

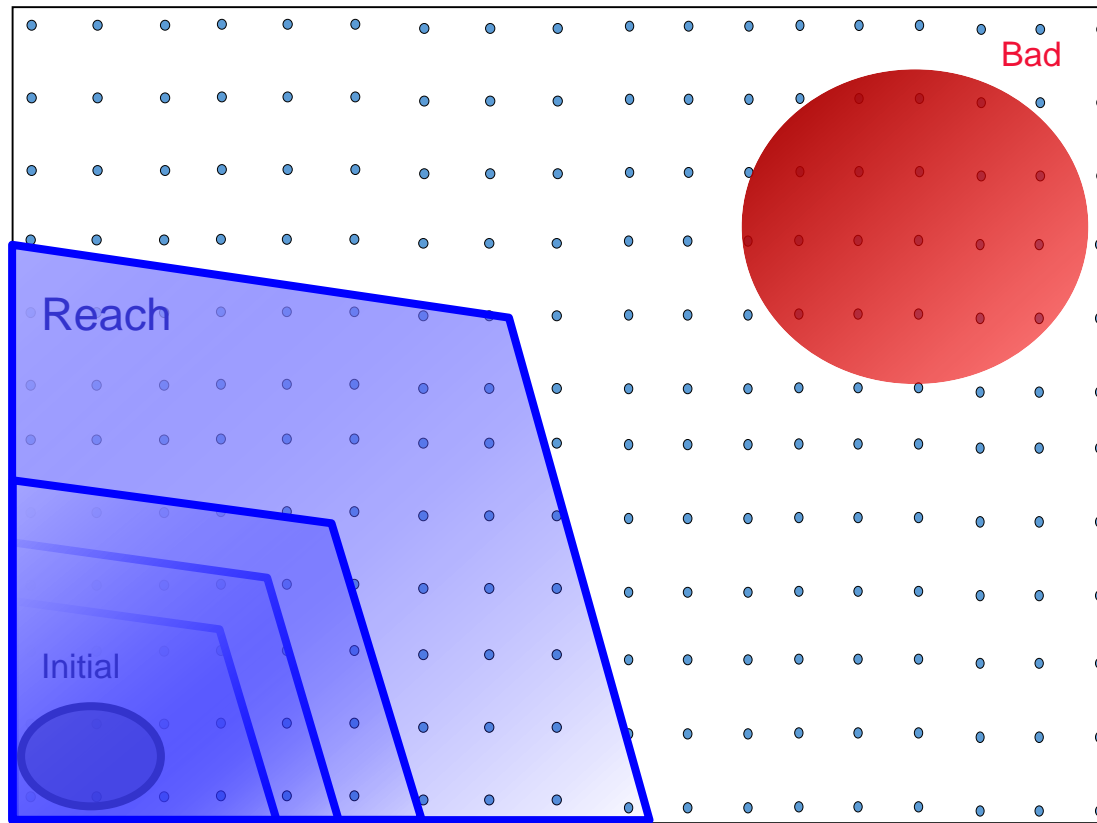
Inferring Inductive Invariants from Phase Structures

Yotam Feldman James R. Wilcox Sharon Shoham Mooly Sagiv

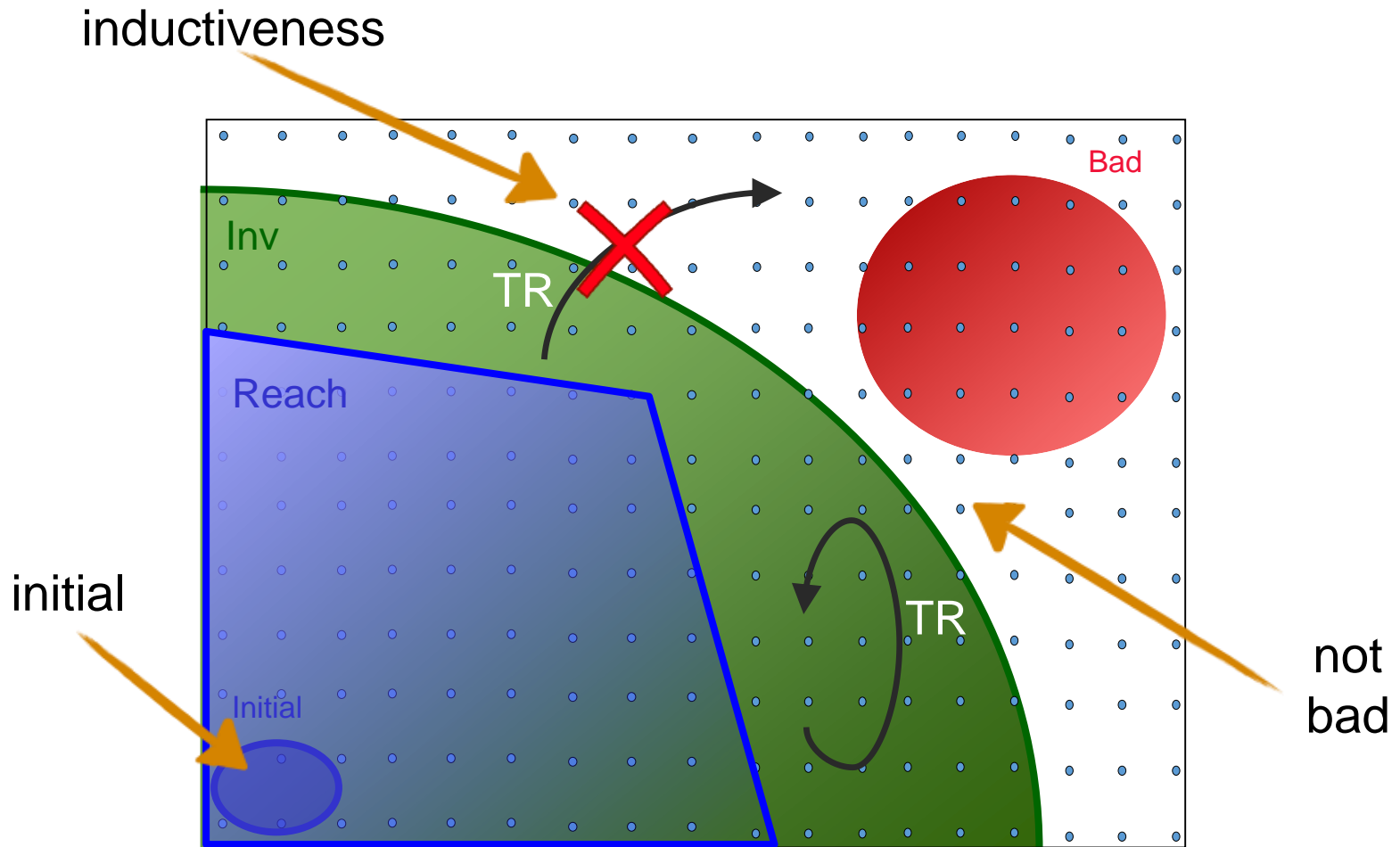


 @yotamfe, @wilcoxjay, @SagivMooly

Safety of Infinite-State Systems



Inductive Invariants



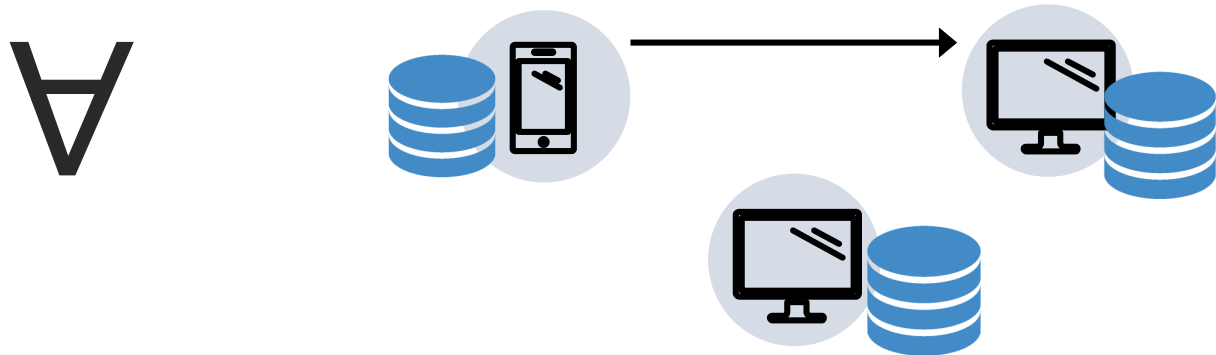
Distributed Protocols in EPR

EPR: A decidable fragment of first order logic

Used for modelling distributed protocols

[Padon et al. PLDI'16, OOPSLA'17, POPL'18, Taube et al. PLDI'18, Berkovits et al. CAV'19]

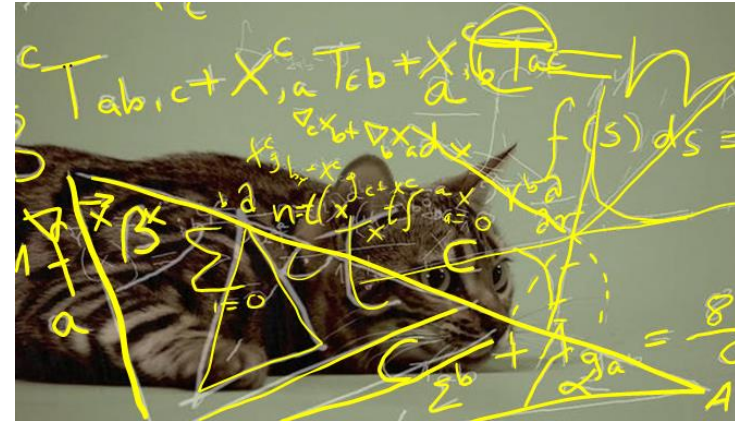
**Our focus: universally quantified invariants
for EPR distributed protocols**



Proving with Inductive Invariants

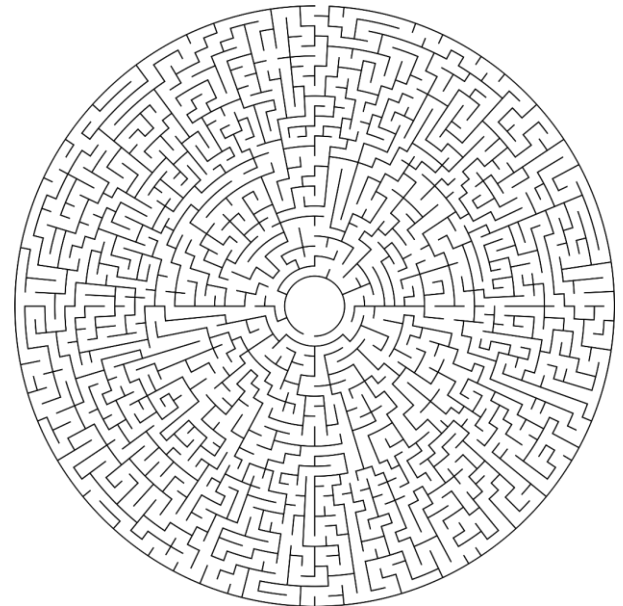
Deductive verification –
manually specify inductive invariant

