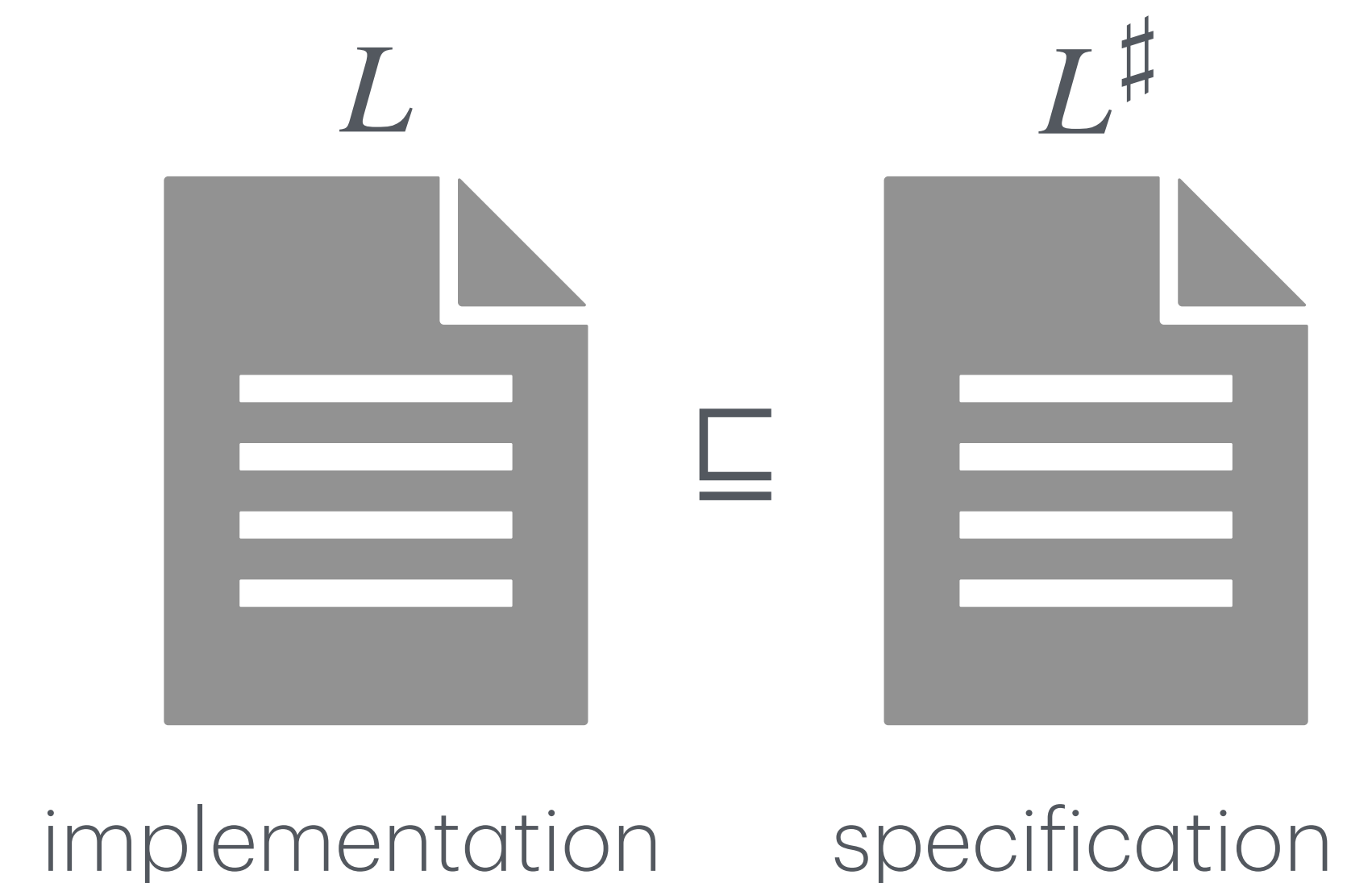
- $\text{Histories}(P)$  denotes the set of histories induced by traces of program  $P$

# A more general abstraction theorem (for SC)

## Theorem

If  $\text{Histories}(MGC[L]) \subseteq \text{Histories}(MGC[L^\#])$ ,  
then for every program  $P$ ,  $\text{Behaviors}(P[L]) \subseteq \text{Behaviors}(P[L^\#])$

- Refinement via linearizability is a particular instance
- This theorem also allows **non-atomic specifications**



# Example: SC assumption is essential!

Assumptions:

- **foo** and **bar** must be called at most once by different threads
- **bar** must be called after **foo** in the execution order

```
foo() {  
  X := 1 rel  
  return()  
}
```

```
bar() {  
  a := X acq  
  return(a)  
}
```

specification

$L^\#$

# Example: SC assumption is essential!

Assumptions:

- **foo** and **bar** must be called at most once by different threads
- **bar** must be called after **foo** in the execution order

```
foo() {  
  return()  
}
```

```
foo() {  
  X := 1 rel  
  return()  
}
```

```
bar() {  
  pick a∈{0,1}  
  return(a)  
}
```

```
bar() {  
  a := X acq  
  return(a)  
}
```

implementation

specification

$L$

$L^\#$

# Example: SC assumption is essential!

Assumptions:

- **foo** and **bar** must be called at most once by different threads
- **bar** must be called after **foo** in the execution order

```
foo() {  
  return()  
}
```

```
bar() {  
  pick a∈{0,1}  
  return(a)  
}
```

implementation

$L$

```
foo() {  
  X := 1 rel  
  return()  
}
```

```
bar() {  
  a := X acq  
  return(a)  
}
```

specification

$L^\#$

Histories( $MGC[L]$ )

=

Histories( $MGC[L^\#]$ )

```
T1:call foo()  
T1:return foo  
T2:call bar()  
T2:return bar 0
```

```
T1:call foo()  
T1:return foo  
T2:call bar()  
T2:return bar 1
```

# Example: SC assumption is essential!

Assumptions:

- **foo** and **bar** must be called at most once by different threads
- **bar** must be called after **foo** in the execution order

```
Z := 1 rlx
foo()
Y := 1 rlx
```

```
a := Y rlx
if (a=1) then
  c := bar() // 1
  d := Z rlx // 0
```

```
foo() {
  return()
}
```

```
foo() {
  X := 1 rel
  return()
}
```

```
bar() {
  pick a∈{0,1}
  return(a)
}
```

```
bar() {
  a := X acq
  return(a)
}
```

implementation

specification

$L$

$L^\#$

This behavior is:

- impossible with the specification  $L^\#$
- but, possible with the implementation  $L$ !



