Complexity of Boolean and Arithmetic Circuits - 236603

Exercise 4

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Please write your solution in LaTeXand submit them by email to shpilka@cs.technion.ac.il.

1. Let $f : \{0,1\}^n \to \{0,1\}$ be a function that can be written both as an s-DNF and as a t-CNF. Show that f has a decision tree of depth $s \cdot t$.
2. Let C be an arithmetic circuit of size s and depth d computing a polynomial f of degree r in n variables. Prove that there exists a homogeneous circuit of size $O(sr^2)$ and depth $O(d \log(r))$ that computes the homogeneous components of f.
3. Let $d = 2k + 1$ be an odd integer. Let $X_1, \ldots, X_d$ be d matrices over $\mathbb{R}$ of dimension n. Let $f(X_1, \ldots, X_d) = \text{trace}(X_1 \cdot X_2 \cdot \ldots \cdot X_d)$. Show that any homogeneous $\Sigma\Pi\Sigma$ for f is of size $\Omega(n^k)$.
4. Show that for the non-commutative model it is no true that $\mathrm{P} = \mathrm{NC}^2$.
5. Recall that a matrix A is (r, k)-rigid if it cannot be written as $A = B + C$ where $\text{rank}(B) \le r$ and C has at most k non zero entries in every row. Show that most $n \times n$ matrices over $GF(2)$ (the field with 2 elements) are $(\frac{n}{100}, \frac{n}{100})$-rigid.