Labor intensive



Invariant inference –
automatically search for invariant

Limited and fragile



Our Approach

User-guided invariant inference –
manually specify high-level **intuition**,
automatically find **full** proof

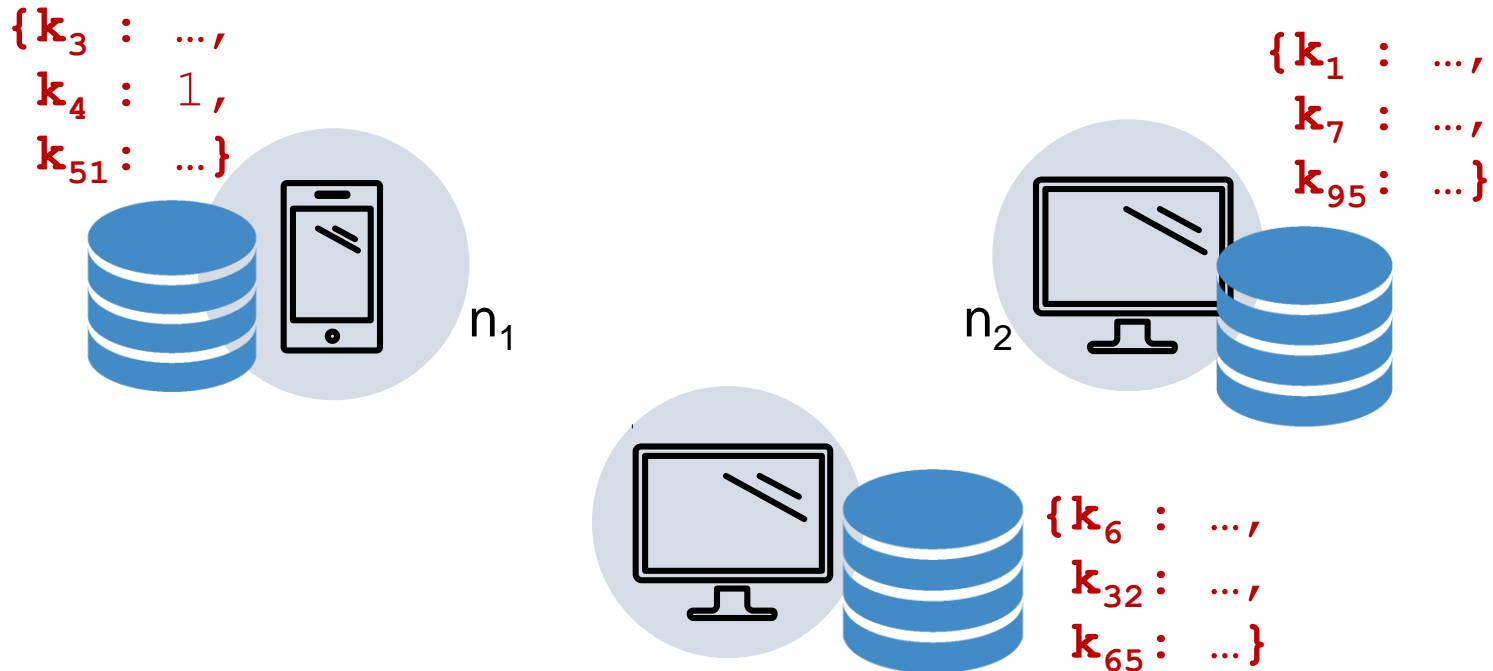
1. Guide invariant inference using **phase structures**
2. Apply to inference of **universally quantified** invariants on **challenging distributed protocols** modelled in EPR



Example: Sharded Key-Value Store

State: modeled over global relations

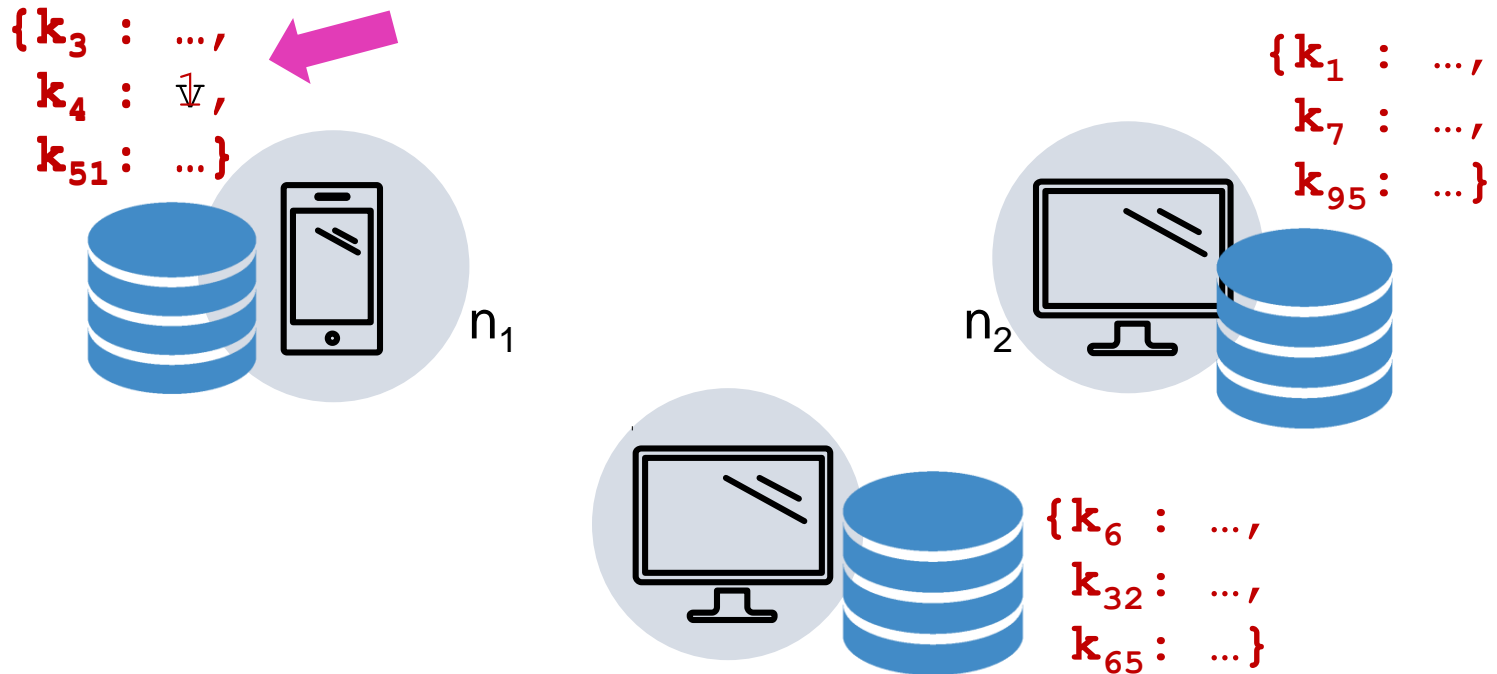
- local state
- network



Example: Sharded Key-Value Store

change local table:

```
table(n1, k4) := v
```

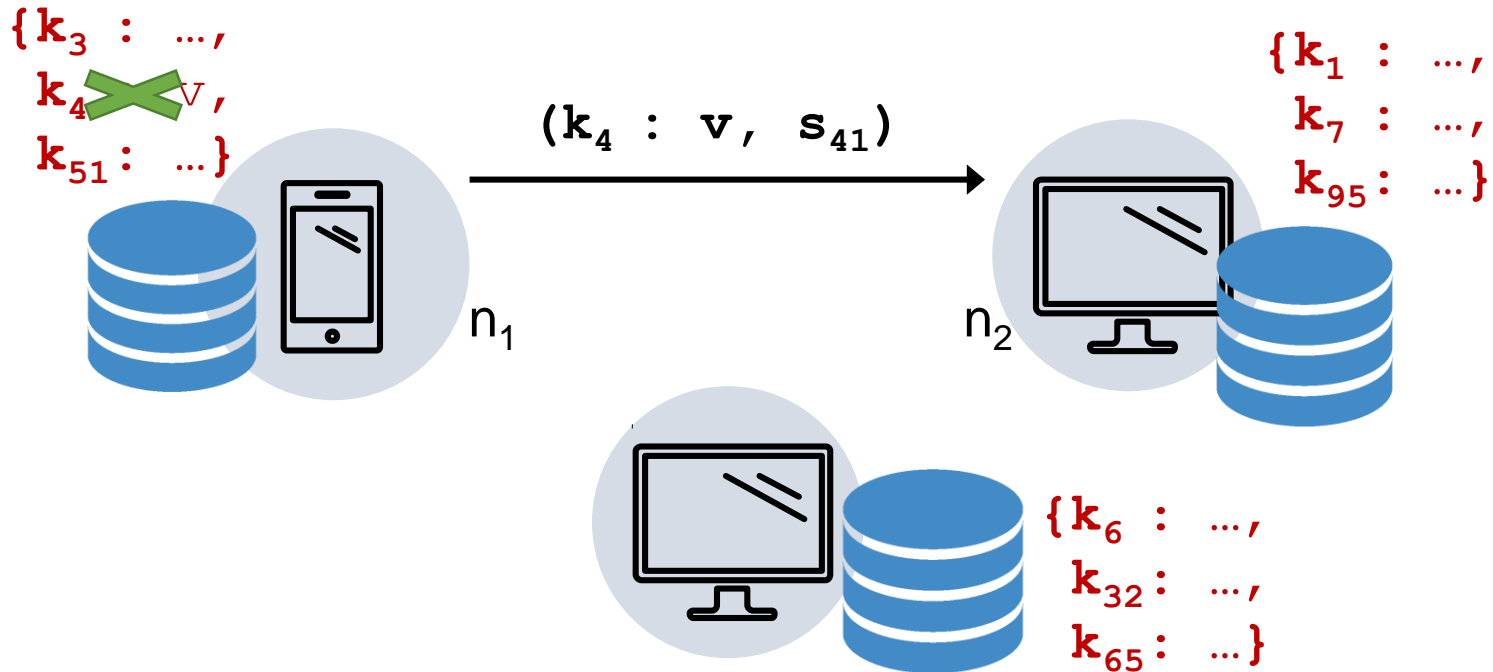


Example: Sharded Key-Value Store

reshard:

$\text{table}(n_1, k_4) := \perp$

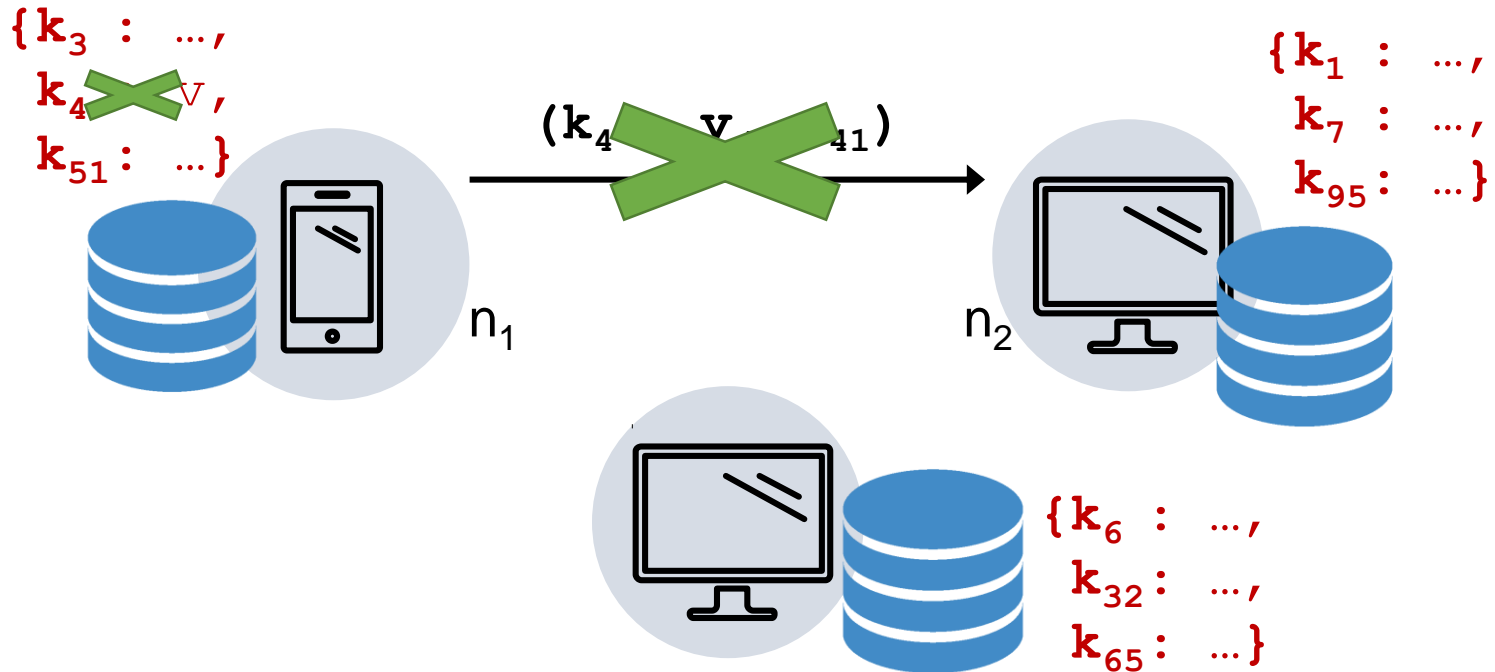
$\text{transfer_msg}(n_1, n_2, k_4, v, s_{41}) := \text{true}$



Example: Sharded Key-Value Store

drop transfer message:

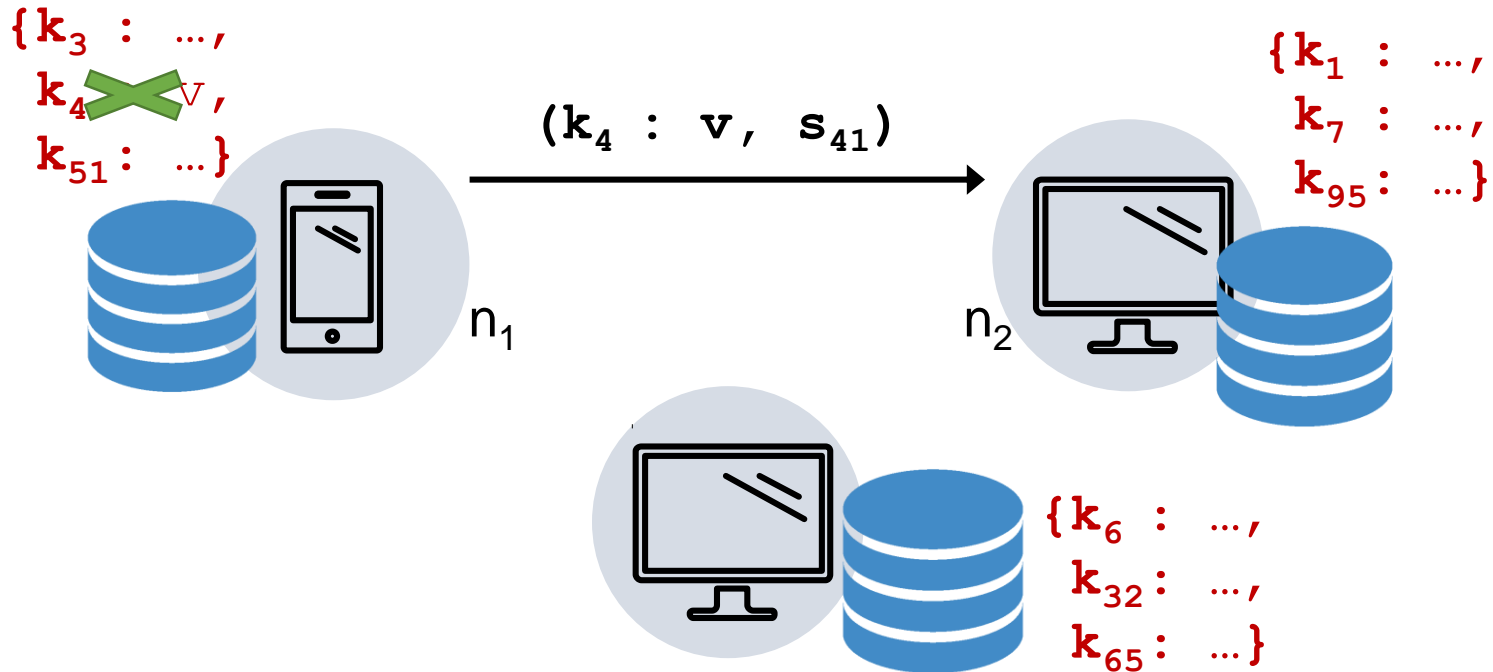
```
transfer_msg(n1, n2, k4, v, s41) := false
```



Example: Sharded Key-Value Store

retransmit:

```
transfer_msg(n1, n2, k4, v, s41) := true
```

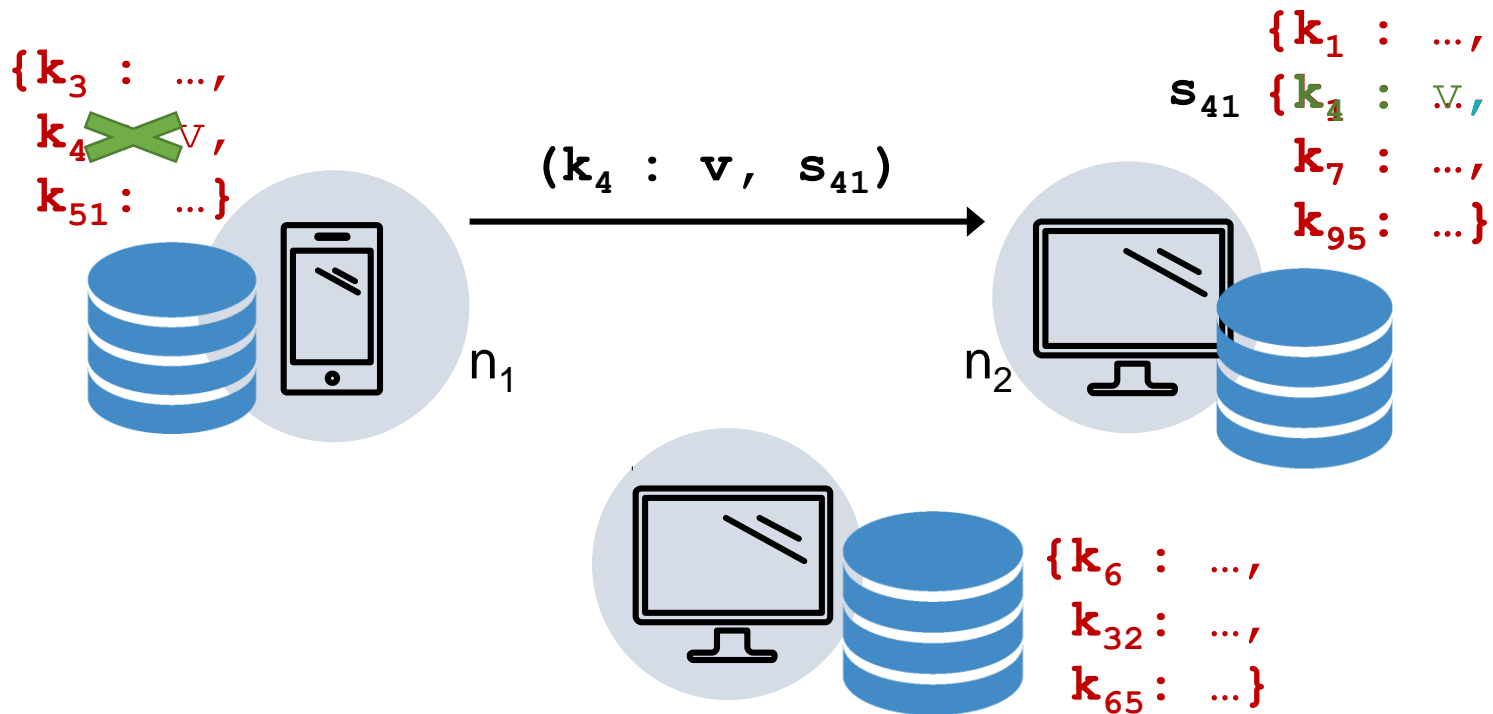


Example: Sharded Key-Value Store

recv transfer message:

$table(n_2, k_4) := v; seq_recvd(n_2, n_1, s_{41}) := true$

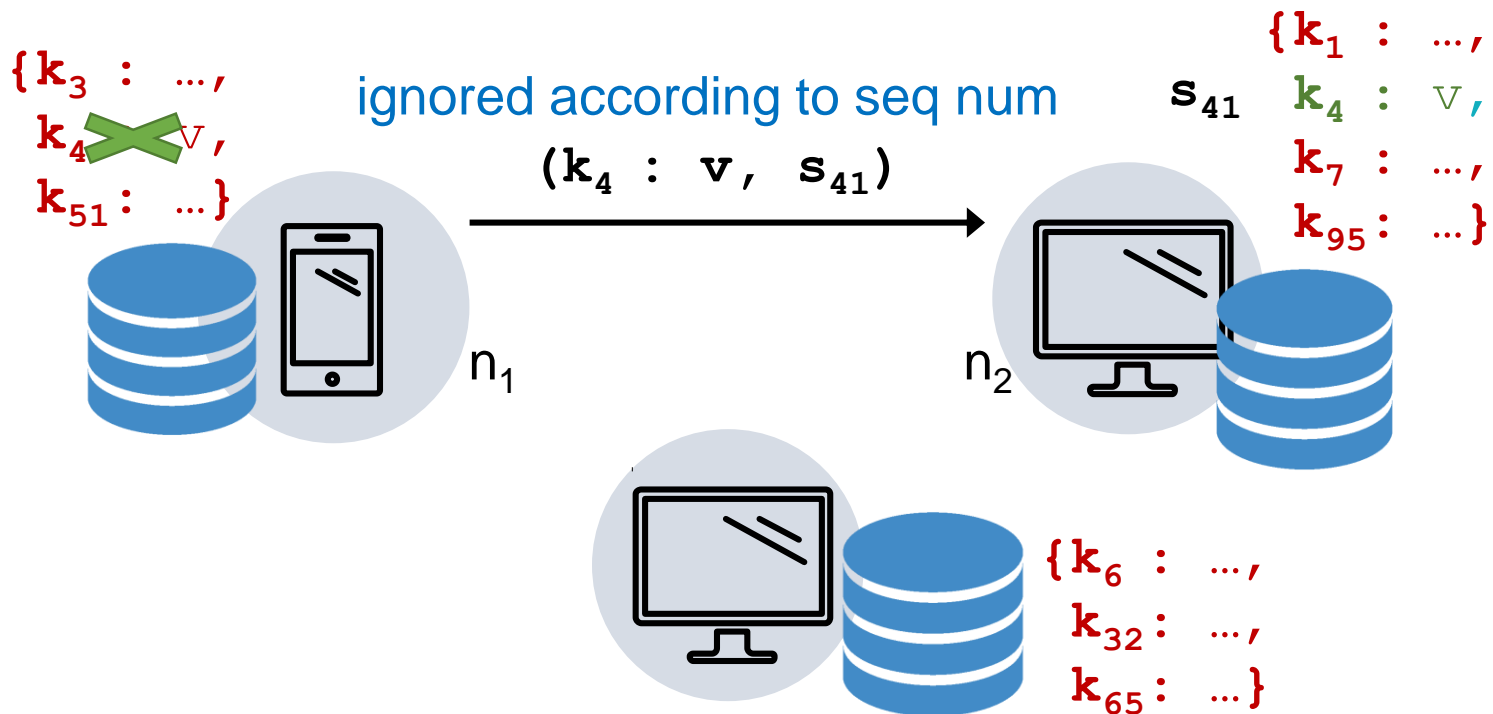
$transfer_msg(n_1, n_2, k_4, v, s_{41}) := false$



Example: Sharded Key-Value Store

retransmit:

```
transfer_msg(n1, n2, k4, v, s41) := true
```

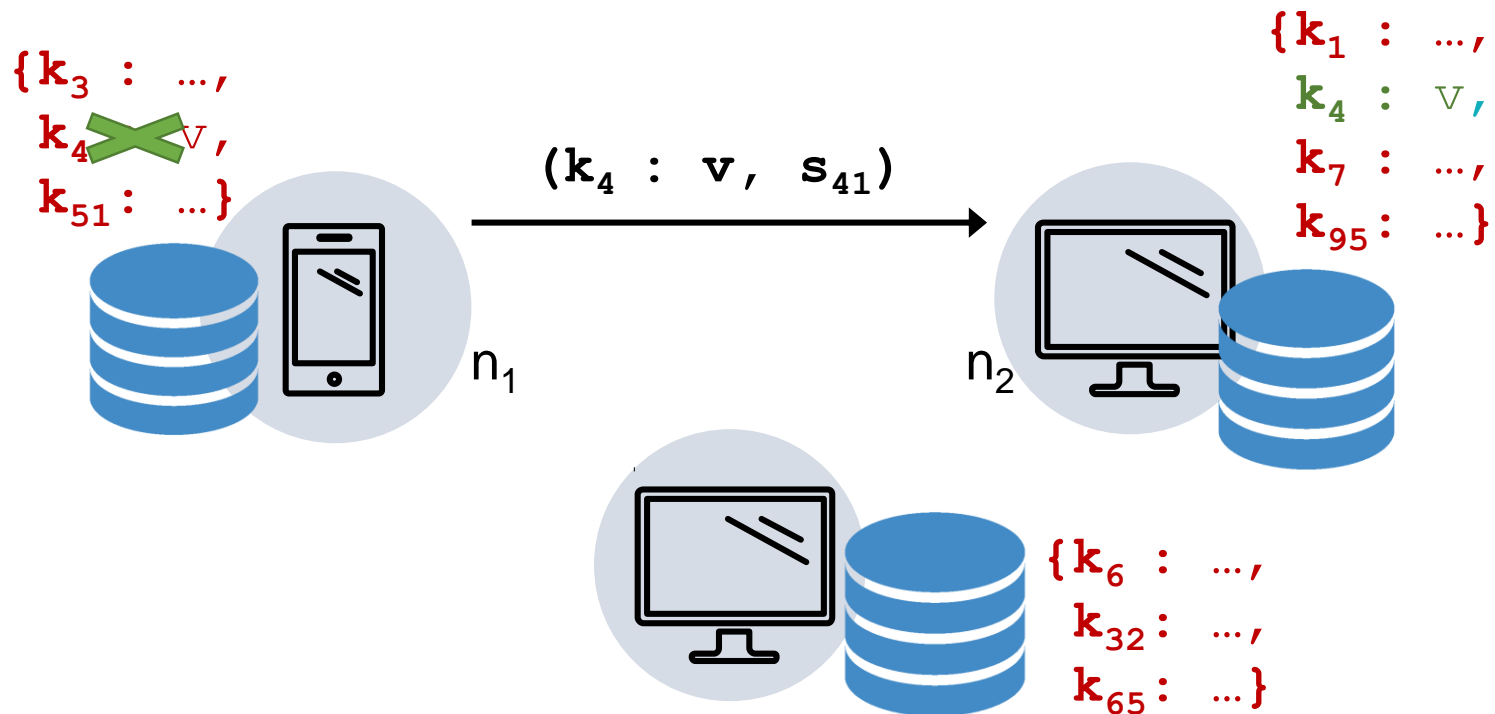


Example: Sharded Key-Value Store

Safety property:

$\forall n_1, n_2, k, v_1, v_2.$

$\text{table}(n_1, k, v_1) \wedge \text{table}(n_2, k, v_2) \rightarrow v_1 = v_2$



Deductive Verification for Sharded KV

invariant $\forall k, n_1, n_2, v_1, v_2. \text{table}(n_1, k, v_1) \wedge \text{table}(n_2, k, v_2) \rightarrow n_1 = n_2 \wedge v_1 = v_2$

invariant $\forall k, n_1, n_2. \text{owner}(n_1, k) \wedge \text{owner}(n_2, k) \rightarrow n_1 = n_2$

invariant $\forall k, n, v. \text{table}(n, k, v) \rightarrow \text{owner}(n, k)$

invariant $\forall k, \text{src}, \text{dst}, v, s, n. \neg (\text{transfer_msg}(\text{src}, \text{dst}, k, v, s) \wedge \neg \text{seqnum_recvd}(\text{dst}, \text{src}, s) \wedge \text{owner}(n, k))$

invariant $\forall k, \text{src}, \text{dst}, v, s, n. \neg (\text{unacked}(\text{src}, \text{dst}, k, v, s) \wedge \neg \text{seqnum_recvd}(\text{dst}, \text{src}, s) \wedge \text{owner}(n, k))$

invariant $\forall k, \text{src}_1, \text{src}_2, \text{dst}_1, \text{dst}_2, v_1, v_2, s_1, s_2. \text{transfer_msg}(\text{src}_1, \text{dst}_1, k, v_1, s_1) \wedge \neg \text{seqnum_recvd}(\text{dst}_1, \text{src}_1, s_1) \wedge \text{transfer_msg}(\text{src}_2, \text{dst}_2, k, v_2, s_2) \wedge \neg \text{seqnum_recvd}(\text{dst}_2, \text{src}_2, s_2) \rightarrow \text{src}_1 = \text{src}_2 \wedge \text{dst}_1 = \text{dst}_2 \wedge v_1 = v_2 \wedge s_1 = s_2$

invariant $\forall k, \text{src}_1, \text{src}_2, \text{dst}_1, \text{dst}_2, v_1, v_2, s_1, s_2. \text{transfer_msg}(\text{src}_1, \text{dst}_1, k, v_1, s_1) \wedge \neg \text{seqnum_recvd}(\text{dst}_1, \text{src}_1, s_1) \wedge \text{unacked}(\text{src}_2, \text{dst}_2, k, v_2, s_2) \wedge \neg \text{seqnum_recvd}(\text{dst}_2, \text{src}_2, s_2) \rightarrow \text{src}_1 = \text{src}_2 \wedge \text{dst}_1 = \text{dst}_2 \wedge v_1 = v_2 \wedge s_1 = s_2$

invariant $\forall \text{src}_1, \text{src}_2, \text{dst}_1, \text{dst}_2, v_1, v_2, s_1, s_2. \text{unacked}(\text{src}_1, \text{dst}_1, k, v_1, s_1) \wedge \neg \text{seqnum_recvd}(\text{dst}_1, \text{src}_1, s_1) \wedge \text{unacked}(\text{src}_2, \text{dst}_2, k, v_2, s_2) \wedge \neg \text{seqnum_recvd}(\text{dst}_2, \text{src}_2, s_2) \rightarrow \text{src}_1 = \text{src}_2 \wedge \text{dst}_1 = \text{dst}_2 \wedge v_1 = v_2 \wedge s_1 = s_2$

invariant $\forall \text{src}_1, \text{src}_2, \text{dst}_1, \text{dst}_2, v_1, v_2, s_1, s_2. \text{unacked}(\text{src}_1, \text{dst}_1, k, v_1, s_1) \wedge \neg \text{seqnum_recvd}(\text{dst}_1, \text{src}_1, s_1) \wedge \text{unacked}(\text{src}_2, \text{dst}_2, k, v_2, s_2) \wedge \neg \text{seqnum_recvd}(\text{dst}_2, \text{src}_2, s_2) \rightarrow \text{src}_1 = \text{src}_2 \wedge \text{dst}_1 = \text{dst}_2 \wedge v_1 = v_2 \wedge s_1 = s_2$

invariant $\forall \text{src}_1, \text{src}_2, \text{dst}_1, \text{dst}_2, v_1, v_2, s_1, s_2. \text{unacked}(\text{src}_1, \text{dst}_1, k, v_1, s_1) \wedge \neg \text{seqnum_recvd}(\text{dst}_1, \text{src}_1, s_1) \wedge \text{unacked}(\text{src}_2, \text{dst}_2, k, v_2, s_2) \wedge \neg \text{seqnum_recvd}(\text{dst}_2, \text{src}_2, s_2) \rightarrow \text{src}_1 = \text{src}_2 \wedge \text{dst}_1 = \text{dst}_2 \wedge v_1 = v_2 \wedge s_1 = s_2$