# Example: SC assumption is essential!

Assumptions:

- **foo** and **bar** must be called at most once by different threads
- **bar** must be called after **foo** in the execution order

```
foo() {  
  return()  
}
```

```
foo() {  
  X := 1 rel  
  return()  
}
```

```
bar() {  
  pick a ∈ {0,1}  
  return(a)  
}
```

```
bar() {  
  a := X acq  
  return(a)  
}
```

implementation

$L$

specification

$L^\#$

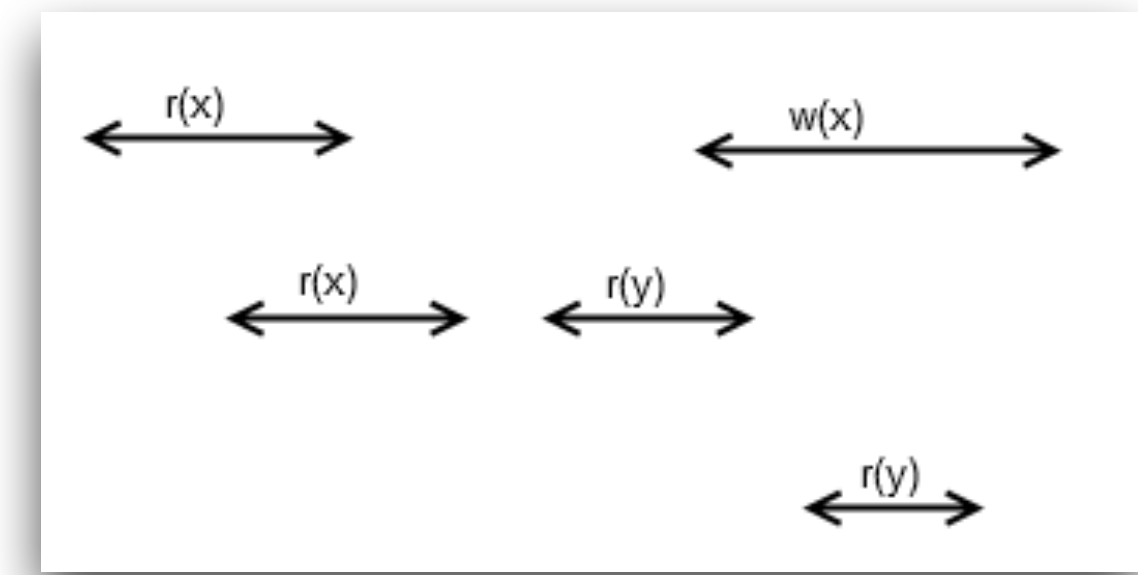
```
foo()  
Y := 1 rel
```

```
a := Y acq  
if (a=1) then  
  c := bar() //  $\emptyset$ 
```

This behavior is:

- impossible with the specification  $L^\#$
- but, possible with the implementation  $L$ !

# What can we do about it?



- Under WMM client-library interaction is **not** fully captured by call/return histories
- We can work with **partial orders** (akin to execution graphs) rather than sequential histories:
  - Batty, Dodds, Gotsman: **Library abstraction for C/C++ concurrency**. POPL 2013. <https://doi.org/10.1145/2429069.2429099>
  - Doherty, Dongol, Wehrheim, Derrick: **Making Linearizability Compositional for Partially Ordered Executions**. IFM 2018. [https://doi.org/10.1007/978-3-319-98938-9\\_7](https://doi.org/10.1007/978-3-319-98938-9_7)
- Or **enrich sequential histories** with more information:
  - Burckhardt, Gotsman, Musuvathi, Yang: **Concurrent Library Correctness on the TSO Memory Model**. ESOP 2012. [https://doi.org/10.1007/978-3-642-28869-2\\_5](https://doi.org/10.1007/978-3-642-28869-2_5)
  - Khyzha, L: **Abstraction for Crash-Resilient Objects**. ESOP 2022. [https://doi.org/10.1007/978-3-030-99336-8\\_10](https://doi.org/10.1007/978-3-030-99336-8_10)

# Enriched histories for RC11

```
foo()  
Y := 1 rel
```

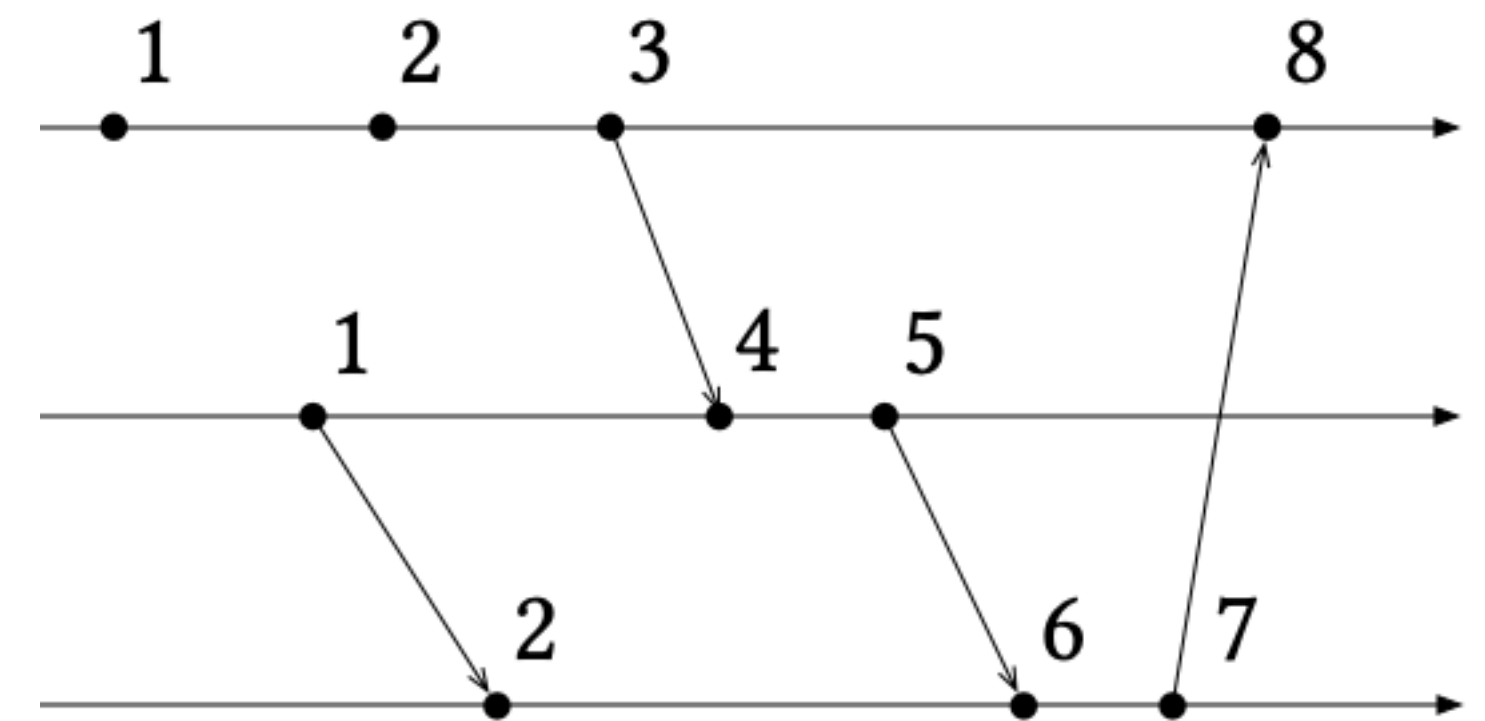
```
a := Y acq  
if (a=1) then  
  c := bar() //  $\emptyset$ 
```

- The read **Y=1** imposes *hb* order, so **T2** must be aware of **foo()**'s effect when **bar()** is called
- We will expose this in histories by including propagations of call/return

real-time order  $\Rightarrow$  happens before

# A propagation semantics for RC11

- A novel **operational semantics** for (a fragment of) RC11
- Explicit **point-to-point propagation transitions** marking when an event of one thread becomes visible to another thread
- We include **propagation of the call/return events** in memory traces



# Example: MP

```
Y := 42 rel  
X := 1 rel
```

```
a := X acq  
if (a=1) then  
  b := Y acq //  $\emptyset$ 
```

some possible traces:

```
T1:Wy42  
T1:Wx1  
T2:Rx0
```

```
T1:Wy42  
T1:Wx1  
T1→T2:Wy42  
T2:Rx0
```

```
T1:Wy42  
T1:Wx1  
T1→T2:Wy42  
T1→T2:Wx1  
T2:Rx1  
T2:Ry42
```

In the Release/Acquire fragment:

- propagation follows *hb*
- read from the *mo*-maximal write that was propagated to the thread

# Example with function calls

```
foo() {  
  X := 1 rel  
  return()  
}
```

```
bar() {  
  a := X acq  
  return(a)  
}
```

```
foo()  
Y := 1 rel
```

```
a := Y acq  
if (a=1) then  
  c := bar()
```

we include propagations of calls/  
returns in histories

a possible trace

```
T1:call foo()  
T1:Wx1  
T1:return foo  
T1:Wy1  
T1→T2:call foo()  
T1→T2:Wx1  
T1→T2:return foo()  
T1→T2:Wy1  
T2:Ry1  
T2:call bar()  
T2:Rx1  
T2:return bar 1
```

# Example

induced *enriched* histories of MGC

```
foo() {  
  return()  
}
```

```
foo() {  
  X := 1 rel  
  return()  
}
```

```
bar() {  
  pick a∈{0,1}  
  return(a)  
}
```

```
bar() {  
  a := X acq  
  return(a)  
}
```

implementation

specification

$L$

$L^\#$

T1:call foo()

T1:return foo

T1→T2:call foo()

T1→T2:return foo()

T2:call bar()

T2:return bar 0

T1:call foo()

T1:return foo

T2:call bar()

T2:return bar 1

possible for  $L$  but not for  $L^\#$  !

# Abstraction theorem for RC11

## Theorem

If  $\text{Histories}(MGC[L]) \subseteq \text{Histories}(MGC[L^\#])$ ,  
then for every program  $P$ ,  $\text{Behaviors}(P[L]) \subseteq \text{Behaviors}(P[L^\#])$  **under SC**

## Theorem

If  $\text{PHistories}(MGC[L]) \subseteq \text{PHistories}(MGC[L^\#])$ ,  
then for every program  $P$ ,  $\text{Behaviors}(P[L]) \subseteq \text{Behaviors}(P[L^\#])$  **under RC11**

where  $\text{PHistories}(P)$  denotes the set of **enriched histories** (with calls/returns/call propagations/return propagations) induced by traces of program  $P$



# Application: RCU



- Simple **lock-based** specification for basic Read-Copy-Update (RCU) primitives under RC11
  - unlike existing **declarative** ad-hoc specifications
  - RCU in client programs on RC11 can be understood **via locks**
- We used the the **FDR4 refinement checker** library correctness for a simple RCU implementation from:

**FDR4**

Alglave, Maranget, McKenney, Parri, Stern: **Frightening Small Children and Disconcerting Grown-ups: Concurrency in the Linux Kernel.** ASPLOS 2018. <https://doi.org/10.1145/3173162.3177156>

# Restricted clients

Libraries often have “calling policies”  
(e.g., single producer, consume only non-empty collections, ...)

public member function

**std::list::pop\_front**

---

```
void pop_front();
```

**Delete first element**

Removes the first element in the `list` container, effectively reducing its `size` by one.

● **Exception safety**

---

If the container is not `empty`, the function never throws exceptions (no-throw guarantee).  
Otherwise, it causes *undefined behavior*.

# Restricted clients

```
push(1)
a := pop()
```

- We would like a stronger theorem:

## Theorem

- If  $\text{PHistories}(MGC_{\text{policy}}[L]) \subseteq \text{PHistories}(MGC_{\text{policy}}[L^\#])$ ,  
then for every program  $P$  that adheres to the policy, **Behaviors** $(P[L]) \subseteq \text{Behaviors}(P[L^\#])$  **under RC11**
- To show that  $P$  adheres to policy, should we use  $L$  or  $L^\#$ ?
- We want it to be  $L^\#$  so that the theorem can be applied **without any knowledge of  $L$** !
- Circular argument? induction works!

# Restricted clients

```
push(1)
a := pop()
```

- We would like a stronger theorem:

## Theorem

- If  $\text{PHistories}(MGC_{\text{policy}}[L]) \subseteq \text{PHistories}(MGC_{\text{policy}}[L^\#])$ ,  
then for every program  $P$  that adheres to the policy,  $\text{Behaviors}(P[L]) \subseteq \text{Behaviors}(P[L^\#])$  **under RC11**
- To show that  $P$  adheres to policy, should we use  $L$  or  $L^\#$ ?
- We want it to be  $L^\#$  so that the theorem can be applied **without any knowledge of  $L$** !
- Circular argument? induction works!

# The final abstraction theorem



- If the following hold:

1.  $\text{PHistories}(MGC_{policy}[L]) \subseteq \text{PHistories}(MGC_{policy}[L^\#])$

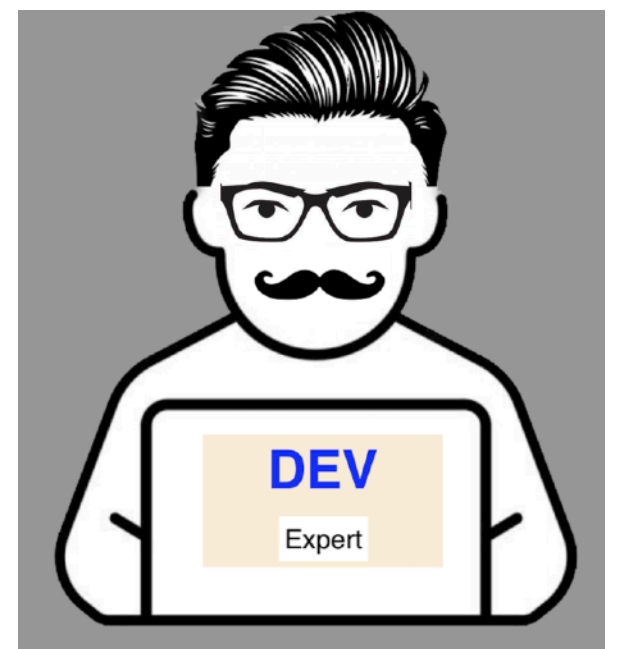
2.  $MGC_{policy}[L]$  is not racy

3.  $\text{PHistories}(P[L^\#]) \subseteq \text{PHistories}(MGC_{policy}[L^\#])$

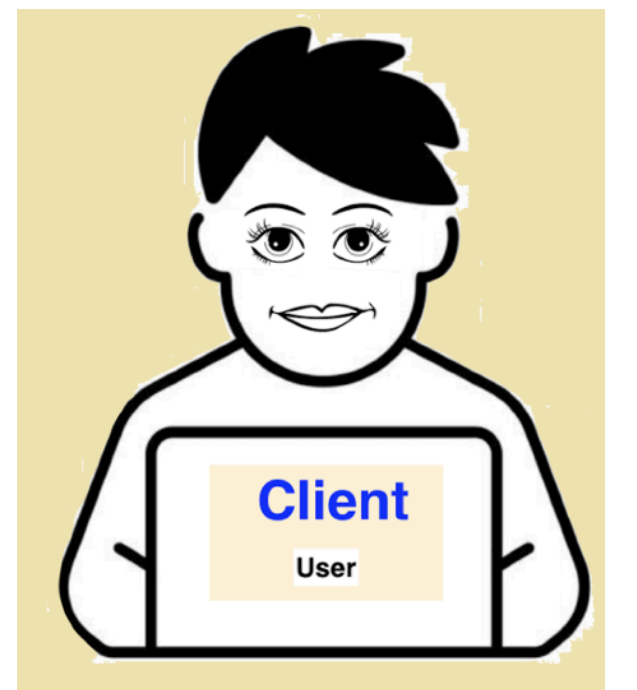
4.  $P[L^\#]$  is not racy

- Then:  $\text{Behaviors}(P[L]) \subseteq \text{Behaviors}(P[L^\#])$

developer  
obligations



client  
obligations



# LDRF-RA via library abstraction

```
writex(v) {
  X := v rel
  return()
}
```

```
writex(v) {
  X := v
  return()
}
```

```
readx() {
  a := X acq
  return(a)
}
```

```
readx() {
  a := X
  return(a)
}
```