Labor intensive

Invariant Inference for Sharded KV

Protocol	Inductive Invariant Inference
...	...
Sharded KV	failed to converge in 1 hour in 13/16 Z3 seeds

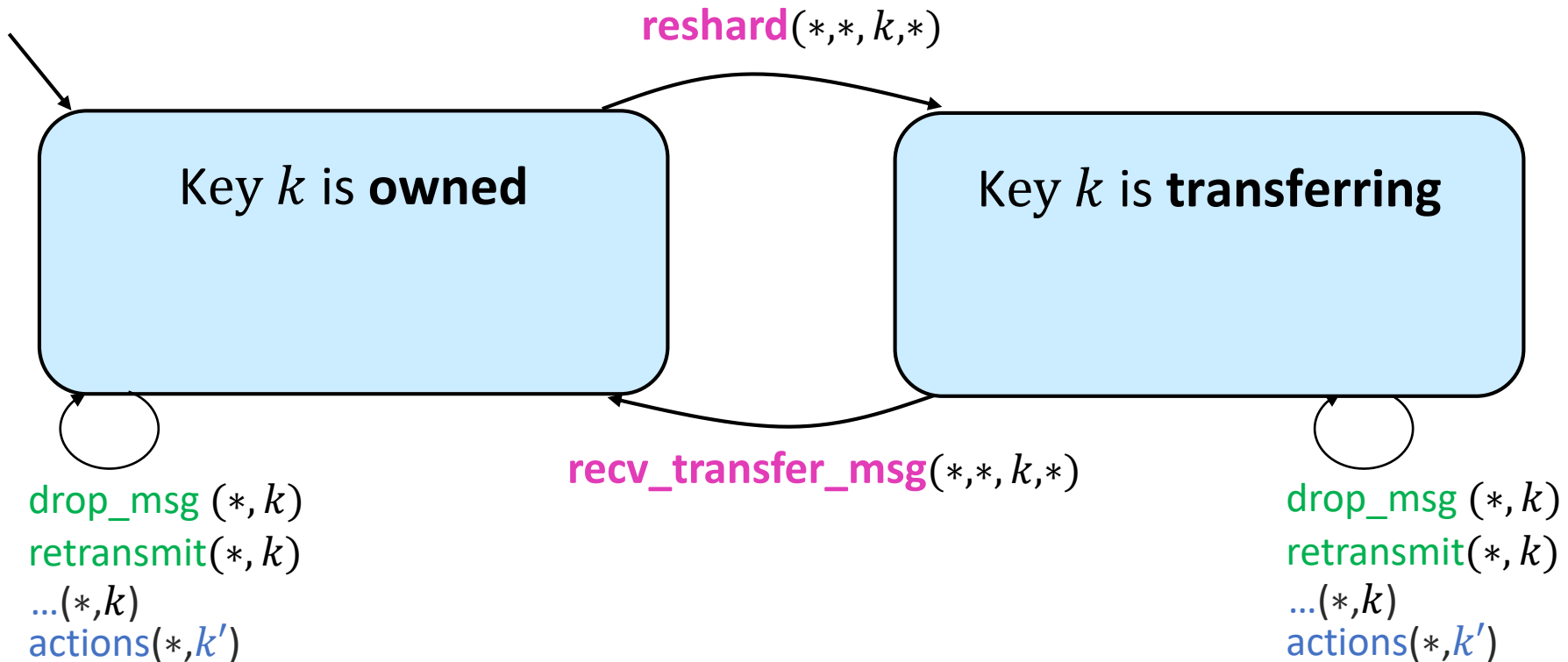
Solution: guide using phase structure

Limited and fragile

[CAV'15, JACM'17] Property-Directed Inference of Universal Invariants or Proving Their Absence, A. Karbyshev, N. Bjorner, S. Itzhaky, N. Rinetzky and S. Shoham.

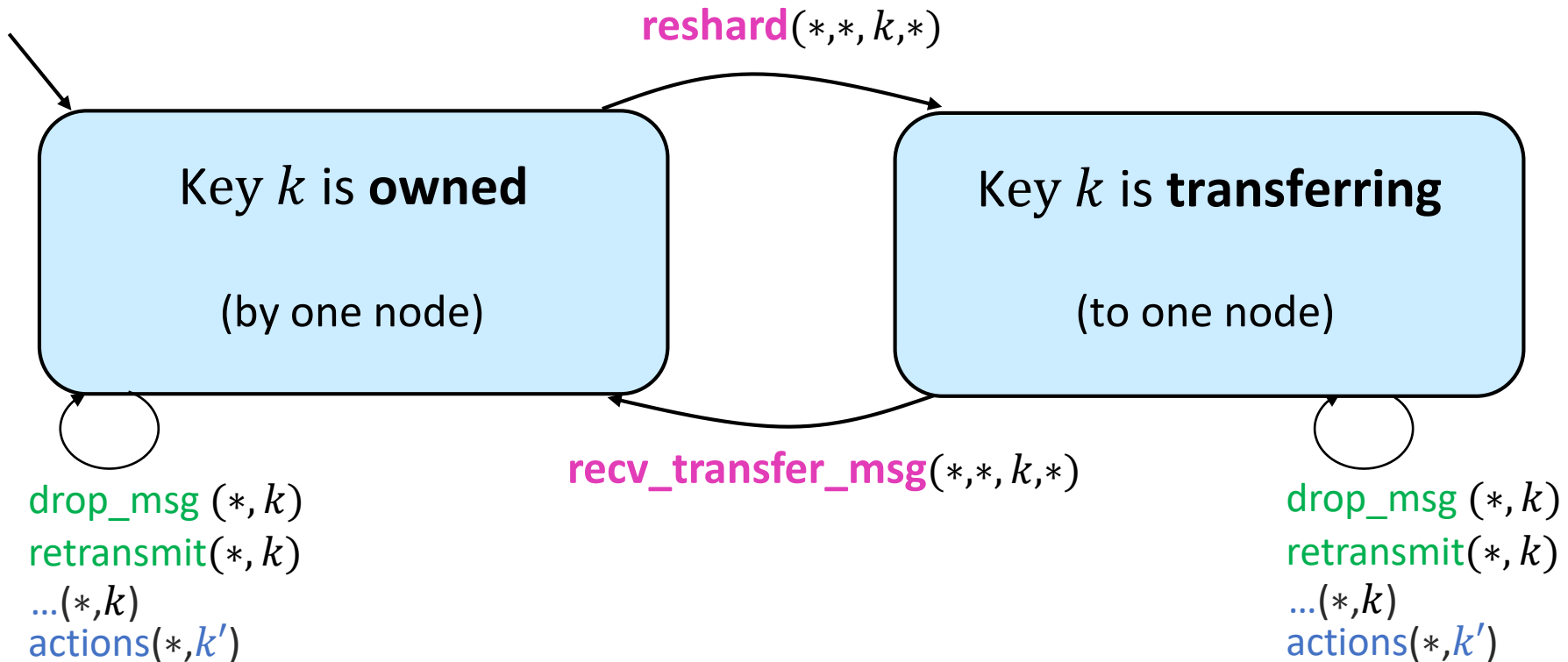
Phase Structure of Distributed KV's Proof

$\forall k.$



Phase Structure of Distributed KV's Proof

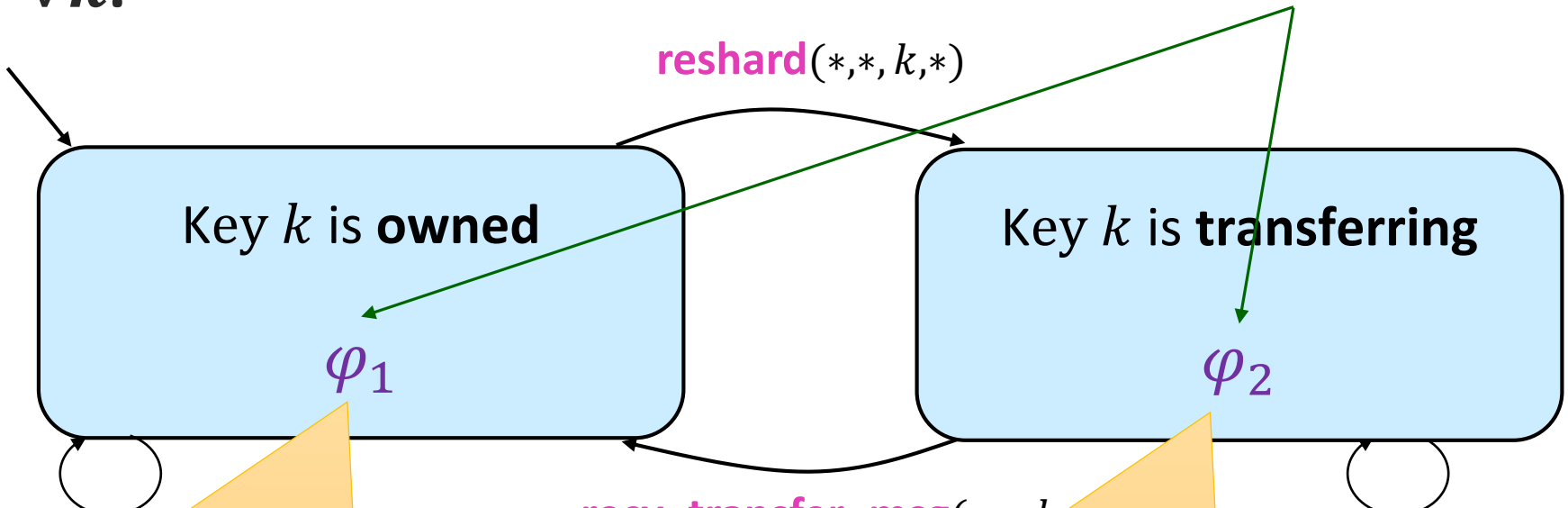
$\forall k.$



Phase Structure of Distributed KV's Proof

$\forall k.$

Phase characterizations



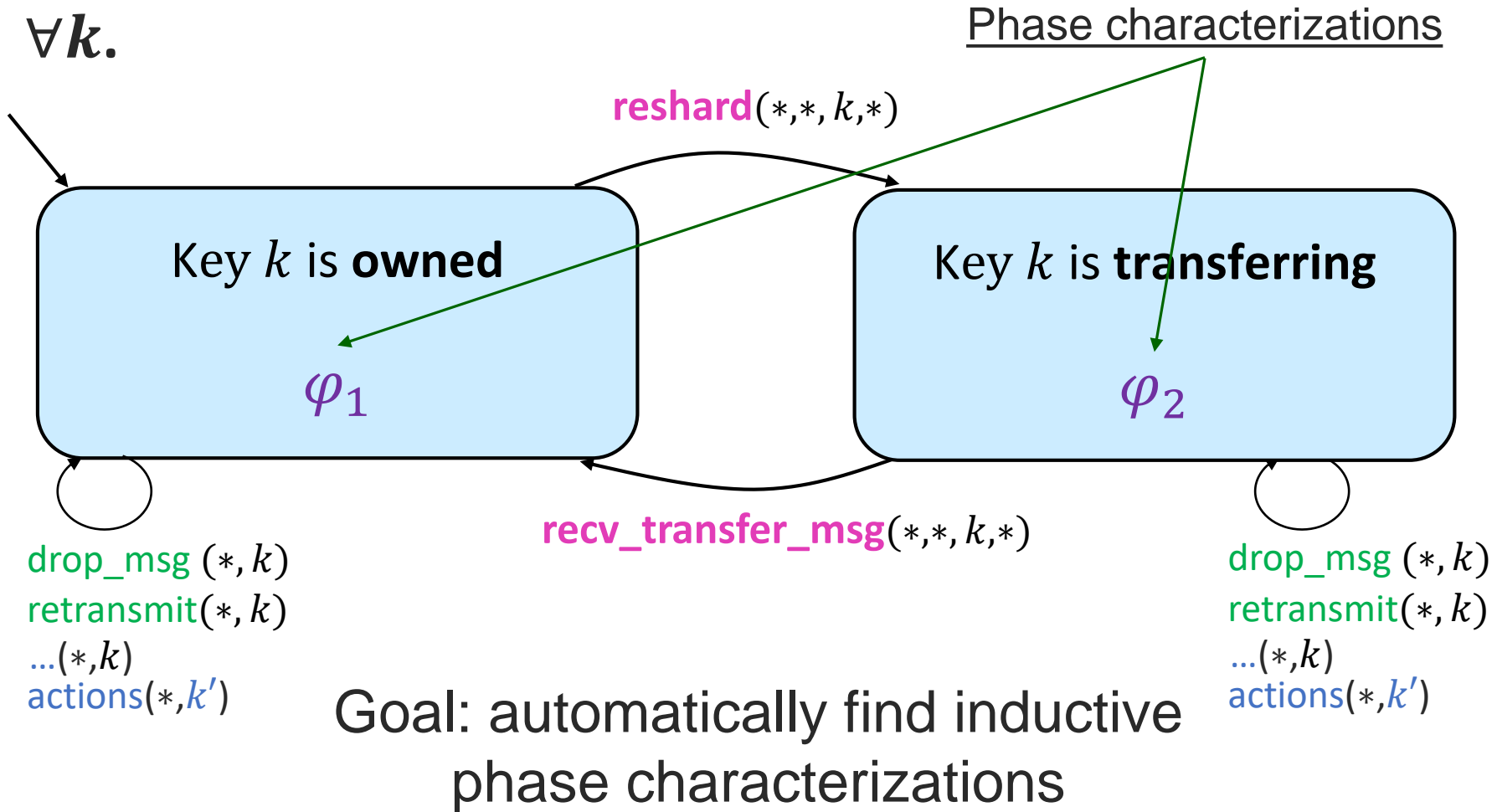
dro
ret
...(>
act

$\forall n_1, n_2, v, s.$
 $\neg(\text{transfer_msg}(n_1, n_2, k, v, s)$
 $\wedge \neg \text{seq_recvd}(n_1, n_2, k, v, s))$
 ...

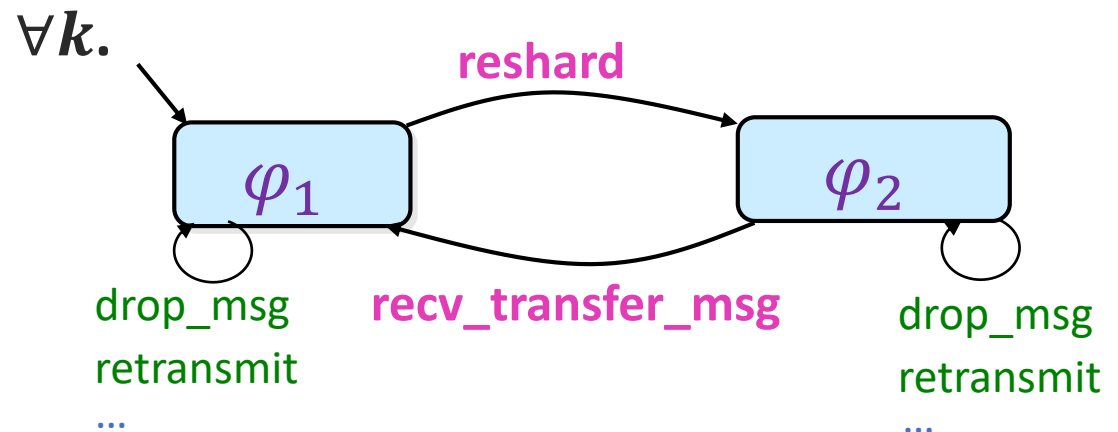
msg_transfer_msg(*,*,k,...)

$\forall n, v. \neg \text{table}(n, k, v)$
 ...

Phase Structure of Distributed KV's Proof

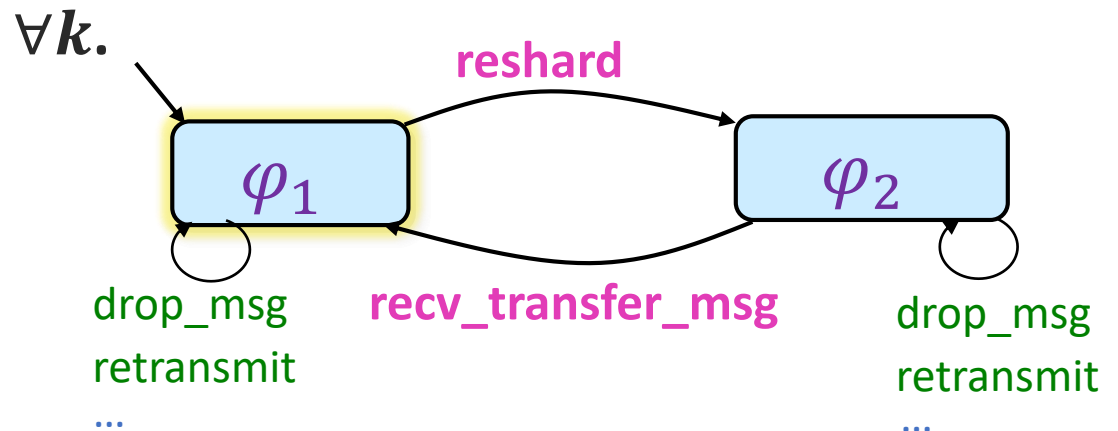


Inductive Phase Invariants



Inductive Phase Invariants

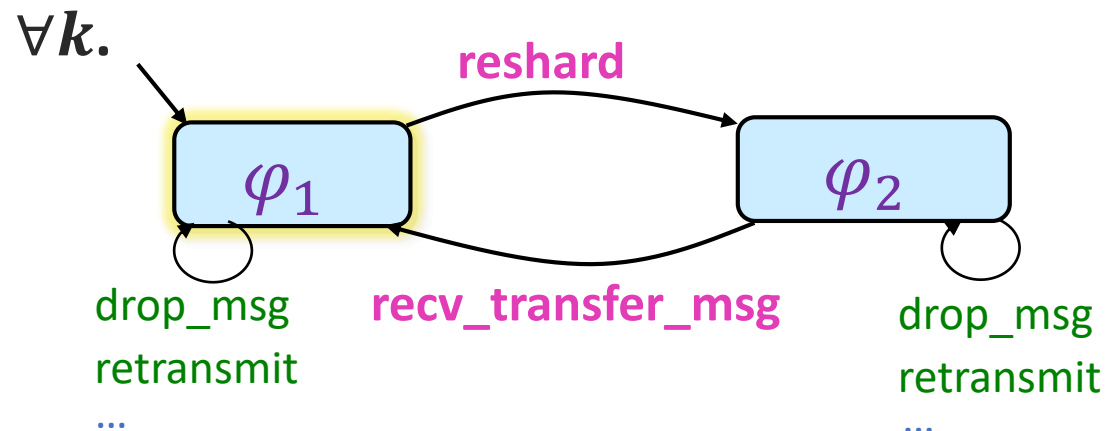
Init $\Rightarrow \varphi_1$



Inductive Phase Invariants

Init $\Rightarrow \varphi_1$

$\varphi_1 \Rightarrow$ Safety, ...