Specification

$L^\#$

Implementation

$L$

all races of  $P[Loc := \mathbf{ra}]$  on locations in  $Loc$  under RC11 semantics are on accesses marked as **rel/acq** in  $P$

$Loc$  = set of locations accesses solely by the library

$MGC_{policy}$  = call methods in a way that avoids races between **write<sub>x</sub>** and **read<sub>x</sub>**

developer obligation: ✓

$\text{PHistories}(MGC_{policy}[L]) \subseteq \text{PHistories}(MGC_{policy}[L^\#])$

client obligation:

$\text{PHistories}(P[L^\#]) \subseteq \text{PHistories}(MGC_{policy}[L^\#])$

conclusion:

$\text{Behaviors}(P[L]) \subseteq \text{Behaviors}(P[L^\#])$

every behavior of  $P$  is a behavior of  $P[Loc := \mathbf{ra}]$

# Library specification under WMM

## Example

- Sequential specifications tell us nothing about the **synchronization** induced by the library

```
X := 42 rlx  
enqueue(q,1)
```

```
a := dequeue(q) // 1  
c := X rlx // 0
```

```
X := 1 rlx
```

```
a := dequeue(q) // ⊥
```

```
enqueue(q,1)
```

```
c := X rlx // 0
```

```
X := 1 rlx  
enqueue(q,1)
```

```
enqueue(q,2)  
A := X rlx // 0
```

```
b := dequeue(q) // 1
```

```
c := dequeue(q) // 2
```

- How to specify the different options?

# Library specification under WMM

- Declarative library specifications with specialized synchronization relations:

Raad, Doko, Rozic, L, Vafeiadis: **On library correctness under weak memory consistency: specifying and verifying concurrent libraries under declarative consistency models**. POPL 2019. <https://doi.org/10.1145/3290381>

- An operational approach?

- What can serve as reference implementation for different options?
- A per-object lock gives us the strongest queue: real-time order  $\Rightarrow$  happens-before



# RC11 library specification constructs

specification construct

- We propose **partial locks**: locks that induce **only intra-library** synchronization
- If we wrap a sequential implementation (using non-atomics) in a partial per-object lock, we obtain a queue that does not provide any synchronization to its clients

e.g., `enqueue(v) { ... }`  $\rightarrow$  `enqueue(v) { locklib(L) { ... } }`

- By using release/acquire accesses in the specification we can express stronger queues

# Verification under WMM

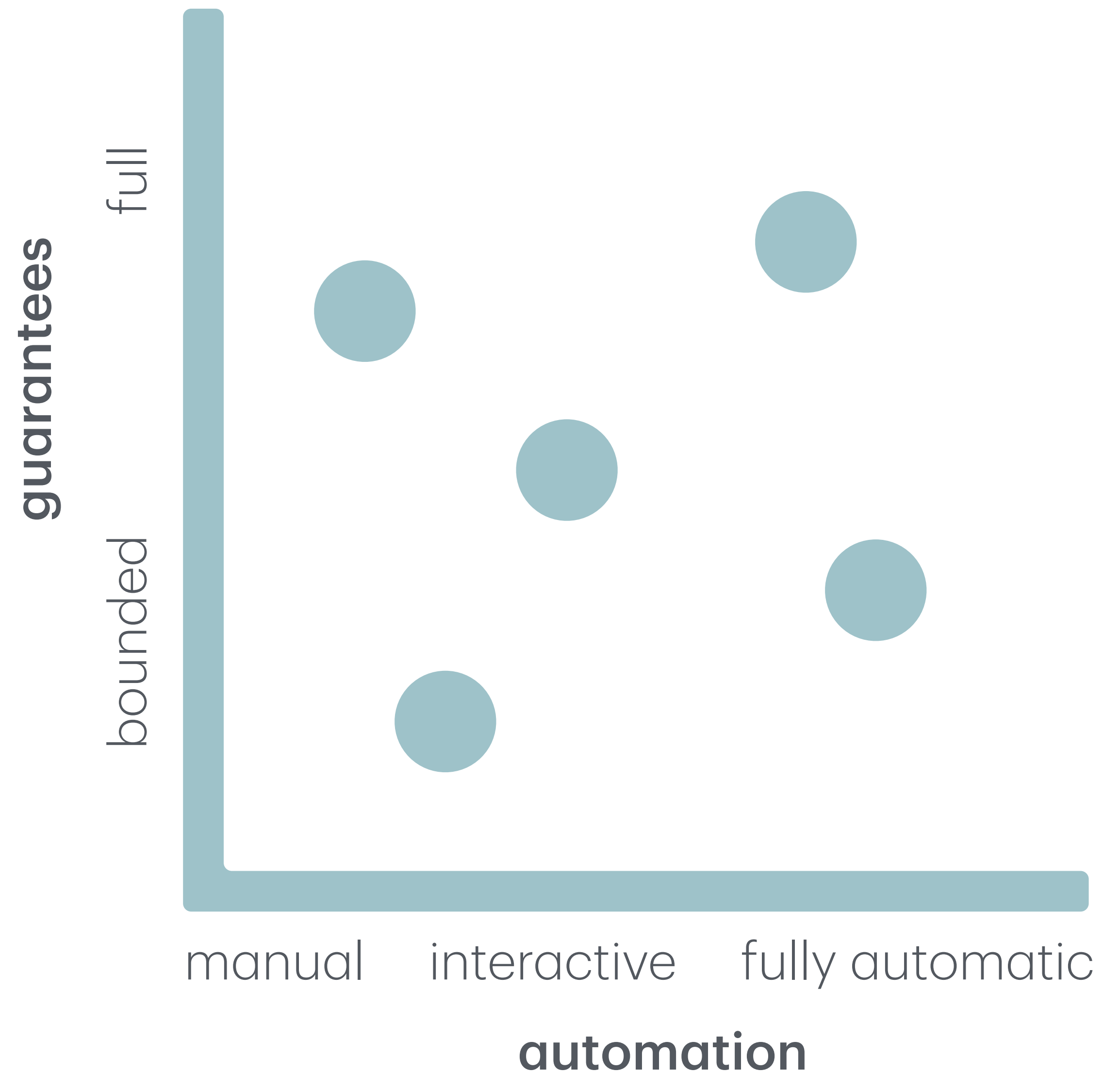
A short (and very partial) survey

# Verification questions and approaches for WMM

What do we verify?

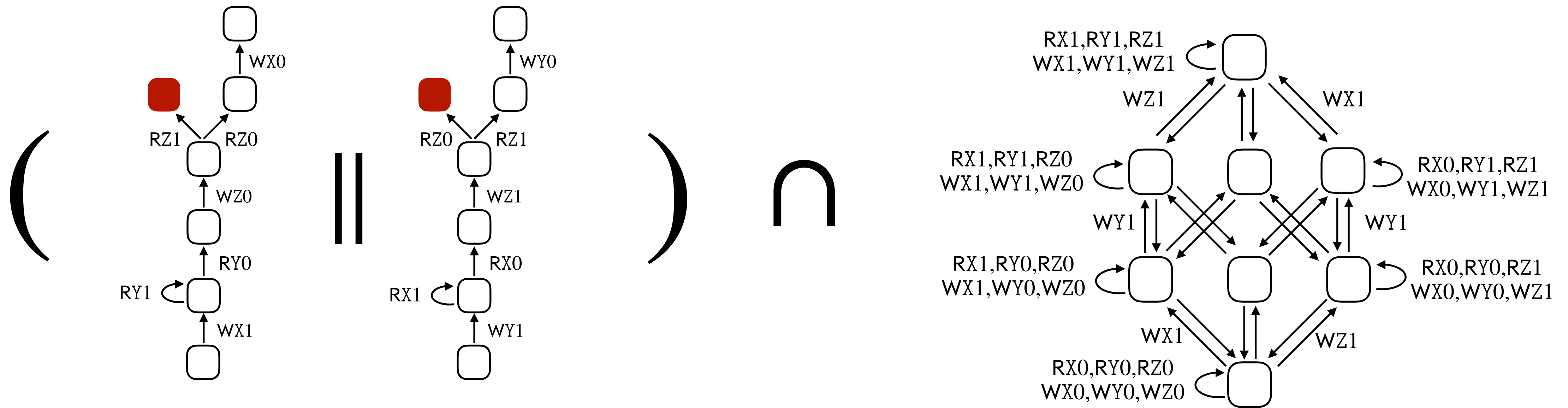
- Program never crashes
- Provides mutual exclusion
- Correctly implements a concurrent data structure
- ...

- 1 Theoretical decidability
- 2 Model checking and testing
- 3 Program logics
- 4 Robustness





# Theoretical verification under SC



For programs with a bounded data domain, this problem is clearly **decidable**:

- Reduction to reachability in finite-state systems
- PSPACE-complete

# Some results for weak memory modes

1

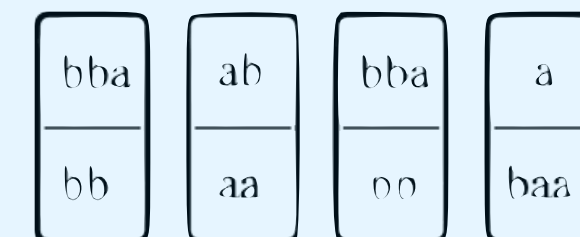
Reachability under **x86-TSO** is decidable:

- via a **dual** semantics (load-buffers instead of store buffers) that forms a WSTS

Abdulla, Atig, Bouajjani, Ngo: **A Load-Buffer Semantics for Total Store Ordering**. Log. Methods Comput. 2018. [https://doi.org/10.23638/LMCS-14\(1:9\)2018](https://doi.org/10.23638/LMCS-14(1:9)2018)

Reachability under **RA** is undecidable:

- reduction from Post correspondence problem

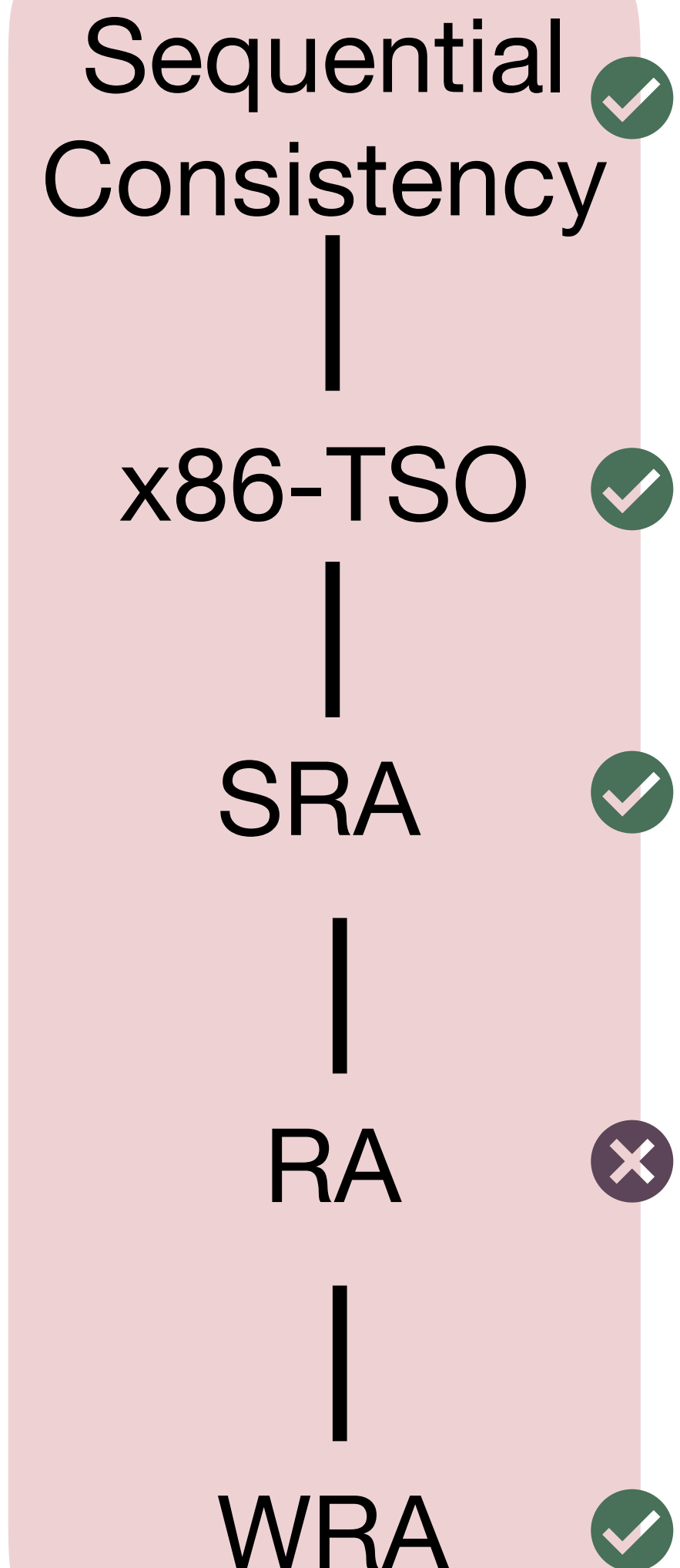


Abdulla, Arora, Atig, Krishna: **Verification of programs under the release-acquire semantics**. PLDI 2019. <https://doi.org/10.1145/3314221.3314649>

Reachability under **WRA/SRA** is decidable:

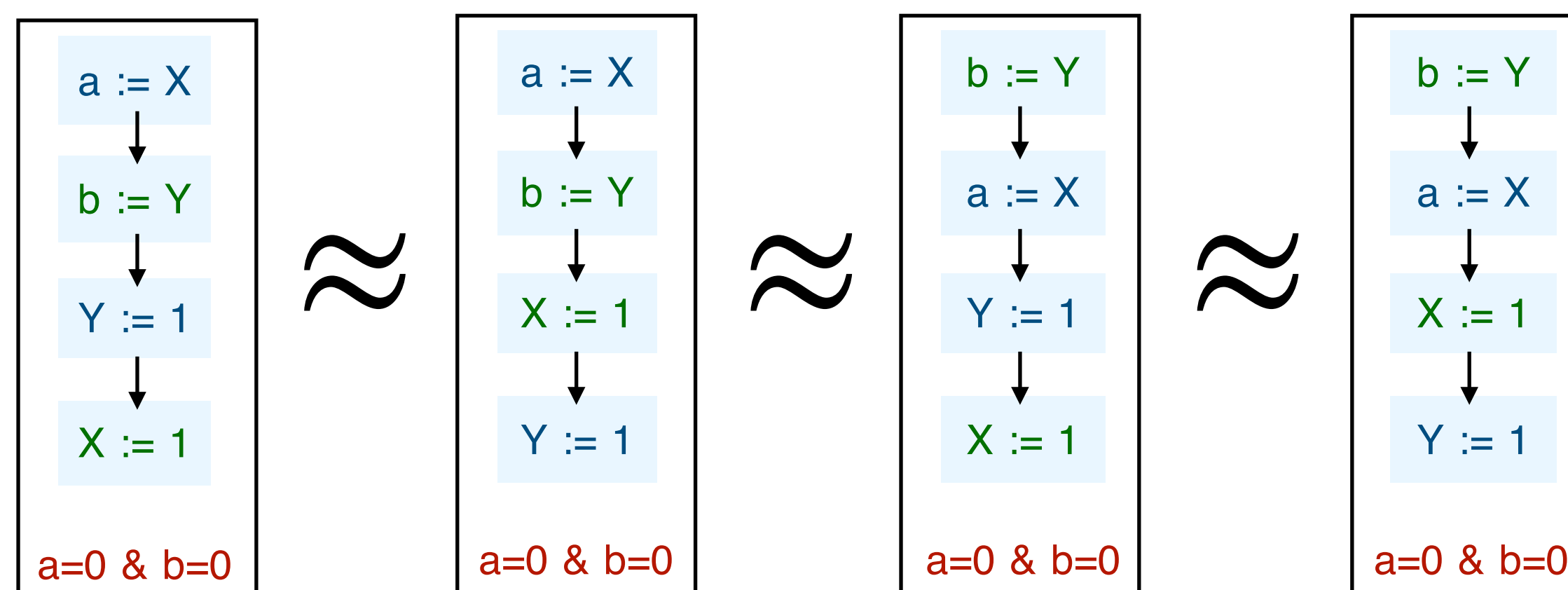
- via a **potential-based** semantics that forms a WSTS

L, Boker: **What's Decidable About Causally Consistent Shared Memory?** ACM Trans. Program. Lang. Syst. 2022. <https://doi.org/10.1145/3505273>



# Model checking

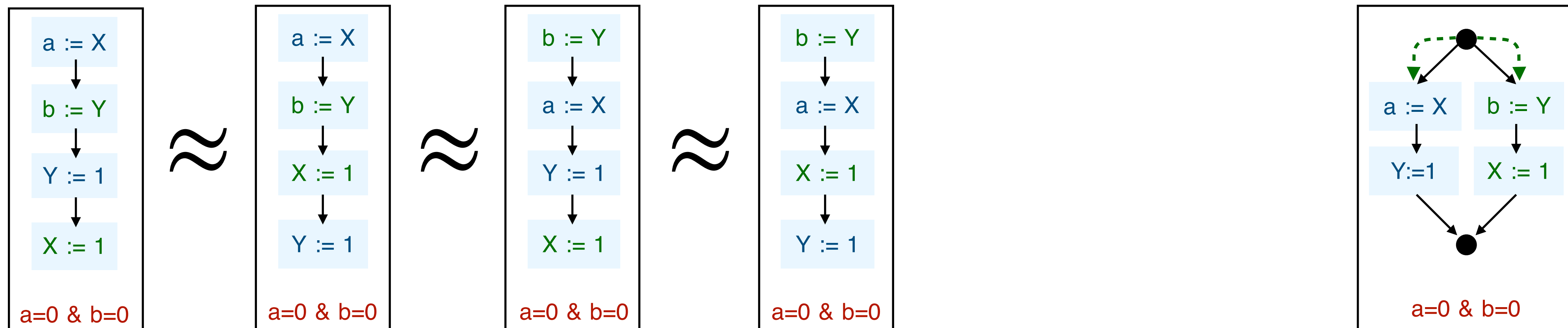
- Given a **loop-free program** (usually after loop unrolling), exhaustively verify that all its runs do not violate safety assertions
- Naively checking all traces is **infeasible** (for both time and memory)
- Remedies:
  - **stateless verification**: explore all executions without storing in memory the executions explored so far
  - **partial order reduction**: explore one candidate from each equivalence class



# Partial order reduction using execution graphs

2

- Explore **consistent execution graphs** rather than traces (also for SC!)
- Execution graphs track less redundant order and **represent equivalence classes**



Kokologiannakis, Raad, Vafeiadis: **Model checking for weakly consistent libraries**. PLDI 2019. <https://doi.org/10.1145/3314221.3314609>