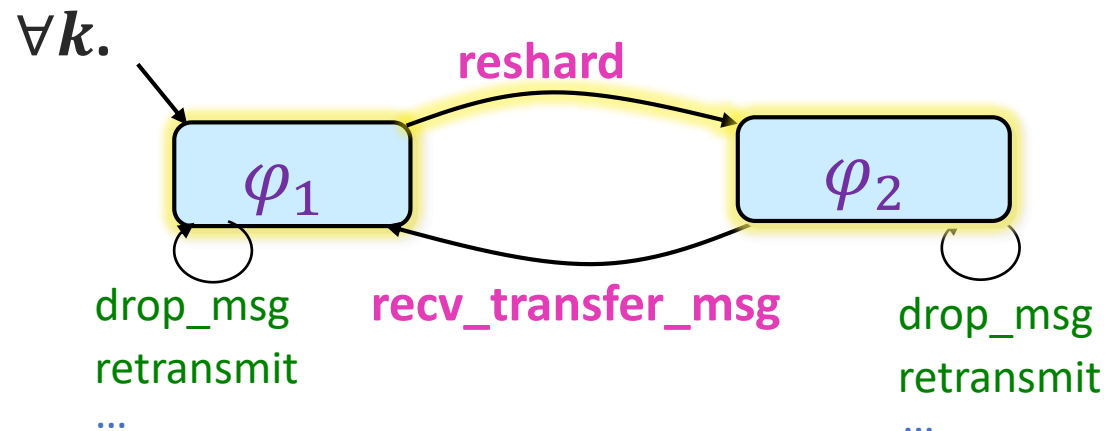


Inductive Phase Invariants

Init $\Rightarrow \varphi_1$

$\varphi_1 \Rightarrow \text{Safety, ...}$

$\varphi_1 \wedge \text{TR}_{\text{reshard}} \Rightarrow \varphi_2', \dots$



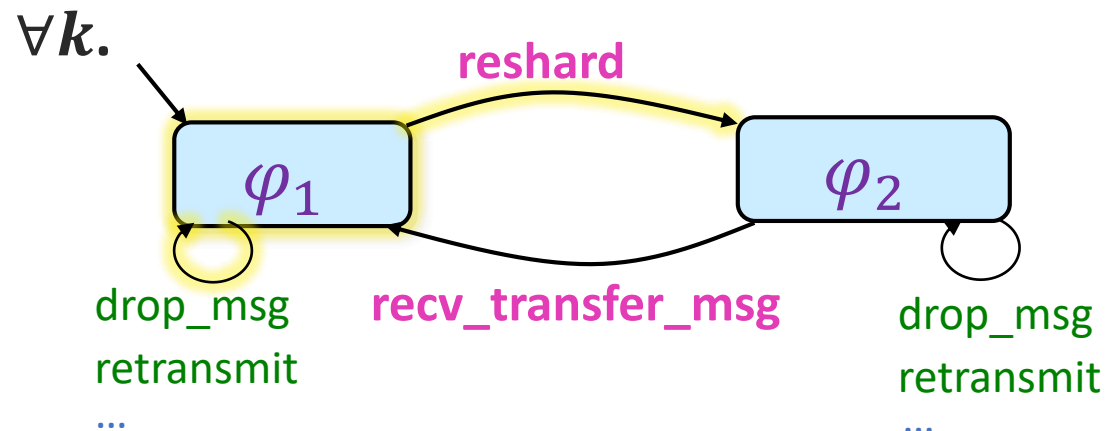
Inductive Phase Invariants

Init $\Rightarrow \varphi_1$

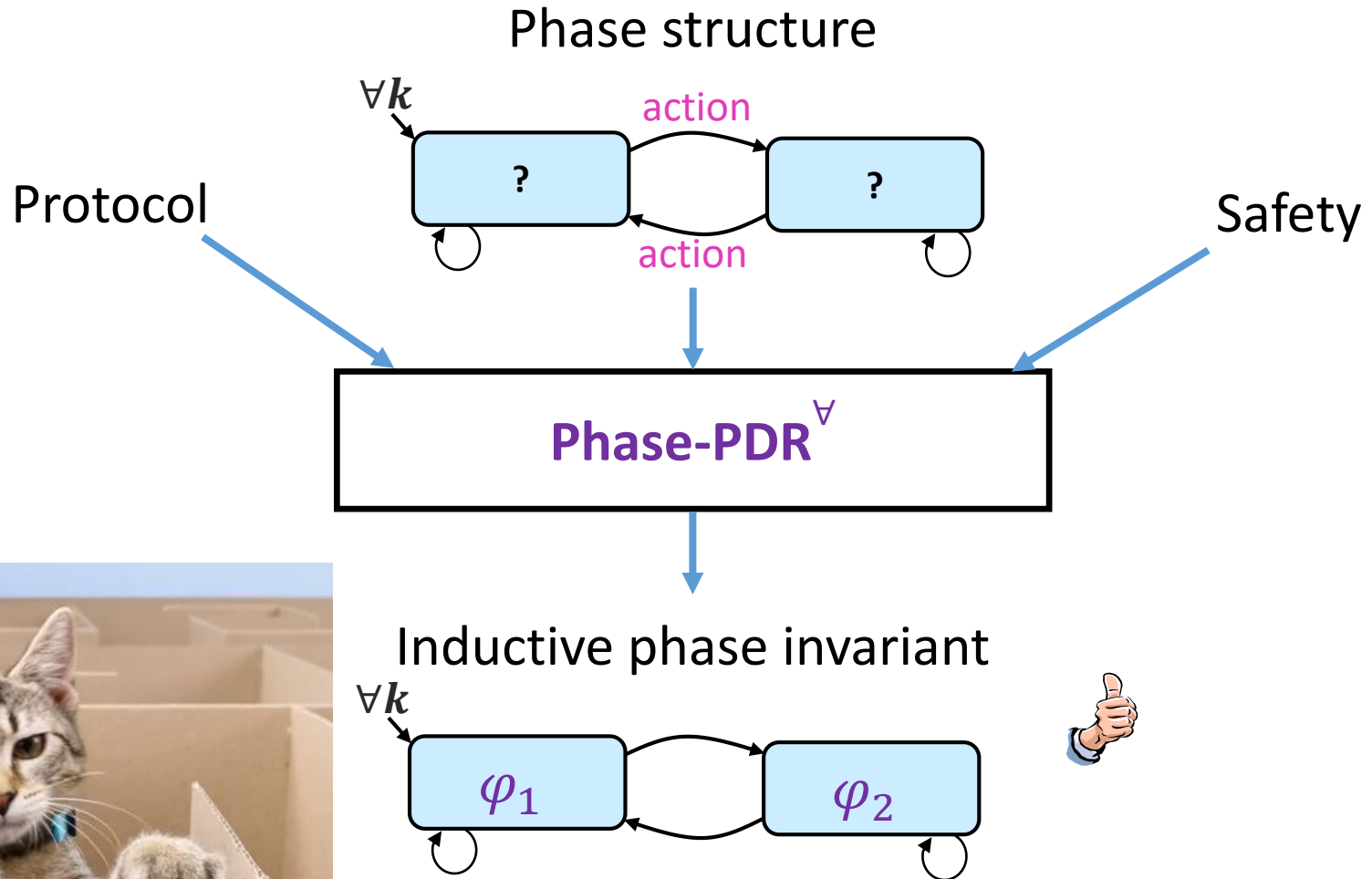
$\varphi_1 \Rightarrow$ Safety, ...

$\varphi_1 \wedge \text{TR}_{\text{reshard}} \Rightarrow \varphi_2', \dots$

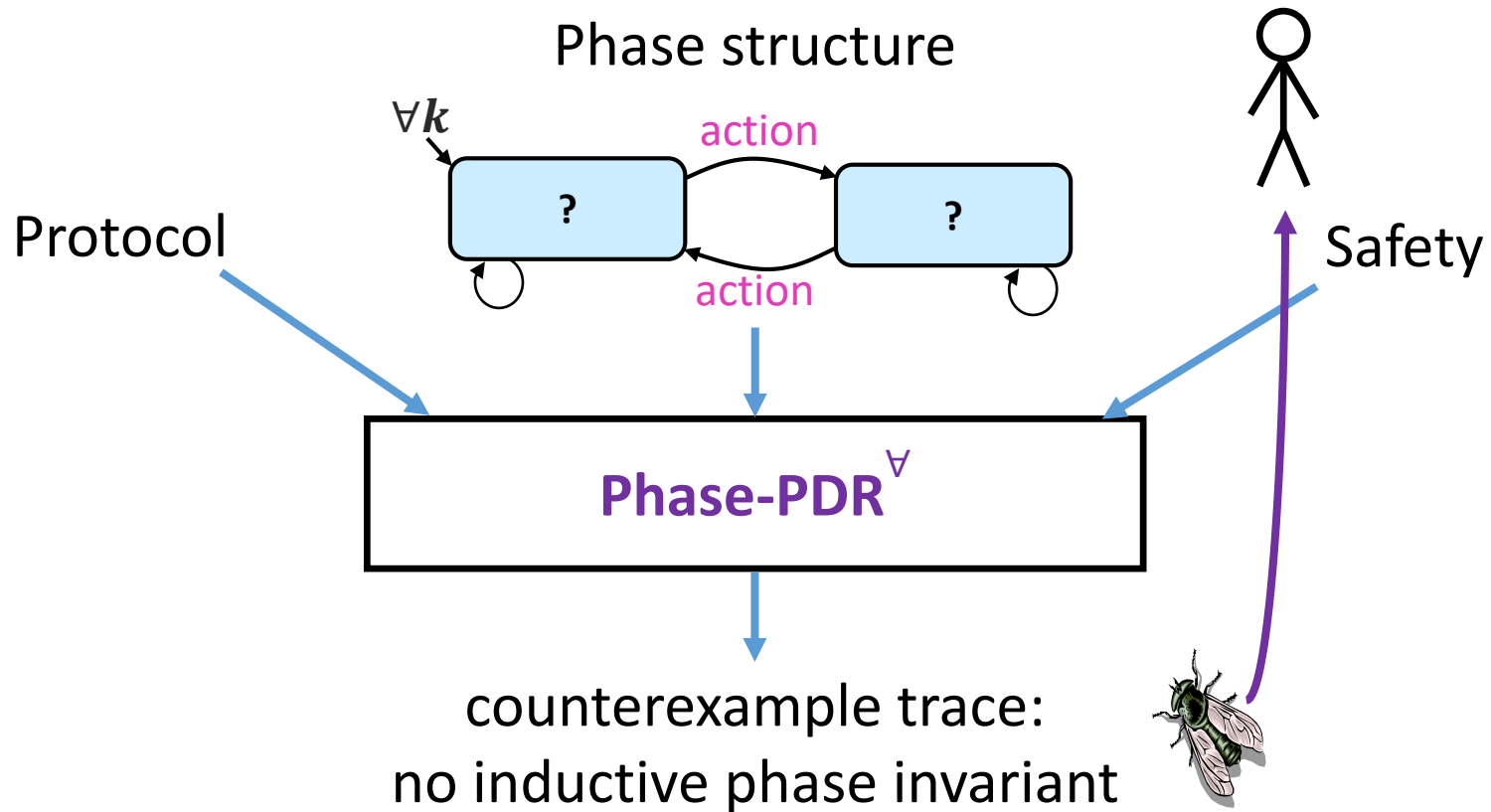
$\varphi_1 \wedge \text{TR} \Rightarrow \text{TR}_{\text{reshard}} \vee \text{TR}_{\text{drop_msg}} \vee \dots, \dots$



Inference Using Phase Structures



Inference Using Phase Structures



Inferring Inductive Phase Invariants

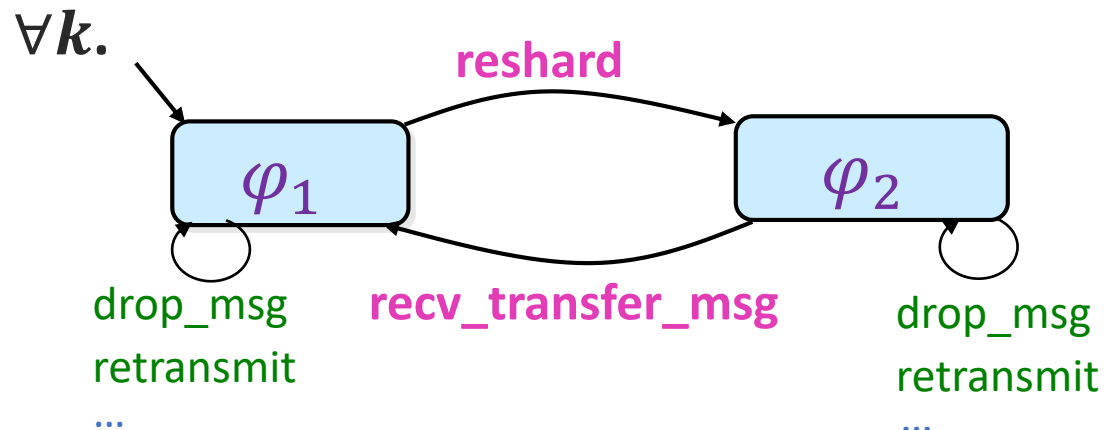
Init $\Rightarrow \varphi_1$

$\varphi_1 \Rightarrow$ Safety, ...