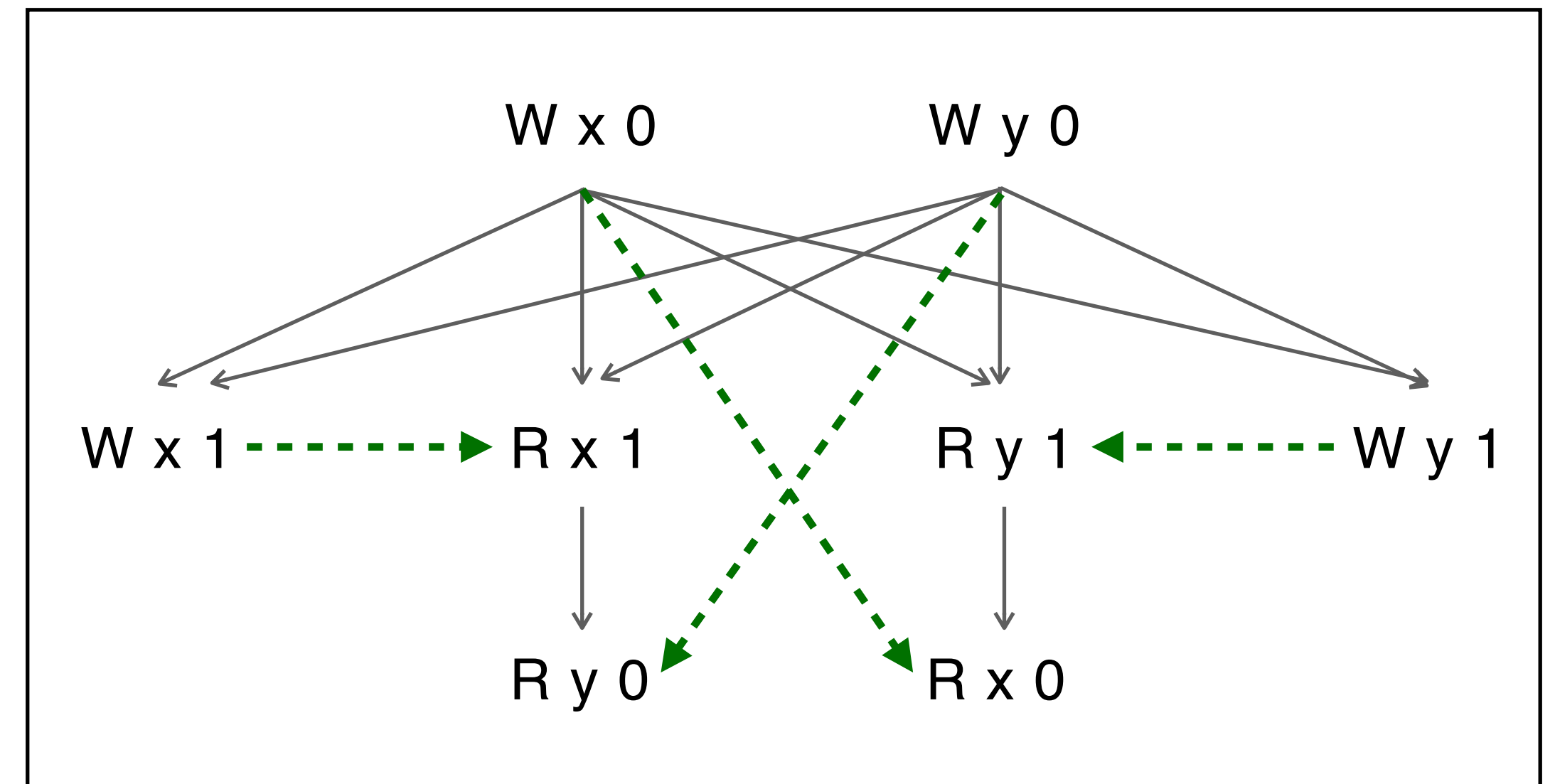
Kokologiannakis, Marmanis, Gladstein, Vafeiadis: **Truly stateless, optimal dynamic partial order reduction**. POPL 2021. <https://doi.org/10.1145/3498711>

Luo, Demsky: **C11Tester: a race detector for C/C++ atomics**. ASPLOS 2021. <https://doi.org/10.1145/3445814.3446711>.



# A related problem

- Given an execution graph  $G$  check whether it is **consistent** under a memory model  $M$
- Some weak memory models make this problem **easier!**
  - Given only program-order  $po$  and **reads-from**  $rf$  relations:
    - Checking for **SC**-consistency is **NP-complete**
    - Checking for **RA**-consistency is in **PTIME**



# Program logics

- A (mostly) manual approach for syntax-guided verification
- Derivation rules that provide **reasoning principles**

$$\frac{\{P\}C_1\{Q\} \quad \{Q\}C_2\{R\}}{\{P\}C_1; C_2\{R\}}$$

$$\frac{\{P \wedge b\}C\{P\}}{\{P\}\text{while } b \text{ do } C\{P \wedge \neg b\}}$$

*Thread 1 proof outline:*

$$\begin{aligned} & \{\text{Seen}(\pi, V_0) * \text{Hist}(x, [(0, -, V_x)]) * V_x \sqsubseteq V_0 * \boxed{\text{Inv}_y(V_0)}\} \\ & x_{[\text{na}]} := 37 \\ & \{\exists V_{37} \sqsupseteq V_0. \text{Seen}(\pi, V_{37}) * \text{Hist}(x, [(37, -, V_{37})])\} \\ & \{\text{Seen}(\pi, V_{37}) * \boxed{\text{Inv}_x(V_{37})}\} \\ & \text{open } \text{Inv}_y \left\{ \begin{array}{l} \{\text{Seen}(\pi, V_{37}) * \exists h. \text{Hist}(y, h) * \dots\} \\ y_{[\text{at}]} := 1 \\ \{\exists V_1 \sqsupseteq V_{37}. \text{Seen}(\pi, V_1) * \text{Hist}(y, h \uplus [(1, -, V_1)]) * \boxed{\text{Inv}_x(V_{37})}\} \end{array} \right. \\ & \{\text{Seen}(\pi, V_1) * \boxed{\text{Inv}_y(V_0)}\} \end{aligned}$$

*Thread 2 proof outline:*

$$\begin{aligned} & \{\text{Seen}(\pi, V_0) * \boxed{\text{Inv}_y(V_0)} * \boxed{\diamond}\} \\ & \text{repeat } y_{[\text{at}]}; \\ & \{\exists V_1, V_{37}, V_2. V_2 \sqsupseteq V_1 \sqsupseteq V_{37} * \text{Seen}(\pi, V_2) * \boxed{\text{Inv}_x(V_{37})} * \boxed{\diamond}\} \\ & \{\text{Seen}(\pi, V_2) * V_{37} \sqsubseteq V_2 * \text{Hist}(x, [(37, -, V_{37})])\} \\ & x_{[\text{na}]} \\ & \{z. \text{Seen}(\pi, V_2) * z = 37 * \text{Hist}(x, [(37, -, V_{37})])\} \end{aligned}$$

$$\begin{aligned} \text{Inv}_y(V_0) &\triangleq \exists h. \text{Hist}(y, h) * (0, -, V_0) \in h * \forall V_1, v_1 \neq 0. (v_1, -, V_1) \in h \Rightarrow \exists V_{37} \sqsubseteq V_1. \boxed{\text{Inv}_x(V_{37})} \\ \text{Inv}_x(V_{37}) &\triangleq \boxed{\diamond} \vee \text{Hist}(x, [(37, -, V_{37})]) \end{aligned}$$

# Owicki-Gries / rely-guarantee logics

```

    Init: d := 0; f := 0;
          {f =1 0 ∧ f =2 0 ∧ d =1 0 ∧ d =2 0}
Thread 1 || Thread 2
{f ≠2 1 ∧ d =1 0}  { [f = 1](d =2 5) }
1 : d := 5;          3 : do r1 ←A f until r1 = 1;
{f ≠2 1 ∧ d =1 5}  { d =2 5 }
2 : f :=R 1;       4 : r2 ← d;
{true}              { r2 = 5 }
                    { r2 = 5 }
  
```

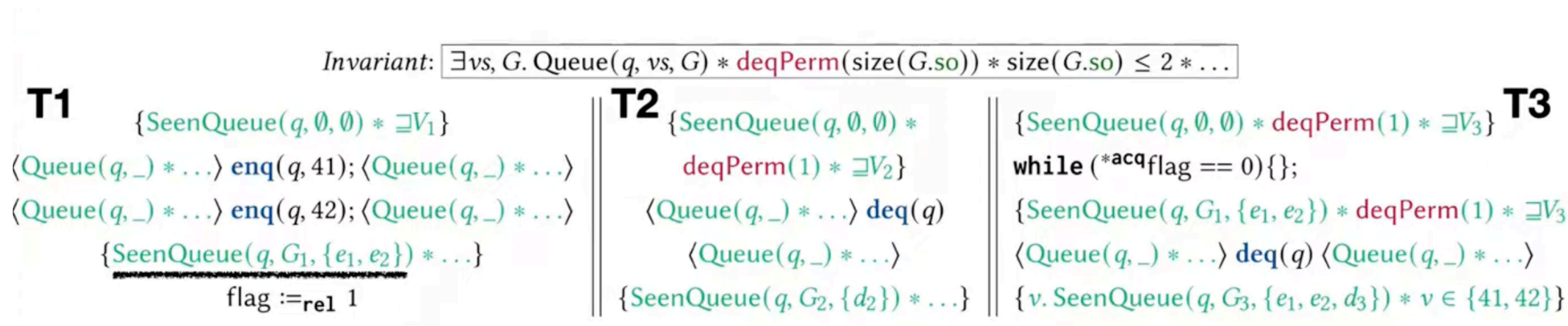
- SC-based reasoning is unsound
- Develop **specialized assertions** for expressing invariants on top of an **operational presentation** of the memory model

Dalvandi, Doherty, Dongol, Wehrheim: **Owicki-Gries Reasoning for C11 RAR**. ECOOP 2020. <https://doi.org/10.4230/LIPIcs.ECOOP.2020.11>

L, Dongol, Wehrheim: **Rely-Guarantee Reasoning for Causally Consistent Shared Memory**. CAV 2023. [https://doi.org/10.1007/978-3-031-37706-8\\_11](https://doi.org/10.1007/978-3-031-37706-8_11)

# Seperation logics

- Concurrent separation logic is designed to reason about DRF programs
- So it is trivially sound under models that satisfy the DRF guarantee
- Extensions allow reasoning about synchronization primitives of RC11
- In particular, ownership transfer is possible via `rel/acq` synchronization



Vafeiadis, Narayan: **Relaxed separation logic: a program logic for C11 concurrency.** OOPSLA 2013. <https://doi.org/10.1145/2509136.2509532>

Dang, Jourdan, Kaiser, Dreyer: **RustBelt meets relaxed memory.** POPL 2020. <https://doi.org/10.1145/3371102>

Dang, Jung, Choi, Nguyen, Mansky, Kang, Dreyer: **Compass: strong and compositional library specifications in relaxed memory separation logic.** PLDI 2022. <https://doi.org/10.1145/3519939.3523451>

# Robustness

- Many (useful) programs are *robust*:

all program behaviors allowed by RC11 are in fact also allowed by SC

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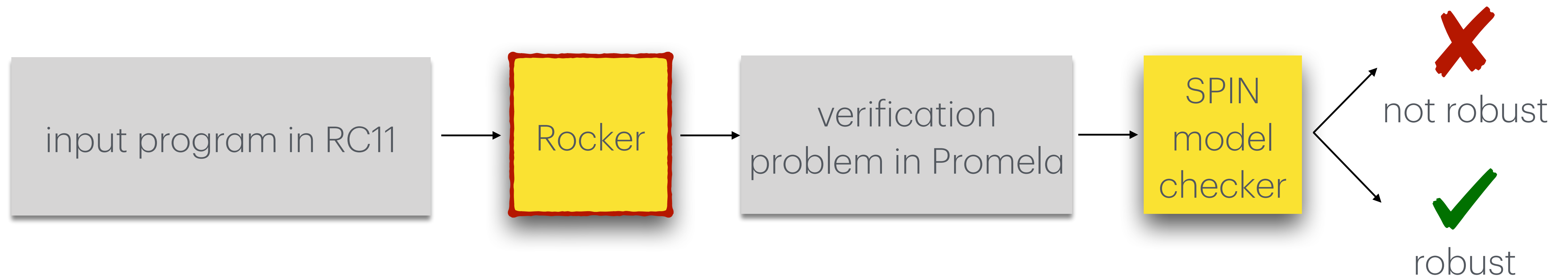
all program behaviors allowed by RC11 are in fact also allowed by SC

$$\text{verification under weak memory} = \text{verification under SC} + \text{robustness}$$

## Theorem

Execution-graph robustness against RC11 is PSPACE-complete

Idea: run an instrumented program under SC that monitors whether some step is allowed under RC11 but not under SC



L, Margalit: **Robustness against release/acquire semantics**. PLDI 2019. <https://doi.org/10.1145/3314221.3314604>

Margalit, L: **Verifying observational robustness against a c11-style memory model**. POPL 2021. <https://doi.org/10.1145/3434285>

# Conclusion

We talked about:

1. The C/C++11 memory model
2. The out-of-thin-air problem & RC11
3. Implementability of (R)C11: compiler optimizations and mapping to hardware
4. Programmability guarantees: DRF theorems, library abstraction
5. Verification

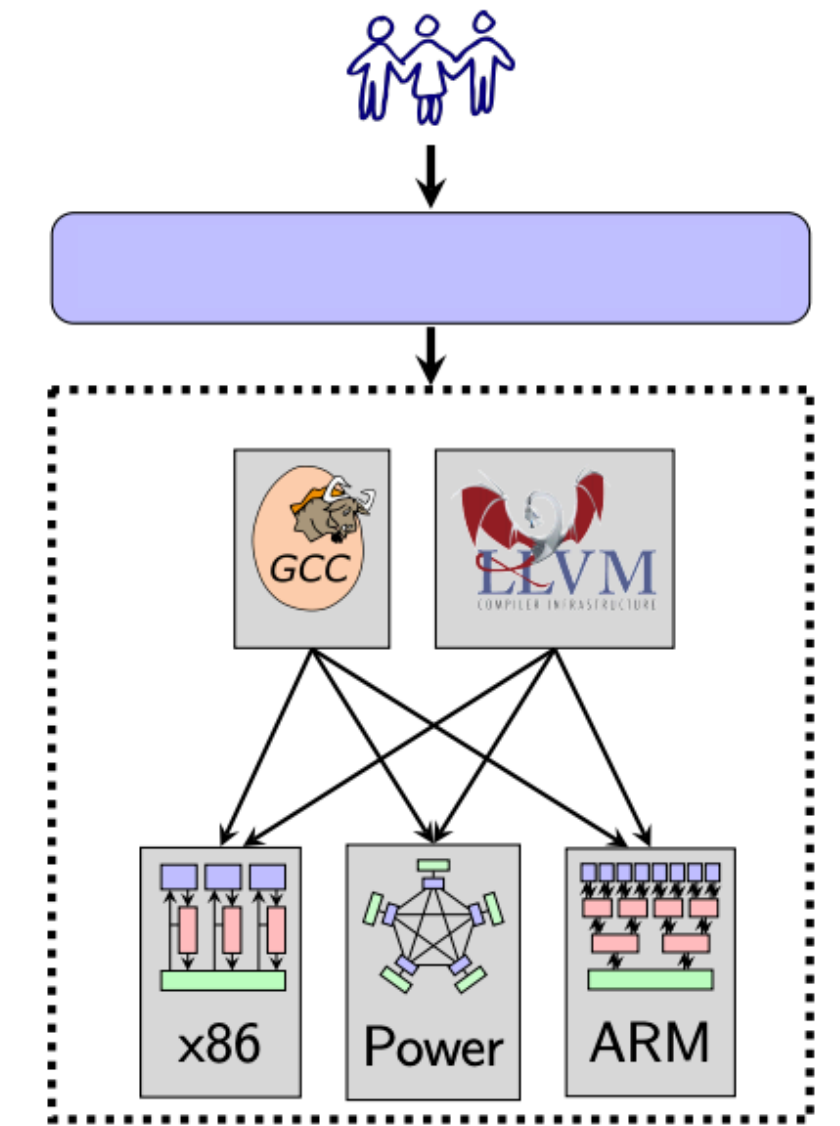
*Weak memory models are not only a **threat**, but also an **opportunity** to better understand concurrency!*



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