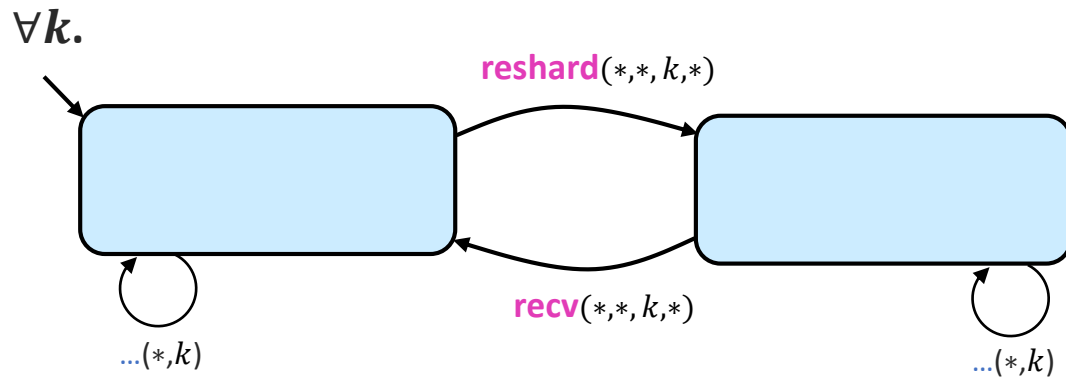
$\varphi_1 \wedge \text{TR}_{\text{reshard}} \Rightarrow \varphi_2', \dots$

$\varphi_1 \wedge \text{TR} \Rightarrow \text{TR}_{\text{reshard}} \vee \text{TR}_{\text{drop_msg}} \vee \dots, \dots$

System of *linear*
Constrained Horn Clauses (CHC)
over unknown predicates $\varphi_1, \varphi_2!$

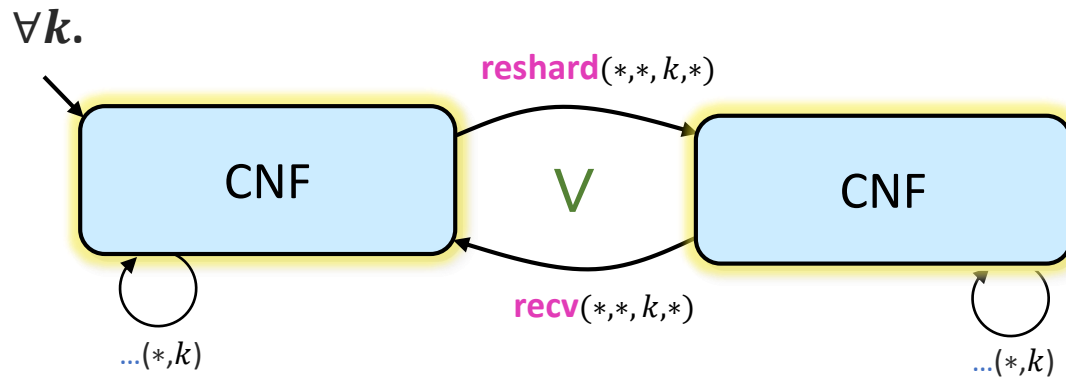


Phases Guide Inference



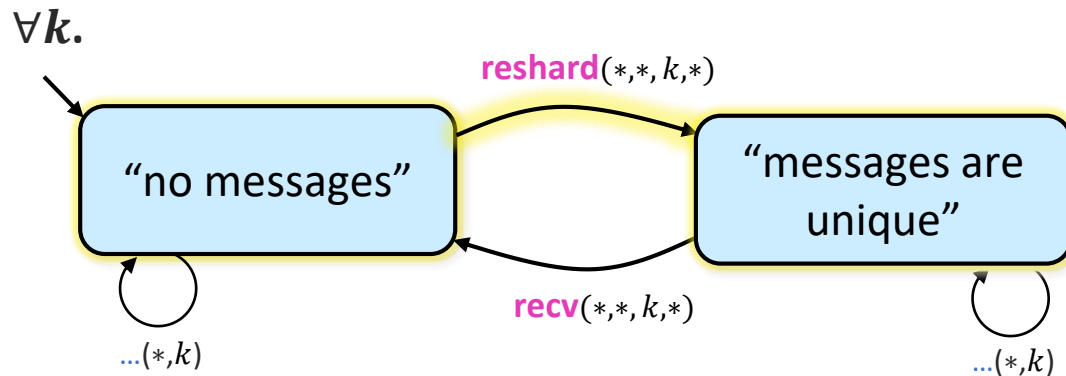
Phases Guide Inference

- Semantic disjunctive template



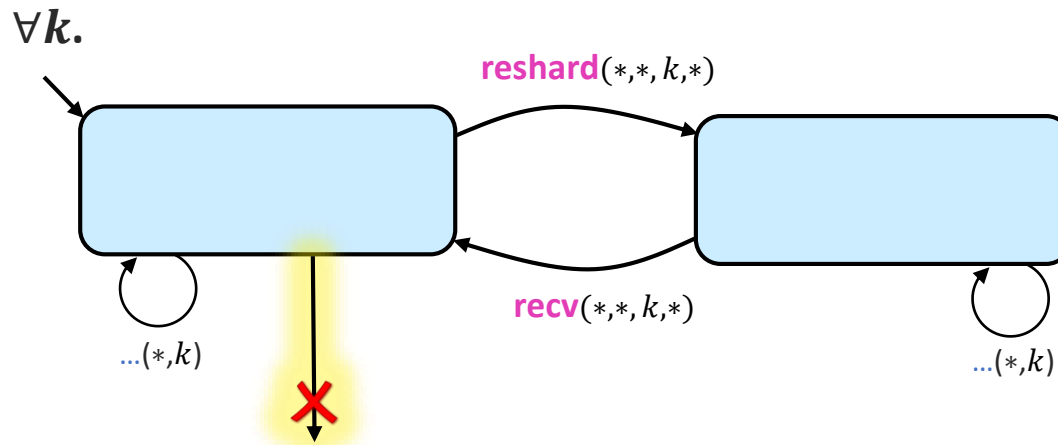
Phases Guide Inference

- Semantic disjunctive template
- **Phase decomposition and incremental construction**



Phases Guide Inference

- Semantic disjunctive template
- Phase decomposition and incremental construction
- **Impossible transitions**



Implementation



wilcoxjay/mypyvy



- **mypyvy**: a tool inspired by Ivy, over Z3
 - Statically-typed Python

Invariant inference:

- Standard **PDR[∇]** for standard inductive invariants
adaptation **Phase-PDR[∇]** for phase invariants

[CAV'15, JACM'17] Property-Directed Inference of Universal Invariants or Proving Their Absence, A. Karbyshev, N. Bjorner, S. Itzhaky, N. Rinetzky and S. Shoham.

Evaluation

Protocol	Inductive Invariant [seconds]	Phase Structure [seconds]
Lock server (single lock)	2.21	0.67
Lock server (multiple locks)	2.73	1.06
Simple consensus	60.54	1355*
Ring leader election	152.44	2.53
Sharded KV (basic)	1.79	1.59
Sharded KV	2070*	372.5
MESI cache coherence	-	90.1

- * not all runs terminated in 1 hour
- no runs terminated in 1 hour

Evaluation

Protocol	Inductive Invariant [seconds]	Phase Structure [seconds]
Lock server (single lock)	2.21	0.67
Lock server (multiple locks)	2.73	1.06
Simple consensus	60.54	1355*
Ring leader election	152.44	2.53
Sharded KV (basic)	1.79	1.59
Sharded KV	2070*	372.5
MESI cache coherence	-	90.1

- * not all runs terminated in 1 hour
- no runs terminated in 1 hour

Evaluation

Protocol	Inductive Invariant [seconds]	Phase Structure [seconds]
Lock server (single lock)	2.21	0.67
Lock server (multiple locks)	2.73	1.06
Simple consensus	60.54	1355*
Ring leader election	152.44	2.53
Sharded KV (basic)	1.79	1.59
Sharded KV	2070*	372.5
MESI cache coherence	-	90.1

- * not all runs terminated in 1 hour
- no runs terminated in 1 hour

Evaluation

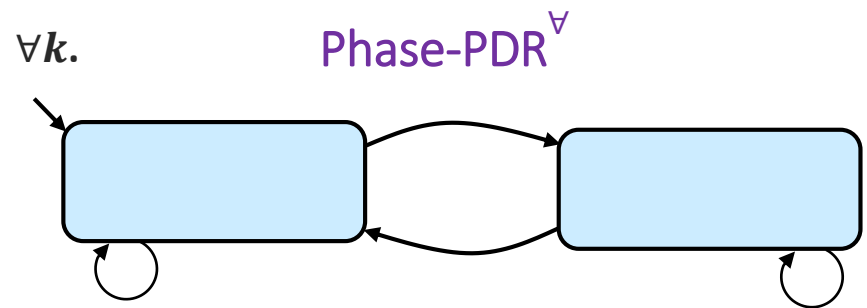
Protocol	Inductive Invariant [seconds]	Phase Structure [seconds]
Lock server (single lock)	2.21	0.67
Lock server (multiple locks)	2.73	1.06
Simple consensus	60.54	1355*
Ring leader election	152.44	2.53
Sharded KV (basic)	1.79	1.59
Sharded KV	2070*	372.5
MESI cache coherence	-	90.1

- * not all runs terminated in 1 hour
- no runs terminated in 1 hour

Summary

User-guided invariant inference by **phase structures**

- Convey high-level intuition
- Direct proof search effectively
 - Semantic disjunctive template
 - Incrementality between phases
 - Disabled transitions
- Facilitate inference beyond the state of the art
- Faster convergence



- **Sketching correctness** of infinite-state systems

